X100/301

NATIONAL
QUALIFICATIONS
2009

THURSDAY, 21 MAY
9.00 AM – 10.30 AM

MATHEMATICS
HIGHER
Paper 1
(Non-calculator)

Read carefully

Calculators may NOT be used in this paper.

Section A – Questions 1–20 (40 marks)

Instructions for completion of **Section A** are given on page two.

For this section of the examination you must use an **HB pencil**.

Section B (30 marks)

1 Full credit will be given only where the solution contains appropriate working.

2 Answers obtained by readings from scale drawings will not receive any credit.

Read carefully

1 Check that the answer sheet provided is for **Mathematics Higher (Section A)**.

2 For this section of the examination you must use an **HB pencil** and, where necessary, an eraser.

3 Check that the answer sheet you have been given has **your name**, **date of birth**, **SCN** (Scottish Candidate Number) and **Centre Name** printed on it.

Do not change any of these details.

4 If any of this information is wrong, tell the Invigilator immediately.

5 If this information is correct, **print** your name and seat number in the boxes provided.

6 The answer to each question is **either** A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space provided (see sample question below).

7 There is **only one correct** answer to each question.

8 Rough working should **not** be done on your answer sheet.

9 At the end of the exam, put the **answer sheet for Section A inside the front cover of your answer book**.

Sample Question

A curve has equation $y = x^3 - 4x$.

What is the gradient at the point where $x = 2$?

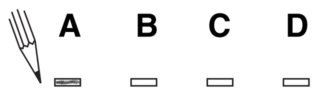
A 8

B 1

C 0

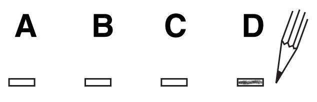
D -4

The correct answer is **A—**8. The answer **A** has been clearly marked in **pencil** with a horizontal line (see below).



Changing an answer

If you decide to change your answer, carefully erase your first answer and, using your pencil, fill in the answer you want. The answer below has been changed to **D**.



FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}|\cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2\sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

[Turn over

SECTION A

ALL questions should be attempted.

1. A sequence is defined by $u_{n+1} = 3u_n + 4$ with $u_1 = 2$.

What is the value of u_3?

A 34

B 21

C 18

D 13

2. A circle has equation $x^2 + y^2 + 8x + 6y - 75 = 0$.

What is the radius of this circle?

A 5

B 10

C $\sqrt{75}$

D $\sqrt{175}$

3. Triangle PQR has vertices at P(–3, –2), Q(–1, 4) and R(3, 6).

PS is a median. What is the gradient of PS?

A -2

B $-\frac{7}{4}$

C 1

D $\frac{7}{4}$

4. A curve has equation $y = 5x^3 - 12x$.

What is the gradient of the tangent at the point (1, –7)?

A -7

B -5

C 3

D 5

5. Here are two statements about the points S(2, 3) and T(5, −1):

(1) The length of ST = 5 units;

(2) The gradient of ST $= \frac{4}{3}$.

Which of the following is true?

A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

6. A sequence is generated by the recurrence relation $u_{n+1} = 0 \cdot 7u_n + 10$.

What is the limit of this sequence as $n \to \infty$?

A $\frac{100}{3}$

B $\frac{100}{7}$

C $\frac{17}{100}$

D $\frac{3}{10}$

7. If the exact value of $\cos x$ is $\frac{1}{\sqrt{5}}$, find the exact value of $\cos 2x$.

A $-\frac{3}{5}$

B $-\frac{2}{\sqrt{5}}$

C $\frac{2}{\sqrt{5}}$

D $\frac{3}{5}$

[Turn over

8. What is the derivative of $\frac{1}{4x^3}$, $x \neq 0$?

A $\quad \frac{1}{12x^2}$

B $\quad -\frac{1}{12x^2}$

C $\quad \frac{4}{x^4}$

D $\quad -\frac{3}{4x^4}$

9. The line with equation $y = 2x$ intersects the circle with equation $x^2 + y^2 = 5$ at the points J and K.

What are the x-coordinates of J and K?

A $\quad x_J = 1,\ x_K = -1$

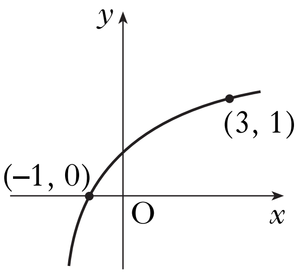
B $\quad x_J = 2,\ x_K = -2$

C $\quad x_J = 1,\ x_K = -2$

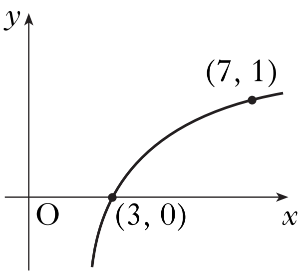
D $\quad x_J = -1,\ x_K = 2$

10. Which of the following graphs has equation $y = \log_5(x - 2)$?

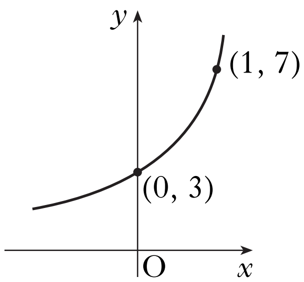
A



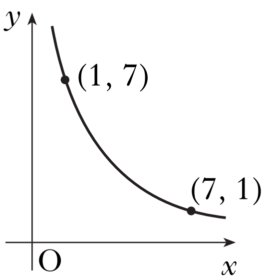
B



C



D



[Turn over

11. How many solutions does the equation

$$(4\sin x - \sqrt{5})(\sin x + 1) = 0$$

have in the interval $0 \le x < 2\pi$?

A 4

B 3

C 2

D 1

12. A function f is given by $f(x) = 2x^2 - x - 9$.

Which of the following describes the nature of the roots of $f(x) = 0$?

A No real roots

B Equal roots

C Real distinct roots

D Rational distinct roots

13. k and a are given by

$$k\sin a^\circ = 1$$
$$k\cos a^\circ = \sqrt{3}$$

where $k > 0$ and $0 \le a < 90$.

What are the values of k and a?

	k	a
A	2	60
B	2	30
C	$\sqrt{10}$	60
D	$\sqrt{10}$	30

14. If $f(x) = 2\sin\left(3x - \frac{\pi}{2}\right) + 5$, what is the range of values of $f(x)$?

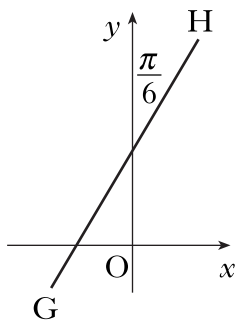
A $-1 \le f(x) \le 11$

B $2 \le f(x) \le 8$

C $3 \le f(x) \le 7$

D $-3 \le f(x) \le 7$

15. The line GH makes an angle of $\frac{\pi}{6}$ radians with the y-axis, as shown in the diagram. What is the gradient of GH?



A $\sqrt{3}$

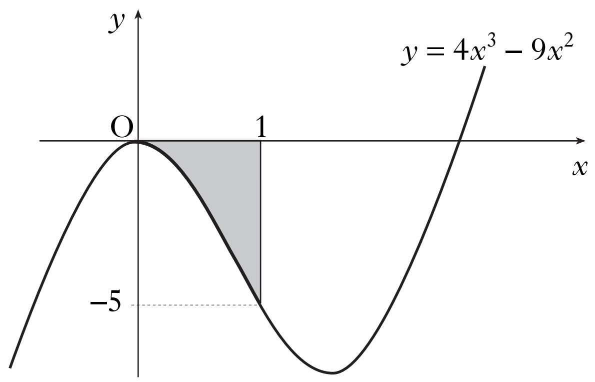
B $\frac{1}{2}$

C $\frac{1}{\sqrt{2}}$

D $\frac{\sqrt{3}}{2}$

16. The graph of $y = 4x^3 - 9x^2$ is shown in the diagram.

Which of the following gives the area of the shaded section?



A $\left[x^4 - 3x^3\right]_{-5}^{0}$

B $-\left[x^4 - 3x^3\right]_{0}^{1}$

C $\left[12x^2 - 18x\right]_{-5}^{0}$

D $-\left[12x^2 - 18x\right]_{0}^{1}$

17. The vector $\boldsymbol{u}$ has components $\begin{pmatrix} -3 \\ 0 \\ 4 \end{pmatrix}$.

Which of the following is a unit vector parallel to $\boldsymbol{u}$?

A $-\frac{3}{5}\boldsymbol{i} + \frac{4}{5}\boldsymbol{k}$

B $-3\boldsymbol{i} + 4\boldsymbol{k}$

C $-\frac{3}{\sqrt{7}}\boldsymbol{i} + \frac{4}{\sqrt{7}}\boldsymbol{k}$

D $-\frac{1}{3}\boldsymbol{i} + \frac{1}{4}\boldsymbol{k}$

18. Given that $f(x) = (4 - 3x^2)^{-\frac{1}{2}}$ on a suitable domain, find $f'(x)$.

A $-3x(4 - 3x^2)^{-\frac{1}{2}}$

B $-\frac{1}{2}(4 - 6x)^{-\frac{3}{2}}$

C $2(4 - 3x^3)^{\frac{1}{2}}$

D $3x(4 - 3x^2)^{-\frac{3}{2}}$

19. For what values of x is $6 + x - x^2 < 0$?

A $x > 3$ only

B $x < -2$ only

C $x < -2$, $x > 3$

D $-3 < x < 2$

20. $A = 2\pi r^2 + 6\pi r$.

What is the rate of change of A with respect to r when $r = 2$?

A 10π

B 12π

C 14π

D 20π

[*END OF SECTION A*]

SECTION B

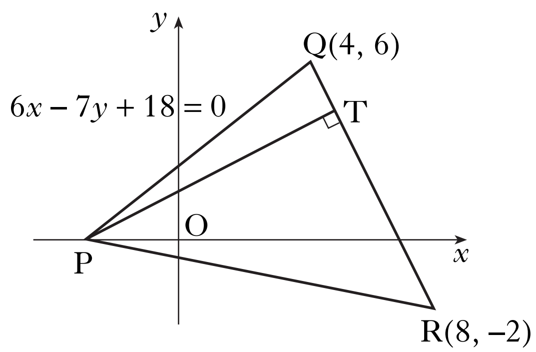
ALL questions should be attempted.

Marks

21. Triangle PQR has vertex P on the x-axis, as shown in the diagram.

Q and R are the points $(4, 6)$ and $(8, -2)$ respectively.

The equation of PQ is $6x - 7y + 18 = 0$.



(a) State the coordinates of P. **1**

(b) Find the equation of the altitude of the triangle from P. **3**

(c) The altitude from P meets the line QR at T. Find the coordinates of T. **4**

22. D, E and F have coordinates $(10, -8, -15)$, $(1, -2, -3)$ and $(-2, 0, 1)$ respectively.

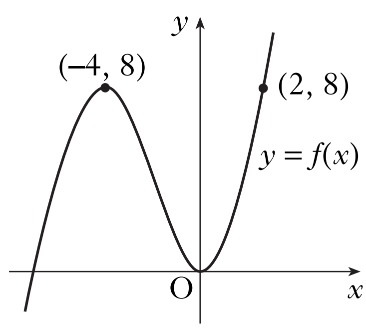
(a) (i) Show that D, E and F are collinear.

(ii) Find the ratio in which E divides DF. **4**

(b) G has coordinates $(k, 1, 0)$.

Given that DE is perpendicular to GE, find the value of k. **4**

23. The diagram shows a sketch of the function $y = f(x)$.



(a) Copy the diagram and on it sketch the graph of $y = f(2x)$. **2**

(b) On a separate diagram sketch the graph of $y = 1 - f(2x)$. **3**

[Turn over for Question 24 on *Page twelve*

Marks

24. (*a*) Using the fact that $\frac{7\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$, find the exact value of $\sin\left(\frac{7\pi}{12}\right)$. **3**

(*b*) Show that $\sin(A + B) + \sin(A - B) = 2\sin A\cos B$. **2**

(*c*) (i) Express $\frac{\pi}{12}$ in terms of $\frac{\pi}{3}$ and $\frac{\pi}{4}$.

(ii) Hence or otherwise find the exact value of $\sin\left(\frac{7\pi}{12}\right) + \sin\left(\frac{\pi}{12}\right)$. **4**

[*END OF SECTION B*]

[*END OF QUESTION PAPER*]

X100/302

NATIONAL
QUALIFICATIONS
2009

THURSDAY, 21 MAY
10.50 AM – 12.00 NOON

MATHEMATICS
HIGHER
Paper 2

Read Carefully

1 **Calculators may be used in this paper.**

2 Full credit will be given only where the solution contains appropriate working.

3 Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos \theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\begin{aligned}
\sin(\mathrm{A} \pm \mathrm{B}) &= \sin \mathrm{A} \cos \mathrm{B} \pm \cos \mathrm{A} \sin \mathrm{B} \\
\cos(\mathrm{A} \pm \mathrm{B}) &= \cos \mathrm{A} \cos \mathrm{B} \mp \sin \mathrm{A} \sin \mathrm{B} \\
\sin 2\mathrm{A} &= 2\sin \mathrm{A} \cos \mathrm{A} \\
\cos 2\mathrm{A} &= \cos^2 \mathrm{A} - \sin^2 \mathrm{A} \\
&= 2\cos^2 \mathrm{A} - 1 \\
&= 1 - 2\sin^2 \mathrm{A}
\end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

ALL questions should be attempted.

Marks

1. Find the coordinates of the turning points of the curve with equation $y = x^3 - 3x^2 - 9x + 12$ and determine their nature. **8**

2. Functions f and g are given by $f(x) = 3x + 1$ and $g(x) = x^2 - 2$.

(*a*) (i) Find $p(x)$ where $p(x) = f(g(x))$.

(ii) Find $q(x)$ where $q(x) = g(f(x))$. **3**

(*b*) Solve $p'(x) = q'(x)$. **3**

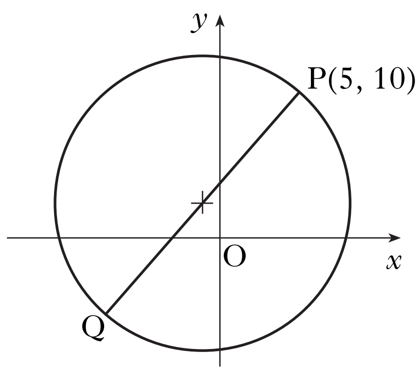
3. (*a*) (i) Show that $x = 1$ is a root of $x^3 + 8x^2 + 11x - 20 = 0$.

(ii) Hence factorise $x^3 + 8x^2 + 11x - 20$ fully. **4**

(*b*) Solve $\log_2(x + 3) + \log_2(x^2 + 5x - 4) = 3$. **5**

4. (*a*) Show that the point P(5, 10) lies on circle C_1 with equation $(x + 1)^2 + (y - 2)^2 = 100$. **1**

(*b*) PQ is a diameter of this circle as shown in the diagram. Find the equation of the tangent at Q. **5**



(*c*) Two circles, C_2 and C_3, touch circle C_1 at Q.

The radius of each of these circles is twice the radius of circle C_1.

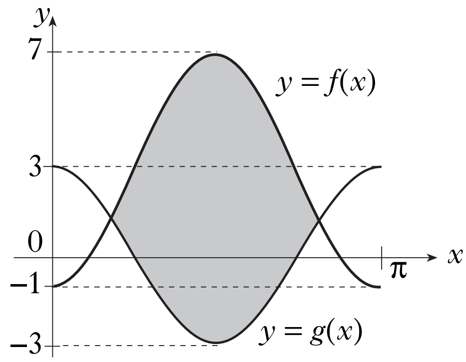
Find the equations of circles C_2 and C_3. **4**

[Turn over

Marks

5. The graphs of $y = f(x)$ and $y = g(x)$ are shown in the diagram.

$f(x) = -4\cos(2x) + 3$ and $g(x)$ is of the form $g(x) = m\cos(nx)$.



(*a*) Write down the values of m and n. **1**

(*b*) Find, correct to one decimal place, the coordinates of the points of intersection of the two graphs in the interval $0 \leq x \leq \pi$. **5**

(*c*) Calculate the shaded area. **6**

6. The size of the human population, N, can be modelled using the equation $N = N_0e^{rt}$ where N_0 is the population in 2006, t is the time in years since 2006, and r is the annual rate of increase in the population.

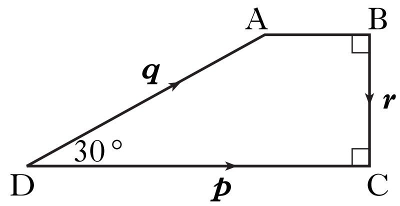
(*a*) In 2006 the population of the United Kingdom was approximately 61 million, with an annual rate of increase of 1·6%. Assuming this growth rate remains constant, what would be the population in 2020? **2**

(*b*) In 2006 the population of Scotland was approximately 5·1 million, with an annual rate of increase of 0·43%.

Assuming this growth rate remains constant, how long would it take for Scotland's population to double in size? **3**

7. Vectors $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$ are represented on the diagram shown where angle ADC = 30°.

It is also given that $|\boldsymbol{p}| = 4$ and $|\boldsymbol{q}| = 3$.



(*a*) Evaluate $\boldsymbol{p}.(\boldsymbol{q} + \boldsymbol{r})$ and $\boldsymbol{r}.(\boldsymbol{p} - \boldsymbol{q})$. **6**

(*b*) Find $|\boldsymbol{q} + \boldsymbol{r}|$ and $|\boldsymbol{p} - \boldsymbol{q}|$. **4**

[*END OF QUESTION PAPER*]

2009 Mathematics

Higher – Paper 1 and Paper 2

Finalised Marking Instructions

General Comments

These marking instructions are for use with the 2009 Higher Mathematics Examination.

For each question the marking instructions are split into two sections, namely the Generic Marking Instructions and the Specific Marking Instructions. The Generic Marking Instructions indicate what evidence must be seen for each mark to be awarded. The Specific Marking Instructions cover the most common methods you are likely to see throughout your marking.
Below these two sections there may be comments, less common methods and common errors.
In general you should use the Specific Marking Instructions together with the comments, less common methods and common errors; only use the Generic Marking Instructions where the candidate has used a method not otherwise covered.

All markers should apply the following general marking principles throughout their marking:

1. Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than marks deducted for what is wrong.

2. Award one mark for each 'bullet' point. Each error should be underlined in RED at the point in the working where it first occurs, and not at any subsequent stage of the working.

3. The working subsequent to an error must be followed through by the marker with possible full marks for the subsequent working, provided that the difficulty involved is approximately similar. Where, subsequent to an error, the working is eased, a deduction(s) of mark(s) should be made. This may happen where a question is divided into parts. In fact, failure to even answer an earlier section does not preclude a candidate from assuming the result of that section and obtaining full marks for a later section.

4.

Tick	✓	Cross	✗	Cross-Tick		Double Cross-Tick	

Correct working should be ticked. This is essential for later stages of the SQA procedures. Where an error occurs, this should be underlined and marked with a cross at the end of the line.
Where working subsequent to an error(s) is correct and scores marks, it should be marked with a crossed tick.
In appropriate cases attention may be directed to work which is not quite correct (e.g. bad form) but which has not been penalised, by underlining with a dotted (or wavy) line.
Work which is correct but inadequate to score any marks should be corrected with a double cross tick.

5. The total mark for each section of a question should be entered in **red** in the **outer** right hand margin, opposite the end of the working concerned.
   - Only the mark should be written, not a fraction of the possible marks.
   - These marks should correspond to those on the question paper and these instructions.

6. Where a candidate has scored zero marks for any question attempted, "0" should be shown against the answer.

7. As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Throughout this paper, unless specifically mentioned in the marking scheme, a correct answer with no working receives no credit.

8 There is no such thing as a transcription error, a trivial error, a casual error or an insignificant error – each one is simply an error. In general, as a consequence of one of these errors, candidates lose the opportunity of gaining the appropriate *ic* or *pd* mark.

9 Normally, do not penalise:

- working subsequent to a correct answer
- omission of units
- legitimate variations in numerical answers
- bad form
- correct working in the "wrong" part of a question

unless specifically mentioned in the marking scheme.

10 No piece of work should be ignored without careful checking - even where a fundamental misunderstanding is apparent early in the answer. Reference should always be made to the marking scheme. Answers which are widely off-beam are unlikely to include anything of relevance but in the vast majority of cases candidates still have the opportunity of gaining the odd mark or two provided it satisfies the criteria for the mark(s).

11 If in doubt between two marks, give an intermediate mark, but without fractions. When in doubt between consecutive numbers, give the higher mark.

12 In cases of difficulty covered neither in detail nor in principle in the Instructions, attention may be directed to the assessment of particular answers by making a referral to the P.A. Please see the general instructions for P.A. referrals.

13 No marks should be deducted at this stage for careless or badly arranged work. In cases where the writing or arrangement is very bad, a note may be made on the upper left-hand corner of the front cover of the script.

14 It is of great importance that the utmost care should be exercised in adding up the marks. Using the Electronic Marks Capture (EMC) screen to tally marks for you is **NOT** recommended. A manual check of the total, using the grid issued with this marking scheme, can be confirmed by the EMC system.

15 Provided that it has not been replaced by another attempt at a solution, working that has been crossed out by the candidate should be marked in the normal way. If you feel that a candidate has been disadvantaged by this action, make a P.A. Referral.

16 **Do not write any comments, words or acronyms on the scripts**.
A revised summary of acceptable notation is given on page 4.

17 **Summary**
Throughout the examination procedures many scripts are remarked. It is essential that markers follow common procedures:

1. Tick correct working.
2. Put a mark in the outer right-hand margin to match the marks allocations on the question paper.
3. Do not write marks as fractions.
4. Put each mark at the end of the candidate's response to the question.
5. Follow through errors to see if candidates can score marks subsequent to the error.
6. Do not write any comments on the scripts.

Higher Mathematics : A Guide to Standard Signs and Abbreviations

Remember - No comments on the scripts. Please use the following and nothing else.

Signs	Comments	Examples	Margins
✓	The tick. You are not expected to tick every line but you must check through the whole of a response.	$\frac{dy}{dx} = 4x - 7$ ✓ •	
$\underline{\quad}$ ✗	The cross and underline. Underline an error and place a cross at the end of the line.	$4x - 7 = \underline{0}$ ✗	
✓✗	The tick-cross. Use this to show correct work where you are following through subsequent to an error.	$x = \frac{7}{4}$ $y = 3\frac{7}{8}$ ✓✗ •	2
		$C = \underline{(1,-1)}$ ✗ $m = \frac{3-(-1)}{4-1}$ $m_{rad} = \frac{4}{3}$ ✓✗ • $m_{tgt} = \frac{-1}{\frac{4}{3}}$ $m_{tgt} = -\frac{3}{4}$ ✓✗ • $y - 3 = -\frac{3}{4}(x-2)$ ✓✗ •	3
∧	The roof. Use this to show something is missing such as a crucial step in a proof or a 'condition' etc.	$x^2 - 3x = 28$ ✓ • ∧ $x = 7$ ✓✗✗	1
✓✗✗	The double cross-tick. Use this to show correct work but which is inadequate to score any marks. This may happen when working has been eased.		
~	Tilde. Use this to indicate a minor transgression which is not being penalised (such as bad form).	$\sin(x) = 0.75$ $= inv\sin(0.75)$ ✓ • $= 48.6°$	1
↓	If a solution continues later on, put an arrow in the marks margin to show this. The mark given should appear at the end.	$x^3 - 4x^2 + 8x - 5 = 0$ $(x-1)(x^2 - 3x + 5) = 0$?	↓

Bullets showing where marks are being allocated may be shown on scripts.

Please use the above and nothing else. All of these are to help us be more consistent and accurate.

Page 5 lists the syllabus coding for each topic. This information is given in the legend above the question. The calculator classification is CN(calculator neutral), CR(calculator required) and NC(non-calculator).

Syllabus Coding by Topic

Unit 1		Unit 2		Unit 3	
A1	*determine range/domain*	A15	*use the general equation of a parabola*	A28	*use the laws of logs to simplify/find equiv. expression*
A2	*recognise general features of graphs:poly,exp,log*	A16	*solve a quadratic inequality*	A29	*sketch associated graphs*
A3	*sketch and annotate related functions*	A17	*find nature of roots of a quadratic*	A30	*solve equs of the form* $A = Be^{kt}$ *for A,B,k or t*
A4	*obtain a formula for composite function*	A18	*given nature of roots, find a condition on coeffs*	A31	*solve equs of the form* $\log_b(a) = c$ *for a,b or c*
A5	*complete the square*	A19	*form an equation with given roots*	A32	*solve equations involving logarithms*
A6	*interpret equations and expressions*	A20	*apply A15-A19 to solve problems*	A33	*use relationships of the form* $y = ax^n$ *or* $y = ab^x$
A7	*determine function(poly,exp,log) from graph & vv*			A34	*apply A28-A33 to problems*
A8	*sketch/annotate graph given critical features*				
A9	*interpret loci such as st.lines,para,poly,circle*				
A10	*use the notation* u_n *for the nth term*	A21	*use Rem Th. For values, factors, roots*	G16	*calculate the length of a vector*
A11	*evaluate successive terms of a RR*	A22	*solve cubic and quartic equations*	G17	*calculate the 3rd given two from A,B and vector AB*
A12	*decide when RR has limit/interpret limit*	A23	*find intersection of line and polynomial*	G18	*use unit vectors*
A13	*evaluate limit*	A24	*find if line is tangent to polynomial*	G19	*use: if* $\mathbf{u}$, $\mathbf{v}$ *are parallel then* $\mathbf{v} = k\mathbf{u}$
A14	*apply A10-A14 to problems*	A25	*find intersection of two polynomials*	G20	*add, subtract, find scalar mult. of vectors*
		A26	*confiirm and improve on approx roots*	G21	*simplify vector pathways*
		A27	*apply A21-A26 to problems*	G22	*interpret 2D sketches of 3D situations*
				G23	*find if 3 points in space are collinear*
				G24	*find ratio which one point divides two others*
G1	*use the distance formula*	G9	*find C/R of a circle from its equation/other data*	G25	*given a ratio, find/interpret 3rd point/vector*
G2	*find gradient from 2 pts,/angle/equ. of line*	G10	*find the equation of a circle*	G26	*calculate the scalar product*
G3	*find equation of a line*	G11	*find equation of a tangent to a circle*	G27	*use: if* $\mathbf{u}$, $\mathbf{v}$ *are perpendicular then* $\mathbf{v.u=0}$
G4	*interpret all equations of a line*	G12	*find intersection of line & circle*	G28	*calculate the angle between two vectors*
G5	*use property of perpendicular lines*	G13	*find if/when line is tangent to circle*	G29	*use the distributive law*
G6	*calculate mid-point*	G14	*find if two circles touch*	G30	*apply G16-G29 to problems eg geometry probs.*
G7	*find equation of median, altitude,perp. bisector*	G15	*apply G9-G14 to problems*		
G8	*apply G1-G7 to problems eg intersect.,concur.,collin.*				
C1	*differentiate sums, differences*	C12	*find integrals of* px^n *and sums/diffs*	C20	*differentiate* $p\sin(ax+b)$, $p\cos(ax+b)$
C2	*differentiate negative & fractional powers*	C13	*integrate with negative & fractional powers*	C21	*differentiate using the chain rule*
C3	*express in differentiable form and differentiate*	C14	*express in integrable form and integrate*	C22	*integrate* $(ax + b)^n$
C4	*find gradient at point on curve & vv*	C15	*evaluate definite integrals*	C23	*integrate* $p\sin(ax+b)$, $p\cos(ax+b)$
C5	*find equation of tangent to a polynomial/trig curve*	C16	*find area between curve and x-axis*	C24	*apply C20-C23 to problems*
C6	*find rate of change*	C17	*find area between two curves*		
C7	*find when curve strictly increasing etc*	C18	*solve differential equations(variables separable)*		
C8	*find stationary points/values*	C19	*apply C12-C18 to problems*		
C9	*determine nature of stationary points*				
C10	*sketch curve given the equation*				
C11	*apply C1-C10 to problems eg optimise, greatest/least*				
T1	*use gen. features of graphs of* $f(x)=k\sin(ax+b)$,	T7	*solve linear & quadratic equations in radians*	T12	*solve sim.equs of form* $k\cos(a)=p$, $k\sin(a)=q$
	$f(x)=k\cos(ax+b)$; *identify period/amplitude*	T8	*apply compound and double angle (c & da) formulae*	T13	*express* $p\cos(x)+q\sin(x)$ *in form* $k\cos(x\pm a)$ *etc*
T2	*use radians inc conversion from degrees & vv*		*in numerical & literal cases*	T14	*find max/min/zeros of* $p\cos(x)+q\sin(x)$
T3	*know and use exact values*	T9	*apply c & da formulae in geometrical cases*	T15	*sketch graph of* $y=p\cos(x)+q\sin(x)$
T4	*recognise form of trig. function from graph*	T10	*use c & da formulaewhen solving equations*	T16	*solve equ of the form* $y=p\cos(rx)+q\sin(rx)$
T5	*interpret trig. equations and expressions*	T11	*apply T7-T10 to problems*	T17	*apply T12-T16 to problems*
T6	*apply T1-T5 to problems*				

Higher Mathematics 2009 v10

For information only

Paper 1 Section A qu.1-10

Qu.	Key	Item no.	solution
1.01	**A**	999	• $u_2 = 3\times 2 + 4 = 10$ • $\therefore u_3 = 3\times 10 + 4 = 34$
1.02	**B**	153	$x^2 + y^2 + 8x + 6y - 75 = 0$ • $r = \sqrt{(-4)^2 + (-3)^2 - (-75)}$ • $r = 10$
1.03	**D**	950	• $S = \left(\frac{-1+3}{2}, \frac{4+6}{2}\right) = (1,5)$ • $m_{PS} = \frac{5--2}{1--3} = \frac{7}{4}$
1.04	**C**	60	• $\frac{dy}{dx} = 15x^2 - 12$ • at $x = 1$, gradient $= 15 - 12 = 3$
1.05	**B**	1201	• $ST = \sqrt{(2-5)^2 + (3--1)^2}$ $ST = 5$ • $m_{ST} = \frac{3--1}{2-5} = -\frac{4}{3}$
1.06	**A**	1239	• $L = 0.7L + 10$ • $L = \frac{10}{0.3} = \frac{100}{3}$
1.07	**A**	63	• $\cos(2x) = 2\cos^2(x) - 1$ • $2\times\left(\frac{1}{\sqrt{5}}\right)^2 - 1 = -\frac{3}{5}$
1.08	**D**	1081	• $f(x) = \frac{1}{4}x^{-3}$ • $f'(x) = -\frac{3}{4}x^{-4}$
1.09	**A**	1901	• $x^2 + (2x)^2 = 5$ • $5x^2 = 5,\quad x = \pm 1$
1.10	**B**	1903	• $x = 3,\quad y = \log(3-2) = 0$ *so B* • $x = 7,\ y = \log_5(7-2) = 1$

Paper 1 Section A qu.11-20

Qu.	Key	Item no.	solution
1.11	**B**	1145	• $\sin x = \frac{\sqrt{5}}{4}$: 2 *solutions* • $\sin x = -1$: 1 *solution*
1.12	**C**	1313	• $b^2 - 4ac = 73 > 0$ • *roots are real and distinct*
1.13	**B**	1146	• $\tan a° = \frac{1}{\sqrt{3}}$ *so* $a = 30$ • $k^2 = 1 + 3$ *so* $k = 2$
1.14	**C**	1172	• $f_{\max} = 2\times 1 + 5 = 7$ • $f_{\min} = 2\times(-1) + 5 = 3$
1.15	**A**	1396	• angle at x-axis $= \frac{\pi}{3}$ • $m_{GH} = \tan\frac{\pi}{3} = \sqrt{3}$
1.16	**B**	1148	• integrate : $x^4 - 3x^3$ • limits : $-[....]_0^1$
1.17	**A**	1133	• $\lvert \boldsymbol{u} \rvert = \sqrt{(-3)^2 + 4^2} = 5$ • *a unit vector* : $\frac{1}{5}(-3\boldsymbol{i} + 4\boldsymbol{j})$
1.18	**D**	394	• $-\frac{1}{2}\left(4 - 3x^2\right)^{-\frac{3}{2}}$ • multiplied by $-6x$
1.19	**C**	1002	• $(2+x)(3-x) < 0$ *solution is either* $-2 < x < 3$ ***or*** $x < -2,\ x > 3$ • $x = 0$ *is FALSE so* $x < -2$ *and* $x > 3$
1.20	**C**	161	• $\frac{dA}{dr} = 4\pi r + 6\pi$ • $\frac{dA}{dr}_{r=2} = 8\pi + 6\pi$ $= 14\pi$

qu		Mark	Code	Cal	Source	ss	pd	ic	C	B	A		U1	U2	U3
1.21	a	1	G4	cn	09013			1					1		
	b	3	G7	cn		1	1	1	3				3		
	c	4	G8	cn		1	2	1	4				4		

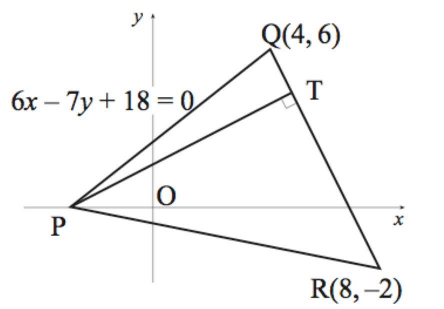
1.21

Triangle PQR has vertex P on the x-axis.
Q and R are the points $(4,6)$ and $(8,-2)$ respectively.
The equation of PQ is $6x-7y+18=0$.

(*a*) State the coordinates of P — 1

(*b*) Find the equation of the altitude of the triangle from P. — 3

(c) The altitude from P meets the line QR at T. Find the coordinates of T. — 4



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$\bullet^1$	ic	interpret x-intercept
$\bullet^2$	pd	find gradient (of QR)
$\bullet^3$	ss	know and use $m_1m_2=-1$
$\bullet^4$	ic	state equ. of altitude
$\bullet^5$	ic	state equ. of line (QR)
$\bullet^6$	ss	prepare to solve sim. equ.
$\bullet^7$	pd	solve for x
$\bullet^8$	pd	solve for y

Primary Method : Give 1 mark for each $\bullet$

$\bullet^1$	$P=(-3,0)$	see Notes 1, 2
$\bullet^2$	$m_{QR}=-2$	or equivalent
$\bullet^3$	$m_{alt}=\frac{1}{2}$	s / i by $\bullet^4$
$\bullet^4$	$alt: y-0=\frac{1}{2}(x+3)$	see Note 4
$\bullet^5$	$QR: y+2=-2(x-8)$ *or* $y-6=-2(x-4)$	
$\bullet^6$	$e.g.\ x-2y=-3\ and\ 2x+y=14$	see Note 5 & Options
$\bullet^7$	$x=5$	
$\bullet^8$	$y=4$	

Notes

1. Without any working;
   accept $(-3,0)$
   accept $x=-3,\ y=0$
   accept $x=-3$ and $y=0$ appearing at $\bullet^4$.
2. $x=-3$ appearing as a consquence of substituting $y=0$ may be awarded $\bullet^1$.
3. At $\bullet^3$, whatever perpendicular gradient is found, it must be in its simplest form either at $\bullet^3$ or $\bullet^4$.
4. $\bullet^4$ is only available as a consequence of attempting to find and use a perpendicular gradient together with whatever coordinates they have for P.

Notes cont

5. $\bullet^6$, $\bullet^7$ and $\bullet^8$ are only available for attempting to solve equations for PT and QR.
6. $\bullet^6$ is a strategy mark for juxtaposing two correctly rearranged equations. Equating zeroes does not gain $\bullet^6$.
7. The answers for $\bullet^7$ and $\bullet^8$ must be of the form of a mixed number or a fraction (vulgar or decimal).

Common Errors

$\bullet^2$	X	$m_{QR}=\ldots=-1$
$\bullet^3$	$X\surd$	$m_{\perp}=1$
$\bullet^4$	$X\surd$	$y-0=1(x+3)$

Option 1 for $\bullet^5$ *to* $\bullet^8$:

$\bullet^5$	$QR: y+2=-2(x-8)$
$\bullet^6$	$\frac{1}{2}(x+3)=-2(x-8)-2$
$\bullet^7$	$x=5$
$\bullet^8$	$y=4$

Option 2 for $\bullet^5$ *to* $\bullet^8$:

$\bullet^5$	$QR: y-6=-2(x-4)$
$\bullet^6$	$\frac{1}{2}(x+3)=-2(x-4)+6$
$\bullet^7$	$x=5$
$\bullet^8$	$y=4$

qu		Mk	Code	cal	Source	ss	pd	ic	C	B	A		U1	U2	U3
1.22	a	4	G23,24	cn	09005	1		3	4						4
	b	4	G27	cn		2	2		4						4

1.22

D, E and F have coordinates $(10,-8,-15)$, $(1,-2,-3)$ and $(-2,0,1)$ respectively.

(*a*) (i) Show that D, E and F are collinear.

(ii) Find the ratio in which E divides DF. 4

(*b*) G has coordinates $(k,1,0)$.

Given that DE is perpendicular to GE, find the value of k. 4

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In this question expressing vectors as coordinates and vice versa is treated as bad form - do not penalise.

$\bullet^1$	ss	use vector approach
$\bullet^2$	ic	compare two vectors
$\bullet^3$	ic	complete proof
$\bullet^4$	ic	state ratio
$\bullet^5$	ss	use vector approach
$\bullet^6$	ss	know scalar product = 0 for $\perp$ vectors
$\bullet^7$	pd	start to solve
$\bullet^8$	pd	complete

Primary Method : Give 1 mark for each •

$\bullet^1$ $\overrightarrow{DE}=\begin{pmatrix}-9\\6\\12\end{pmatrix}$ ***or*** $\overrightarrow{EF}=\begin{pmatrix}-3\\2\\4\end{pmatrix}$ **see Note 1**

$\bullet^2$ 2nd column vector ***and*** $\overrightarrow{DE}=3\overrightarrow{EF}$ (*or equiv.*)

$\bullet^3$ $\overrightarrow{DE}$ and $\overrightarrow{EF}$ have common point and common direction
hence D, E and F collinear **see Note 2**

$\bullet^4$ $3:1$ **stated explicitly**

$\bullet^5$ $\overrightarrow{GE}=\begin{pmatrix}1-k\\-3\\-3\end{pmatrix}$

$\bullet^6$ $\overrightarrow{DE}.\overrightarrow{GE}=0$ **s / i by $\bullet^7$**

$\bullet^7$ $-9(1-k)+6\times(-3)+12\times(-3)$

$\bullet^8$ $k=7$

Notes

1. $\overrightarrow{DE}$ & $\overrightarrow{DF}$ or $\overrightarrow{EF}$ & $\overrightarrow{DF}$ are alternatives to $\overrightarrow{DE}$ & $\overrightarrow{EF}$.
2. $\bullet^3$ can **only** be awarded if a candidate has stated
 * "common point",
 * "common direction" (or "parallel")
 * and "collinear"
3. The "=0" shown at $\bullet^6$ must appear somewhere before $\bullet^8$.
4. In (b) "G.E" $=\begin{pmatrix}k\\1\\0\end{pmatrix}.\begin{pmatrix}1\\-2\\-3\end{pmatrix}=0$ leading to $k=2$, award 1 mark.
5. If $\boldsymbol{a}$ and $\boldsymbol{b}$ are not defined, then merely quoting $\boldsymbol{a}.\boldsymbol{b}=0$ does not gain $\bullet^6$.

Common Error 1 for (b)

$\bullet^5$ √ $\overrightarrow{GE}=\begin{pmatrix}1-k\\-3\\-3\end{pmatrix}$

$\bullet^6$ X $\overrightarrow{DE}.\overrightarrow{GE}=-1$

$\bullet^7$ X√ $-9(1-k)+6\times(-3)+12\times(-3)=-1$

$\bullet^8$ X√ $k=\frac{64}{9}$

Common Error 2 for (b)

$\bullet^5$ X $\begin{pmatrix}k\\1\\0\end{pmatrix}$

$\bullet^6$ X√ $\begin{pmatrix}k\\1\\0\end{pmatrix}.\begin{pmatrix}-9\\6\\12\end{pmatrix}=0$

• X√ $\ldots\ldots k=\frac{2}{3}$ **i.e. 2 marks**

Common Error 3 for (b)

$\bullet^5$ X $\begin{pmatrix}k\\1\\0\end{pmatrix}$

$\bullet^6$ X $\begin{pmatrix}k\\1\\0\end{pmatrix}.\begin{pmatrix}-9\\6\\12\end{pmatrix}=-1$

• X√ $\ldots\ldots k=\frac{7}{9}$ **i.e. 1mark**

Options for $\bullet^1$ *to* $\bullet^3$:

1

$\bullet^1$ $\overrightarrow{DE}=\begin{pmatrix}-9\\6\\12\end{pmatrix}$ $\bullet^2$ $\overrightarrow{DF}=\begin{pmatrix}-12\\8\\16\end{pmatrix}=\frac{4}{3}\overrightarrow{DE}$

$\bullet^3$ $\overrightarrow{DE}$ and $\overrightarrow{DF}$ have common point and common direction
hence D, E and F collinear

2

$\bullet^1$ $\overrightarrow{EF}=\begin{pmatrix}-3\\2\\4\end{pmatrix}$ $\bullet^2$ $\overrightarrow{DF}=\begin{pmatrix}-12\\8\\16\end{pmatrix}=4\overrightarrow{EF}$

$\bullet^3$ $\overrightarrow{EF}$ and $\overrightarrow{DF}$ have common point and common direction
hence D, E and F collinear

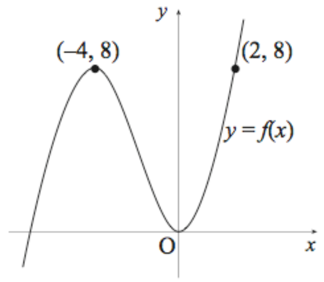
qu		Mk	Code	cal	Source	ss	pd	ic	C	B	A		U1	U2	U3
1.23	a	2	A3	cn	09016			2		2			2		
	b	3	A3	cn		1		2		3			3		

1.23

The diagram shows a sketch of the function $y = f(x)$.

(*a*) Copy the diagram and on it sketch the graph of $y = f(2x)$. 2

(*b*) On a separate diagram sketch the graph of $y = 1 - f(2x)$. 3



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This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

•1 ic scaling parallel to x-axis

•2 ic annotate graph

•3 ss correct order for refl(x) & trans

•4 ic start to annotate final sketch

•5 ic complete annotation

Primary Method : Give 1 mark for each •

3 points : the origin, (1, 8) and (–2, 8)

•1 sketch and 1 point correct

•2 other two points correct

•3 reflect in x-axis, then vertical trans. **s / i by •4**

final points : (0, 1), (1, -7) and (–2, -7)

•4 sketch and 1 final point correct

•5 the other two final points correct

Notes

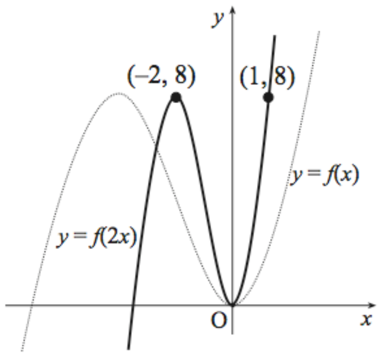
1. In (a) sketching $y = f\left(\frac{1}{2}x\right)$ loses •1 but may gain •2 with appropriate annotation.
2. In (a) no marks are awarded for any other function.
3. Do not penalise omission of the original function in the candidate's sketch for (a).
4. In (b)

	X *refl*	X *refl*	√ *refl*	√ *refl*
	√ *trans*	X *trans*	X *trans*	X *trans*
			$\begin{pmatrix} 0 \\ -1 \end{pmatrix}$	$\begin{pmatrix} \pm 1 \\ 0 \end{pmatrix}$
max	1	0	2	1

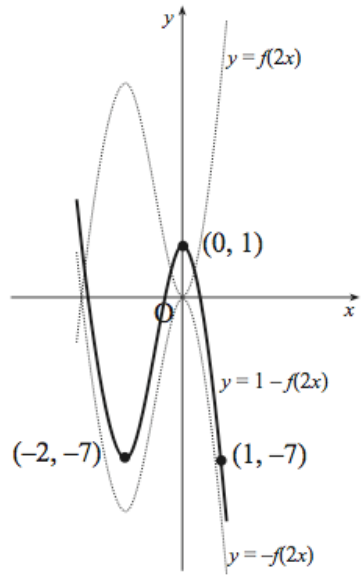
5. In (b): if a candidate does not use their solution for $y = f(2x)$, a maximum of two marks may be awarded for a "correct" solution.
6. In (b):

   No marks are available in (b) unless ***both*** a reflection and a translation have been carried out.

Solution to (a)



Solution to (b)



qu		Mk	Code	Cal	Source	ss	pd	ic	C	B	A		U1	U2	U3
1.24	a	3	T8,T3	nc	09002	1	1	1	3					3	
	b	2	T8	cn				2	2					2	
	c	4	T11	nc		1	1	2	1	3				4	

1.24

(a) Using the fact that $\frac{7\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$, find the exact value of $\sin\left(\frac{7\pi}{12}\right)$. 3

(b) Show that $\sin(A+B) + \sin(A-B) = 2\sin A\cos B$. 2

(c) (i) Express $\frac{\pi}{12}$ in terms of $\frac{\pi}{3}$ and $\frac{\pi}{4}$.

(ii) Hence or otherwise find the exact value of $\sin\left(\frac{7\pi}{12}\right) + \sin\left(\frac{\pi}{12}\right)$. 4

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$\bullet^1$	ss	expand compound angle
$\bullet^2$	ic	substitute exact values
$\bullet^3$	pd	process to a single fraction
$\bullet^4$	ic	start proof
$\bullet^5$	ic	complete proof
$\bullet^6$	ss	identify steps
$\bullet^7$	ic	start process (identify 'A' & 'B')
$\bullet^8$	ic	substitute
$\bullet^9$	pd	process

Primary Method : Give 1 mark for each •

$\bullet^1$ $\sin\frac{\pi}{3}\cos\frac{\pi}{4} + \cos\frac{\pi}{3}\sin\frac{\pi}{4}$ **s / i by $\bullet^2$**

$\bullet^2$ $\frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}} + \frac{1}{2}\times\frac{1}{\sqrt{2}}$

$\bullet^3$ $\dfrac{\sqrt{3}+1}{2\sqrt{2}}$ *or equivalent*

$\bullet^4$ $\sin A\cos B + \cos A\sin B + \ldots$

$\bullet^5$ $\ldots\ldots + \sin A\cos B - \cos A\sin B$ and complete

$\bullet^6$ $\frac{\pi}{12} = \frac{\pi}{3} - \frac{\pi}{4}$ **stated explicitly**

and A *is* $\frac{\pi}{3}$, B *is* $\frac{\pi}{4}$ **s / i by $\bullet^7$**

$\bullet^7$ $2\sin\frac{\pi}{3}\cos\frac{\pi}{4}$

$\bullet^8$ $2\times\dfrac{\sqrt{3}}{2}\times\dfrac{1}{\sqrt{2}}$

$\bullet^9$ $\dfrac{\sqrt{6}}{2}$ $\left(\text{accept } \sqrt{\dfrac{3}{2}} \text{ or } \dfrac{\sqrt{3}}{\sqrt{2}} \textbf{ but not } \dfrac{2\sqrt{3}}{2\sqrt{2}}\right)$

Notes

1. Candidates who work throughout in degrees can gain all the marks.
2. In (a)
   $\sin\left(\frac{\pi}{3}+\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{3}\right) + \sin\left(\frac{\pi}{4}\right)$ *etc*
   cannot be awarded any marks.
   i.e. $\bullet^1$, $\bullet^2$ and $\bullet^3$ are not available.
3. In (b), candidates who use numerical values for A and B earn no marks.
4. In (c)
   $\sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right) = \sin\left(\frac{\pi}{3}\right) - \sin\left(\frac{\pi}{4}\right)$ *etc*
   cannot be awarded any marks.
   i.e. $\bullet^7$, $\bullet^8$ and $\bullet^9$ are not available.

Common Errors

1. $\frac{7\pi}{12} = \frac{\pi}{3} + \frac{\pi}{4}$
   $\therefore\ \frac{\pi}{12} = \frac{1}{7}\left(\frac{\pi}{3}+\frac{\pi}{4}\right)$ does not gain $\bullet^6$.

Alternatives

1. for $\bullet^6$ to $\bullet^8$

$\bullet^6$ $\sin\left(\frac{\pi}{12}\right) = \sin\frac{\pi}{3}\cos\frac{\pi}{4} - \cos\frac{\pi}{3}\sin\frac{\pi}{4}$

$\bullet^7$ $\dfrac{\sqrt{3}}{2}\times\dfrac{1}{\sqrt{2}} - \dfrac{1}{2}\times\dfrac{1}{\sqrt{2}}$

$\bullet^8$ $\dfrac{\sqrt{3}-1}{2\sqrt{2}}$ *or equivalent*

qu		Mk	Code	cal	Source	ss	pd	ic	C	B	A		U1	U2	U3
2.01		8	C8,C9	cn	08507	3	4	1	8				8		

2.01

Find the coordinates of the turning points of the curve with equation $y = x^3 - 3x^2 - 9x + 12$ and determine their nature. 8

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$\bullet^1$	ss	know to differentiate
$\bullet^2$	pd	differentiate
$\bullet^3$	ss	set derivative to zero
$\bullet^4$	pd	factorise
$\bullet^5$	pd	solve for x
$\bullet^6$	pd	evaluate y-coordinates
$\bullet^7$	ss	know to, and justify turning points
$\bullet^8$	ic	interpret result

Primary Method : Give 1 mark for each $\bullet$

$\bullet^1$ $\frac{dy}{dx} = \ldots(1\ \textit{term correct})$

$\bullet^2$ $3x^2 - 6x - 9$

$\bullet^3$ $\frac{dy}{dx} = 0$

$\bullet^4$ $3(x+1)(x-3)$

	$\bullet^5$	$\bullet^6$
$\bullet^5$	$x = -1$	$x = 3$
$\bullet^6$	$y = 17$	$y = -15$

		$\bullet^7$			$\bullet^8$	
x	$\ldots$	-1	$\ldots$	$\ldots$	3	$\ldots$
$\bullet^7$ $\frac{dy}{dx}$	$+$	0	$-$	$-$	0	$+$
$\bullet^8$		max			min	

Notes

1. The "=0" (shown at $\bullet^3$) ***must*** occur at least once before $\bullet^5$.
2. $\bullet^4$ is only available as a consequence of solving $\frac{dy}{dx} = 0$.
3. The nature table must reflect previous working from $\bullet^4$.
4. For $\bullet^4$, accept $(x+1)(x-3)$.
5. The use of the 2nd derivative is an acceptable strategy.
6. As shown in the Primary Method, ($\bullet^5$ and $\bullet^6$) and ($\bullet^7$ and $\bullet^8$) can be marked horizontally or vertically.
7. $\bullet^1$,$\bullet^2$ and $\bullet^3$ are the only marks available to candidates who solve $3x^2 - 6x = 9$.

Notes cont

8. If $\bullet^7$is not awarded, $\bullet^8$ is only available as follow-through if there is clear evidence of where the signs at the $\bullet^7$ stage have been obtained.
9. For $\bullet^7$ and $\bullet^8$

   The completed nature table is worth 2 marks if correct.

   If the labels "x" and/or "$\frac{dy}{dx}$" are missing from an otherwise correct table then **award 1 mark.**

   If the labels "x" and/or "$\frac{dy}{dx}$" are missing from a table where either $\bullet^7$ or $\bullet^8$ (vertically) would otherwise have been awarded, then **award 0 marks.**

Alternatives

This would be fairly common:

$\bullet^1$ √ $\frac{dy}{dx} = \ldots(1\ \textit{term correct})$

$\bullet^2$ √ $3x^2 - 6x - 9$

$\bullet^3$,$\bullet^4$ √√ $(3x-9)(x+1) = 0$

or $(3x+3)(x-3) = 0$

Min. requirements of a nature table

x	$\ldots$	-1	$\ldots$
$\frac{dy}{dx}$	$+$	0	$-$
		max	

Preferred nature table

x	$\ldots$	-1	$\ldots$
$\frac{dy}{dx}$	$+$	0	$-$
	/	—	\
		max	

qu		Mk	Code	cal	Source	ss	pd	ic	C	B	A		U1	U2	U3
2.02	a	3	A4	cn	09011	1		2	3				3		
	b	3	C1	cn		2	1		3				3		

2.02

Functions f and g are given by $f(x) = 3x + 1$ and $g(x) = x^2 - 2$.

(a) (i) Find $p(x)$ where $p(x) = f\left(g(x)\right)$

(ii) Find $q(x)$ where $q(x) = g\left(f(x)\right)$. 3

(b) Solve $p'(x) = q'(x)$. 3

The primary method m.s is based on the following generic m.s.

This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

$\bullet^1$	ss	substitute for $g(x)$ in $f(x)$
$\bullet^2$	ic	complete
$\bullet^3$	ic	sub. and complete for $q(x)$
$\bullet^4$	ss	simplify
$\bullet^5$	pd	differentiate
$\bullet^6$	pd	solve

Primary Method : Give 1 mark for each $\bullet$

$\bullet^1$ $f(x^2 - 2)$ **s / i by $\bullet^2$**

$\bullet^2$ $3(x^2 - 2) + 1$

$\bullet^3$ $(3x + 1)^2 - 2$

	$\bullet^4$	$\bullet^5$
$\bullet^4$	$3x^2 - 5$	$9x^2 + 6x - 1$
$\bullet^5$	$6x$	$18x + 6$ *or equiv.*

s / i by $\bullet^5$

$\bullet^6$ $x = -\frac{1}{2}$

Notes

1. In (a)
   2 marks are available for finding either $f\left(g(x)\right)$ or $g\left(f(x)\right)$ and 1 mark for finding the other.
2. In (b)
   candidates who start by equating $p(x)$ and $q(x)$ and then differentiate may earn $\bullet^4$ and $\bullet^6$ only.

Common Errors

1

$p(x)$ and $q(x)$ switched round:

X	$\bullet^1$	$p(x) = g(3x + 1)$
$X\surd$	$\bullet^2$	$p(x) = (3x + 1)^2 - 2$
$X\surd$	$\bullet^3$	$q(x) = \ldots\ldots = 3(x^2 - 2) + 1$

2

Candidates who find $f(f(x))$ and $g(g(x))$ can earn no marks in (a) but

$X\surd$	$\bullet^4$	$9x + 4$ *and* $x^4 - 4x^2 + 2$
$X\surd$	$\bullet^5$	$9 = 4x^3 - 8x$
XX	$\bullet^6$	*not available*

3

X	$\bullet^4$	$3x^2 - 1$ *and* $9x^2 + 6x - 1$
$X\surd$	$\bullet^5$	$6x$ *and* $18x + 6$
$X\surd$	$\bullet^6$	$x = -\frac{1}{2}$

Alternative for $\bullet^1$ *to* $\bullet^3$:

$\bullet^1$ $f\left(g(x)\right) = 3 \times g(x) + 1$

$\bullet^2$ $f\left(g(x)\right) = 3\left(x^2 - 2\right) + 1$

$g\left(f(x)\right) = \left(f(x)\right)^2 - 2$

$\bullet^3$ $g\left(f(x)\right) = (3x + 1)^2 - 2$

qu		Mk	Code	cal	Source	ss	pd	ic	C	B	A		U1	U2	U3
2.03	a	4	A21	cn	09008	1	1	2	4					4	
	b	5	A32	cn		2	1	2		5					5

2.03

(*a*) (i) Show that $x = 1$ is a root of $x^3 + 8x^2 + 11x - 20 = 0$.

(ii) Hence factorise $x^3 + 8x^2 + 11x - 20$ fully. 4

(*b*) Solve $\log_2(x+3) + \log_2(x^2 + 5x - 4) = 3$. 5

The primary method m.s is based on the following generic m.s.

This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

$\bullet^1$	ss	know and use $f(a) = 0 \Leftrightarrow a$ is a root
$\bullet^2$	ic	start to find quadratic factor
$\bullet^3$	ic	complete quadratic factor
$\bullet^4$	pd	factorise fully
$\bullet^5$	ss	use log laws
$\bullet^6$	ss	know to & convert to exponential form
$\bullet^7$	ic	write cubic in standard form
$\bullet^8$	pd	solve cubic
$\bullet^9$	ic	interpret valid solution

Primary Method : Give 1 mark for each •

$\bullet^1$	$f(1) = 1 + 8 + 11 - 20 = 0$ *so* $x = 1$ *is a root*	**See Note 1**
$\bullet^2$	$(x-1)(x^2 \ldots\ldots\ldots)$	
$\bullet^3$	$(x^2 + 9x + 20)$	
$\bullet^4$	$(x-1)(x+4)(x+5)$	**Stated explicitly**
$\bullet^5$	$\log_2\left((x+3)(x^2+5x-4)\right)$	**s / i by** $\bullet^6$
$\bullet^6$	$(x+3)(x^2+5x-4) = 2^3$	
$\bullet^7$	$x^3 + 8x^2 + 11x - 20 = 0$	
$\bullet^8$	$x = 1$ *or* $x = -4$ *or* $x = -5$	**Stated explicitly here**
$\bullet^9$	$x = 1$ *only*	

Notes

1. For candidates evaluating the function, some acknowledgement of the resulting zero must be shown in order to gain $\bullet^1$.
2. For candidates using synthetic division (shown in Alt. box), some acknowledgement of the resulting zero must be shown in order to gain $\bullet^2$.
3. In option 2 the "zero" has been highlighted by underlining. This can also appear in colour, bold or boxed. Some acknowledgement of the resulting zero must be shown in order to gain $\bullet^1$ as indicated in each option.

Common Errors

1

$\bullet^5$ X	$\log_2 \dfrac{x^2+5x-4}{x+3} = 3$
$\bullet^6$ $X\checkmark$	$\dfrac{x^2+5x-4}{x+3} = 2^3$
$\bullet^7$ X	$x^2 - 3x - 28 = 0$
$\bullet^8$ X	$x = 7$ *or* -4
$\bullet^9$ $X\checkmark$	$x = 7$ **ONLY**

Options

Alternative for $\bullet^1$ to $\bullet^2$.

1

$\bullet^1$
$$\begin{array}{c|cccc} & 1 & 8 & 11 & -20 \\ 1 & & 1 & & \\ \hline & 1 & 9 & & \end{array}$$

$\bullet^2$
$$\begin{array}{c|cccc} & 1 & 8 & 11 & -20 \\ 1 & & 1 & 9 & 20 \\ \hline & 1 & 9 & 20 & 0 \end{array} \quad rem. = 0$$

so $x = 1$ *is root*

see note 2

2

$\bullet^1$
$$\begin{array}{c|cccc} & 1 & 8 & 11 & -20 \\ 1 & & 1 & & \\ \hline & 1 & 9 & & \end{array}$$

$\bullet^2$
$$\begin{array}{c|cccc} & 1 & 8 & 11 & -20 \\ 1 & & 1 & 9 & 20 \\ \hline & 1 & 9 & 20 & \underline{0} \end{array} \quad so\ x = 1\ is\ root$$

see note 3

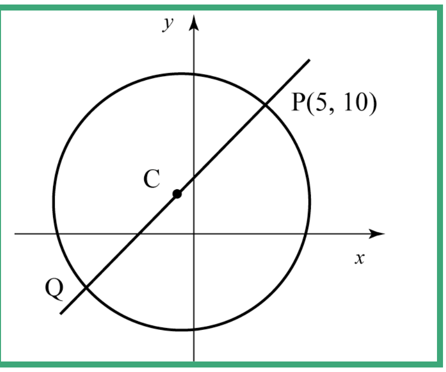
qu		Mk	Code	Cal	Source	ss	pd	ic	C	B	A		U1	U2	U3
2.04	a	1	A6	cn	08026		1		1				1		
	b	5	G11	cn		2		3	5					5	
	c	4	G15	nc		1	1	2			4			4	

2.04

(*a*) Show that the point P(5, 10) lies on circle C_1 with equation $(x+1)^2+(y-2)^2=100$. 1

(*b*) PQ is a diameter of this circle as shown in the diagram. Find the equation of the tangent at Q. 5

(*c*) Two circles, C_2 and C_3, touch circle C_1 at Q. The radius of each of these circles is twice the radius of circle C_1. Find the equations of circles C_2 and C_3. 4



The primary method m.s is based on the following generic m.s.
This generic marking scheme may be used as an equivalence g
but only where a candidate does not use the primary method or
alternative method shown in detail in the marking scheme.

•1	pd	substitute
•2	ic	find centre
•3	ss	use mid-point result for Q
•4	ss	know to, and find gradient of radi
•5	ic	find gradient of tangent
•6	ic	state equation of tangent
•7	ic	state radius
•8	ss	know how to find centre
•9	ic	state equation of one circle
•10	ic	state equation of the other circle

Primary Method : Give 1 mark for each •

•1	$(5+1)^2+(10-2)^2=100$	
•2	$centre=(-1,2)$	
•3	$Q=(-7,-6)$	**(no evidence requ.)**
•4	$m_{rad}=\frac{8}{6}$	
•5	$m_{tgt}=-\frac{3}{4}$	**s / i by •6**
•6	$y-(-6)=-\frac{3}{4}(x-(-7))$	
•7	$radius=20$	**s / i by •9 or •10**
•8	$centre=(5,10)$	**s / i by •9**
•9	$(x-5)^2+(y-10)^2=400$	
•10	$(x+19)^2+(y+22)^2=400$	

Notes

1. In (a), candidates may choose to show that distance CP = the radius. Markers should note that evidence for •2, which is in (b), may appear in (a).
2. The minimum requirement for •1 is as shown in the Primary Method.
3. •6 is only available as a conseqence of attempting to find a perp. gradient.
4. For candidates who choose a Q *ex nihilo*, •6 is only available if the chosen Q lies in the 3rd quadrant.

Notes cont

5. •9 and/or •10 are only available as follow-through if a centre with numerical coordinates has been stated explicitly.
6. •10 is not available as a follow-through; it must be correct.

Alternative for •8, •9 and •10

•8	$centre=(-19,-22)$	**s / i by •9**
•9	$(x+19)^2+(y+22)^2=400$	
•10	$(x-5)^2+(y-10)^2=400$	

qu		Mk	Code	Cal	Source	ss	pd	ic	C	B	A		U1	U2	U3
2.05	a	1	T4	cn	09026			1	1				1		
	b	5	T6	cr		1	3	1	5					5	
	c	6	C17,23	cr		1	3	2		6				6	

2.05

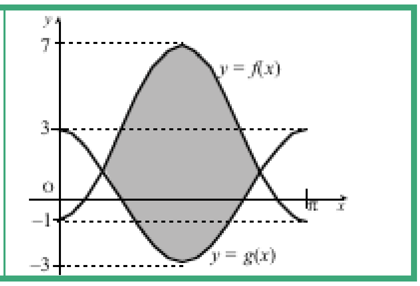
The graphs of $y = f(x)$ and $y = g(x)$ are shown in the diagram.

$f(x) = -4\cos\left(2x\right) + 3$ and $g(x)$ is of the form $g(x) = m\cos\left(nx\right)$.

(*a*) Write down the values of m and n. 1

(*b*) Find, correct to 1 decimal place, the coordinates of the points of intersection of the two graphs in the interval shown. 5

(*c*) Calculate the shaded area. 6



The primary method m.s is based on the following generic m.s.

This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

•1	ic	interprets graph
•2	ss	knows how to find intersection
•3	pd	starts to solve
•4	pd	finds x-coordinate in the 1st quadrant
•5	pd	finds x-coordinate in the 2nd quadrant
•6	pd	finds y-coordinates
•7	ss	knows how to find area
•8	ic	states limits
•9	pd	integrate
•10	pd	integrate
•11	ic	substitute limits
•12	pd	evaluate area

Primary Method : Give 1 mark for each •

•1 $m = 3 \text{ and } n = 2$

•2 $3\cos 2x = -4\cos 2x + 3$

•3 $\cos 2x = \dfrac{3}{7}$

•4 $x = 0.6$

•5 $x = 2.6$

•6 $y = 1.3,\ 1.3$

•7 $\displaystyle\int \left(-4\cos 2x + 3 - 3\cos 2x\right)\ dx$

•8 $\displaystyle\int_{0.6}^{2.6}$

•9 $"-7\sin 2x"$

•10 $3x - \frac{7}{2}\sin 2x$

•11 $(3 \times 2.6 - \frac{7}{2}\sin 5.2) - (3 \times 0.6 - \frac{7}{2}\sin 1.2)$

•12 12.4

Continued on next page

Continued on next page

Question 2.05 cont.

Notes 1

1. Answers which are not rounded should be treated as "bad form" and not penalised.
2. If $n = 1$ from (a), then in (b) the follow-through solution is 0.697 and 5.586. $\bullet^5$ is not available in (b) and $\bullet^8$ is not available in (c).
3. If $n = 3$ from (a), then in (b) only $\bullet^2$ is available.
4. At $\bullet^5$: $x = 2.5$ can only come from calculating $\pi - 0.6$. For this to be accepted, candidates must state that it comes from symmetry of the graph.
5. For $\bullet^6$ Acceptable values of y will lie in the range 1.1 to 1.6 (due to early rounding !!)
6. Values of x used for the limits must lie between 0 and π, i.e $0 < \text{limits} < \pi$, else $\bullet^8$ is lost.
7. $\bullet^8, \bullet^{11}$ and $\bullet^{12}$ are not available to candidates who use -3 and 7 as the limits.
8. Candidates must deal appropriately with any extraneous negative signs which may appear before $\bullet^{12}$ can be awarded.

   It is considered inappropriate to write ……… = –12.4 = 12.4

Common Errors

1. For candidates who work in degrees throughout this question, the following marks are available:

In (b)		In (c)	
$\bullet^2$	√	$\bullet^7$	√
$\bullet^3$	√	$\bullet^8$	X
$\bullet^4$	X	$\bullet^9$	X
$\bullet^5$	X √	$\bullet^{10}$	X √
$\bullet^6$	√	$\bullet^{11}$	X
		$\bullet^{12}$	X

2. In (c) candidates who deal with $f(x)$ and $g(x)$ separately and **add** can only earn at most
   - $\bullet^8$ correct limits
   - $\bullet^9$ for correct integral of $f(x)$
   - $\bullet^{10}$ for correct integral of $g(x)$
   - $\bullet^{11}$ for correct substitution.

Alternative for $\bullet^3, \bullet^4, \bullet^5$

Option 1

$\bullet^3$ $\cos^2 x = \frac{10}{14}$

$\bullet^4$ $\cos x = \sqrt{\frac{10}{14}}, \quad \cos x = -\sqrt{\frac{10}{14}}$

$\bullet^5$ $x = 0.6 \qquad x = 2.6$

Option 2

$\bullet^3$ $\cos^2 x = \frac{10}{14}$

$\bullet^4$ $\cos x = \sqrt{\frac{10}{14}} \quad \textit{and} \quad x = 0.6$

$\bullet^5$ $\cos x = -\sqrt{\frac{10}{14}} \quad \textit{and} \quad x = 2.6$

Option 3

$\bullet^3$ $\sin^2 x = \frac{4}{14}$

$\bullet^4$ $\sin x = \sqrt{\frac{4}{14}}$

$\bullet^5$ $x = 0.6, \quad x = 2.6$

Alternative for $\bullet^9, \bullet^{10}$

$\bullet^9$ $-4\sin 2x - 3\sin 2x$

$\bullet^{10}$ $3x - \frac{4}{2}\sin 2x - \frac{3}{2}\sin 2x$

qu		Mk	Code	cal	Source	ss	pd	ic	C	B	A		U1	U2	U3
2.06	a	2	A30,34	cr	08532		1	1		2					2
	b	3	A30,34	cr		1	1	1			3				3

2.06

The size of the human population, N, can be modelled using the equation $N = N_0 e^{rt}$ where N_0 is the population in 2006, t is the time in years since 2006, and r is the annual rate of increase in the population.

(*a*) In 2006 the population of the United Kingdom was approximately 61 million, with an annual rate of increase of 1·6%. Assuming this growth rate remains constant, what would be the population in 2020 ? 2

(*b*) In 2006 the population of Scotland was approximately 5·1 million, with an annual rate of increase of 0·43%. Assuming this growth rate remains constant, how long would it take for Scotland's population to double in size ? 3

The primary method m.s is based on the following generic m.s.

This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

$\bullet^1$	ic	substitute into equation
$\bullet^2$	pd	evaluate exponential expression
$\bullet^3$	ic	interpret info and substitute
$\bullet^4$	ss	convert expo. equ. to log. equ.
$\bullet^5$	pd	process

Primary Method : Give 1 mark for each •

$\bullet^1$ $61e^{0.016\times 14}$

$\bullet^2$ 76 million *or equiv.*

$\bullet^3$ $10.2 = 5.1e^{0.0043t}$

$\bullet^4$ $0.0043t = \ln 2$

$\bullet^5$ $t = 161.2$ years

Notes

1. For $\bullet^2$, do not accept 76. Accept any answer which rounds to 76 million and was obtained from legitimate sources.
2. $\bullet^5$ is for a rounded up answer or implying a rounded-up answer. Acceptable answers would include 162 and 161.2 but not 161.
3. **Cave** Beware of poor imitations which yield results similar/same to that given in the paradigm, e.g.
   compound percentage
   or recurrence relations.
   These can receive no credit but see Common Error 2 for exception.

Common Errors

1 Candidates who misread the rate of increase:

$\bullet^1$ X $61e^{1.6\times 14}$

$\bullet^2$ $X\surd$ 3.26×10^{11} *million*

$\bullet^3$ $X\surd$ $10.2 = 5.1e^{0.43t}$

$\bullet^4$ $X\surd$ $0.43t = \ln 2$

$\bullet^5$ $X\surd$ $t = 1.612$

2

$\bullet^1$ X 61×1.016^{14}

$\bullet^2$ X 76 *million*

$\bullet^3$ X $10.2 = 5.1\times 1.0043^{t}$

$\bullet^4$ $X\surd$ $t\ln 1.0043 = \ln 2$

$\bullet^5$ $X\surd$ $t = 162$

i.e. award 2 marks

Options

1

$\bullet^1$ $61000000e^{0.016\times 14}$

$\bullet^2$ 76000000

2

$\bullet^1$ $(61 \text{ million})\times e^{0.016\times 14}$

$\bullet^2$ 76 million

3

$\bullet^1$ $61000000e^{0.224}$

$\bullet^2$ 76 million

4

$\bullet^1$ $(61 \text{ million})\times e^{0.224}$

$\bullet^2$ 76000000

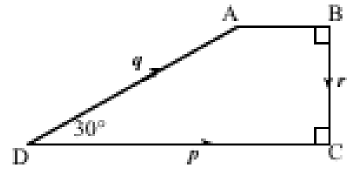
qu		Mk	Code	cal	Source	ss	pd	ic	C	B	A		U1	U2	U3
2.07	a	6	G29,26	cn	09031	1	2	3		6					6
	b	4	G21,30	cr		1	1	2		2	2				4

2.07

Vectors $\boldsymbol{p}$, $\boldsymbol{q}$ and $\boldsymbol{r}$ are represented on the diagram shown where angle ADC = 30°. It is also given that $|\boldsymbol{p}| = 4$ and $|\boldsymbol{q}| = 3$.

(*a*) Evaluate $\boldsymbol{p}.(\boldsymbol{q}+\boldsymbol{r})$ and $\boldsymbol{r}.(\boldsymbol{p}-\boldsymbol{q})$. 6

(*b*) Find $|\boldsymbol{q}+\boldsymbol{r}|$ and $|\boldsymbol{p}-\boldsymbol{q}|$. 4



The primary method m.s is based on the following generic m.s.

This generic marking scheme may be used as an equivalence guide but only where a candidate does not use the primary method or any alternative method shown in detail in the marking scheme.

- •1 ss use distributive law
- •2 ic interpret scalar product
- •3 pd processing scalar product
- •4 ic interpret perpendicularity
- •5 ic interpret scalar product
- •6 pd complete processing
- •7 ic interpret vectors on a 2-D diagram
- •8 pd evaluate magnitude of vector sum
- •9 ic interpret vectors on a 2-D diagram
- •10 pd evaluate magnitude of vector difference

Primary Method : Give 1 mark for each •

- •1 $\boldsymbol{p}.\boldsymbol{q}+\boldsymbol{p}.\boldsymbol{r}$ **s / i by (•2 and •4)**
- •2 $4\times 3\cos 30°$ **s / i by •3**
- •3 $6\sqrt{3}$ (10.4)
- •4 $\boldsymbol{p}.\boldsymbol{r}=0$ **explicitly stated**
- •5 $-|\boldsymbol{r}|\times 3\cos 120°$
- •6 $r=\frac{3}{2}$ *and* ... $\frac{9}{4}$
- •7 $\boldsymbol{q}+\boldsymbol{r}\equiv$ from D to the projection of A onto DC
- •8 $|\boldsymbol{q}+\boldsymbol{r}|=\frac{3\sqrt{3}}{2}$
- •9 $\boldsymbol{p}-\boldsymbol{q}\equiv\overrightarrow{AC}$
- •10 $|\boldsymbol{p}-\boldsymbol{q}|=\sqrt{\left(4-\frac{3\sqrt{3}}{2}\right)^2+\left(\frac{3}{2}\right)^2}$ (2.05)

Notes

1. $\boldsymbol{p}.(\boldsymbol{q}+\boldsymbol{r})=pq+pr$ gains no marks unless the "vectors" are treated correctly further on. In this case treat this as bad form.
2. The evidence for •7 and •9 will likely appear in a diagram with the vectors $\boldsymbol{q}+\boldsymbol{r}$ and $\boldsymbol{p}-\boldsymbol{q}$ **clearly marked.**

Common Errors

1 For •1 to •4

$\boldsymbol{p}.(\boldsymbol{q}+\boldsymbol{r})=\boldsymbol{p}.\boldsymbol{q}+\boldsymbol{p}.\boldsymbol{r}$

$=4\times 3+4\times\frac{3}{2}$

$=18$

can only be awarded •1.

Alternatives 1

1 For •7 and •8 :

•7 √ $\boldsymbol{p}.(\boldsymbol{q}+\boldsymbol{r})=|\boldsymbol{p}|\,|\boldsymbol{q}+\boldsymbol{r}|\cos 0$

$6\sqrt{3}\;=4|\boldsymbol{q}+\boldsymbol{r}|\times 1$

•8 √ $|\boldsymbol{q}+\boldsymbol{r}|=\frac{6\sqrt{3}}{4}=\frac{3\sqrt{3}}{2}$

2 For •9, •10 :

Using right-angled Δ ABC

•9 $\overrightarrow{AC}=\boldsymbol{p}-\boldsymbol{q}$,

and $\left|\overrightarrow{AB}\right|=4-\frac{3\sqrt{3}}{2}$, $\left|\overrightarrow{BC}\right|=\frac{3}{2}$

and $A\hat{C}B=43.06°$

•10 *use* $\boldsymbol{r}.(\boldsymbol{p}-\boldsymbol{q})=\frac{9}{4}$

to get $|\boldsymbol{p}-\boldsymbol{q}|=2.05$

Alternatives 2

3

For •7, •8, •9, •10 :

Set up a coord system with origin at D

•7 $C=(4,0), A=\left(\frac{3\sqrt{3}}{2},\frac{3}{2}\right), B=\left(4,\frac{3}{2}\right)$

•8 $\boldsymbol{p}=\begin{pmatrix}4\\0\end{pmatrix}$, $\boldsymbol{q}=\begin{pmatrix}\frac{3\sqrt{3}}{2}\\\frac{3}{2}\end{pmatrix}$, $\boldsymbol{r}=\begin{pmatrix}0\\-\frac{3}{2}\end{pmatrix}$

•9 $\boldsymbol{q}+\boldsymbol{r}=\begin{pmatrix}\frac{3\sqrt{3}}{2}\\0\end{pmatrix}$ *and* $|\boldsymbol{q}+\boldsymbol{r}|=2.60$

•10 $\boldsymbol{p}-\boldsymbol{q}=\begin{pmatrix}4-\frac{3\sqrt{3}}{2}\\-\frac{3}{2}\end{pmatrix}$ *and* $|\boldsymbol{p}-\boldsymbol{q}|=2.05$

Marks : May 2009

Centre/group cand no.																totals
21a	1						**21a**	1						**21a**	1	
21b	3						**21b**	3						**21b**	3	
21c	4						**21c**	4						**21c**	4	
22a	4						**22a**	4						**22a**	4	
22b	4						**22b**	4						**22b**	4	
23a	2						**23a**	2						**23a**	2	
23b	3						**23b**	3						**23b**	3	
24a	3						**24a**	3						**24a**	3	
24b	2						**24b**	2						**24b**	2	
24c	4						**24c**	4						**24c**	4	
1	8						**1**	8						**1**	8	
2a	3						**2a**	3						**2a**	3	
2b	3						**2b**	3						**2b**	3	
3a	4						**3a**	4						**3a**	4	
3b	5						**3b**	5						**3b**	5	
4a	1						**4a**	1						**4a**	1	
4b	5						**4b**	5						**4b**	5	
4c	4						**4c**	4						**4c**	4	
5a	1						**5a**	1						**5a**	1	
5b	5						**5b**	5						**5b**	5	
5c	6						**5c**	6						**5c**	6	
6a	2						**6a**	2						**6a**	2	
6b	3						**6b**	3						**6b**	3	
7a	6						**7a**	6						**7a**	6	
7b	4						**7b**	4						**7b**	4	
	totals							totals								

Centre/group cand.no																totals
21a	1						**21a**	1						**21a**	1	
21b	3						**21b**	3						**21b**	3	
21c	4						**21c**	4						**21c**	4	
22a	4						**22a**	4						**22a**	4	
22b	4						**22b**	4						**22b**	4	
23a	2						**23a**	2						**23a**	2	
23b	3						**23b**	3						**23b**	3	
24a	3						**24a**	3						**24a**	3	
24b	2						**24b**	2						**24b**	2	
24c	4						**24c**	4						**24c**	4	
1	8						**1**	8						**1**	8	
2a	3						**2a**	3						**2a**	3	
2b	3						**2b**	3						**2b**	3	
3a	4						**3a**	4						**3a**	4	
3b	5						**3b**	5						**3b**	5	
4a	1						**4a**	1						**4a**	1	
4b	5						**4b**	5						**4b**	5	
4c	4						**4c**	4						**4c**	4	
5a	1						**5a**	1						**5a**	1	
5b	5						**5b**	5						**5b**	5	
5c	6						**5c**	6						**5c**	6	
6a	2						**6a**	2						**6a**	2	
6b	3						**6b**	3						**6b**	3	
7a	6						**7a**	6						**7a**	6	
7b	4						**7b**	4						**7b**	4	
	totals							totals								

the end