X100/301

NATIONAL
QUALIFICATIONS
2007

TUESDAY, 15 MAY
9.00 AM – 10.10 AM

MATHEMATICS
HIGHER
Units 1, 2 and 3
Paper 1
(Non-calculator)

Read Carefully

1 **Calculators may NOT be used in this paper.**

2 Full credit will be given only where the solution contains appropriate working.

3 Answers obtained by readings from scale drawings will not receive any credit.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos \theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2\sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

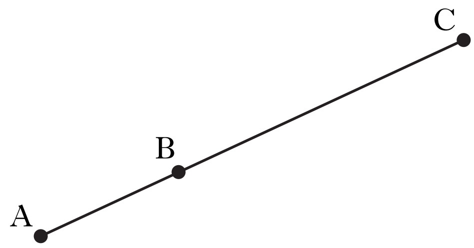
ALL questions should be attempted.

Marks

1. Find the equation of the line through the point (–1, 4) which is parallel to the line with equation $3x - y + 2 = 0$. **3**

2. Relative to a suitable coordinate system A and B are the points (–2, 1, –1) and (1, 3, 2) respectively.

A, B and C are collinear points and C is positioned such that BC = 2AB.



Find the coordinates of C. **4**

3. Functions f and g, defined on suitable domains, are given by $f(x) = x^2 + 1$ and $g(x) = 1 - 2x$.

Find:

(*a*) $g(f(x))$; **2**

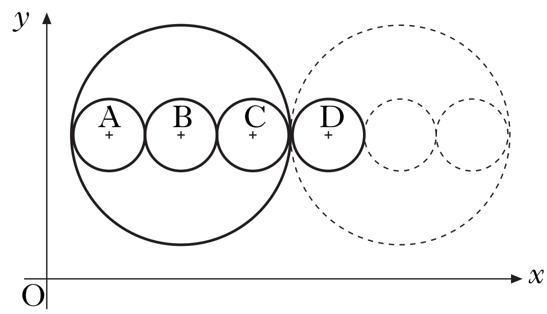
(*b*) $g(g(x))$. **2**

4. Find the range of values of k such that the equation $kx^2 - x - 1 = 0$ has no real roots. **4**

5. The large circle has equation $x^2 + y^2 - 14x - 16y + 77 = 0$.

Three congruent circles with centres A, B and C are drawn inside the large circle with the centres lying on a line parallel to the x-axis.

This pattern is continued, as shown in the diagram.



Find the equation of the circle with centre D. **5**

[Turn over

6. Solve the equation $\sin 2x° = 6\cos x°$ for $0 \leq x \leq 360$. **4**

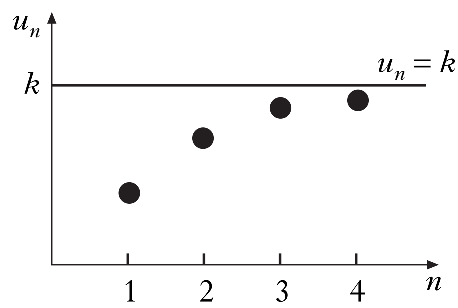
7. A sequence is defined by the recurrence relation

$$u_{n+1} = \frac{1}{4}u_n + 16,\ u_0 = 0.$$

(*a*) Calculate the values of u_1, u_2 and u_3. **3**

Four terms of this sequence, u_1, u_2, u_3 and u_4 are plotted as shown in the graph.

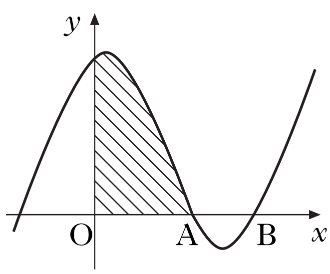
As $n \rightarrow \infty$, the points on the graph approach the line $u_n = k$, where k is the limit of this sequence.



(*b*) (i) Give a reason why this sequence has a limit.

(ii) Find the exact value of k. **3**

8. The diagram shows a sketch of the graph of $y = x^3 - 4x^2 + x + 6$.



(*a*) Show that the graph cuts the x-axis at (3, 0). **1**

(*b*) Hence or otherwise find the coordinates of A. **3**

(*c*) Find the shaded area. **5**

9. A function f is defined by the formula $f(x) = 3x - x^3$.

(*a*) Find the exact values where the graph of $y = f(x)$ meets the x- and y-axes. **2**

(*b*) Find the coordinates of the stationary points of the function and determine their nature. **7**

(*c*) Sketch the graph of $y = f(x)$. **1**

Marks

10. Given that $y = \sqrt{3x^2 + 2}$, find $\dfrac{dy}{dx}$. **3**

11. (*a*) Express $f(x) = \sqrt{3}\cos x + \sin x$ in the form $k\cos(x - a)$, where $k > 0$ and $0 < a < \dfrac{\pi}{2}$. **4**

(*b*) Hence or otherwise sketch the graph of $y = f(x)$ in the interval $0 \leq x \leq 2\pi$. **4**

[*END OF QUESTION PAPER*]

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X100/303

NATIONAL QUALIFICATIONS 2007

TUESDAY, 15 MAY
10.30 AM – 12.00 NOON

MATHEMATICS
HIGHER
Units 1, 2 and 3
Paper 2

Read Carefully

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$\cos ax$	$\frac{1}{a}\sin ax + C$

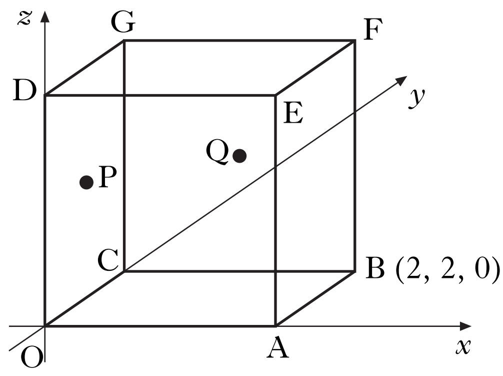
ALL questions should be attempted.

Marks

1. OABCDEFG is a cube with side 2 units, as shown in the diagram.

B has coordinates (2, 2, 0).

P is the centre of face OCGD and Q is the centre of face CBFG.

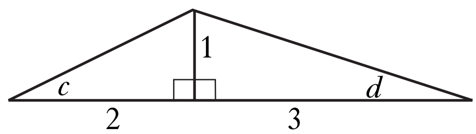


(a) Write down the coordinates of G. **1**

(b) Find $\boldsymbol{p}$ and $\boldsymbol{q}$, the position vectors of points P and Q. **2**

(c) Find the size of angle POQ. **5**

2. The diagram shows two right-angled triangles with angles c and d marked as shown.



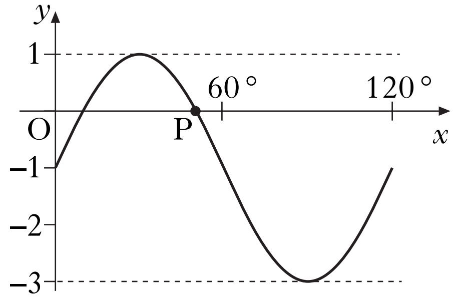
(a) Find the exact value of $\sin(c + d)$. **4**

(b) (i) Find the exact value of $\sin 2c$.

(ii) Show that $\cos 2d$ has the same exact value. **4**

3. Show that the line with equation $y = 6 - 2x$ is a tangent to the circle with equation $x^2 + y^2 + 6x - 4y - 7 = 0$ and find the coordinates of the point of contact of the tangent and the circle. **6**

4. The diagram shows part of the graph of a function whose equation is of the form $y = a\sin(bx°) + c$.

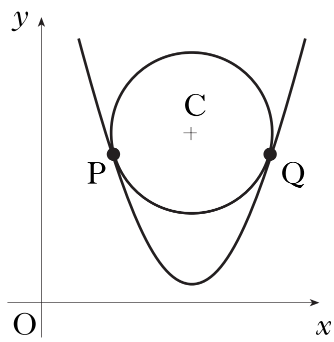


(a) Write down the values of a, b and c. **3**

(b) Determine the exact value of the x-coordinate of P, the point where the graph intersects the x-axis as shown in the diagram. **3**

[Turn over

5. A circle centre C is situated so that it touches the parabola with equation $y = \frac{1}{2}x^2 - 8x + 34$ at P and Q.



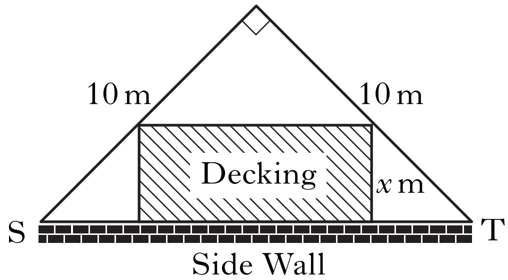
(*a*) The gradient of the tangent to the parabola at Q is 4. Find the coordinates of Q. **5**

(*b*) Find the coordinates of P. **2**

(*c*) Find the coordinates of C, the centre of the circle. **2**

6. A householder has a garden in the shape of a right-angled isosceles triangle.

It is intended to put down a section of rectangular wooden decking at the side of the house, as shown in the diagram.



(*a*) (i) Find the exact value of ST.

(ii) Given that the breadth of the decking is x metres, show that the area of the decking, A square metres, is given by

$$A = \left(10\sqrt{2}\right)x - 2x^2.$$ **3**

(*b*) Find the dimensions of the decking which maximises its area. **5**

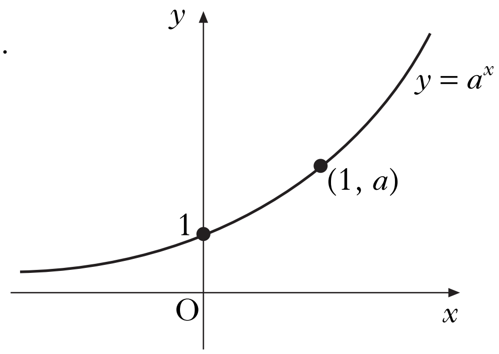
7. Find the value of $\int_0^2 \sin(4x+1)\,dx$. **4**

8. The curve with equation $y = \log_3(x - 1) - 2{\cdot}2$, where $x > 1$, cuts the x-axis at the point $(a, 0)$.

Find the value of a. **4**

Marks

9. The diagram shows the graph of $y = a^x$, $a > 1$.

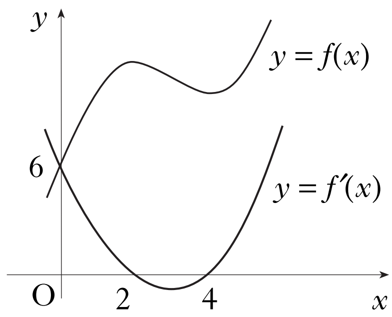


On separate diagrams, sketch the graphs of:

(a) $y = a^{-x}$; **2**

(b) $y = a^{1-x}$. **2**

10. The diagram shows the graphs of a cubic function $y = f(x)$ and its derived function $y = f'(x)$.



Both graphs pass through the point (0, 6).

The graph of $y = f'(x)$ also passes through the points (2, 0) and (4, 0).

(a) Given that $f'(x)$ is of the form $k(x - a)(x - b)$:

(i) write down the values of a and b;

(ii) find the value of k. **3**

(b) Find the equation of the graph of the cubic function $y = f(x)$. **4**

11. Two variables x and y satisfy the equation $y = 3 \times 4^x$.

(a) Find the value of a if (a, 6) lies on the graph with equation $y = 3 \times 4^x$. **1**

(b) If $(-\frac{1}{2}, b)$ also lies on the graph, find b. **1**

(c) A graph is drawn of $\log_{10}y$ against x. Show that its equation will be of the form $\log_{10}y = Px + Q$ and state the gradient of this line. **4**

[*END OF QUESTION PAPER*]

HIGHER 2007 PAPER 1 SOLUTIONS

1. $P(-1,4)$

$$3x - y + 2 = 0$$

$$y = 3x + 2$$

$$\Rightarrow m = 3$$

$$y - b = m(x-a)$$

$$y - 4 = 3(x+1)$$

$$y - 4 = 3x + 3$$

$$y = 3x + 7$$

2.

$BC = 2AB$

$(-2,1,-1)$ $(1,3,2)$

$$\vec{AC} = 3\vec{AB} = 3\begin{pmatrix}3\\2\\3\end{pmatrix} = \begin{pmatrix}9\\6\\9\end{pmatrix} \Rightarrow C(7,7,8)$$

OR

$$\underset{\sim}{c} - \underset{\sim}{a} = 3(\underset{\sim}{b} - \underset{\sim}{a})$$

$$\underset{\sim}{c} = \underset{\sim}{a} + 3\underset{\sim}{b} - 3\underset{\sim}{a}$$

$$\underset{\sim}{c} = 3\underset{\sim}{b} - 2\underset{\sim}{a}$$

3. $f(x) = x^2 + 1$ $g(x) = 1 - 2x$

a)

$$g(f(x)) = g(x^2+1)$$

$$= 1 - 2(x^2+1)$$

$$= 1 - 2x^2 - 2$$

$$= -1 - 2x^2$$

b)

$$g(g(x)) = g(1-2x)$$

$$= 1 - 2(1-2x)$$

$$= 1 - 2 + 4x$$

$$= 4x - 1$$

4. $kx^2 - x - 1 = 0$

$a = k \quad b = -1 \quad c = -1$

for no real roots $b^2 - 4ac < 0$

$b^2 - 4ac = (-1)^2 - 4(k)(-1)$

$= 1 + 4k$

$\Rightarrow \quad 1 + 4k < 0$

$4k < -1$

$k < \frac{-1}{4}$

5. $x^2 + y^2 - 14x - 16y + 77 = 0$

$C(7, 8) \qquad r = \sqrt{49 + 64 - 77}$

$r = \sqrt{36}$

$r = 6$

$\Rightarrow$ radius of circle D $= 2$

Centre of B equal to large circle, ie $(7,8)$

Centre of D is equivalent to 4 radii to the right, ie. 8

$\Rightarrow C_D(15, 8)$

$\Rightarrow$ equation of circle D is $(x-15)^2 + (y-8)^2 = 4$

6. $\sin 2x = 6\cos x^\circ \qquad 0 \leqslant x \leqslant 360$

$$2\sin x \cos x = 6\cos x^\circ$$

$$2\sin x^\circ \cos x^\circ - 6\cos x^\circ = 0$$

$$2\cos x^\circ(\sin x^\circ - 3) = 0$$

either $2\cos x^\circ = 0$ or $\sin x - 3 = 0$

$x = 90, 270 \qquad \sin x = 3$

undefined

$\Rightarrow \quad x \in \{90^\circ, 270^\circ\}$

7. $u_{n+1} = \frac{1}{4}u_n + 16 \qquad u_0 = 0$

a) $u_1 = 16 \qquad u_2 = \frac{1}{4}(16) + 16 \qquad u_3 = \frac{1}{4}(20) + 16$

$= 20 \qquad = 21$

b) (i) A limit exists as $-1 < m < 1$, ie $-1 < \frac{1}{4} < 1$

(ii) $L = \frac{c}{1-m}$ or. at limit $L = \frac{1}{4}L + 16$

$L = \frac{16}{1-1/4} \qquad \frac{3}{4}L = 16$

$L = \frac{16}{3/4} \qquad L = \frac{16}{3/4}$ etc.

$L = \frac{4}{3}.16 \qquad \Rightarrow K = \frac{64}{3}$

$L = \frac{64}{3}$

8. a)

$$\begin{array}{c|cccc} 3 & 1 & -4 & 1 & 6 \\ & & 3 & -3 & -6 \\ \hline & 1 & -1 & -2 & \boxed{0} \end{array}$$

$\Rightarrow x = 3$ is a root

or when $x = 3$,

$y = (3)^3 - 4(3)^2 + (3) + 6$

$y = 27 - 4.9 + 9$

$y = 27 - 36 + 9$

$y = 0$

b) Quotient $= x^2 - x - 2$

$\Rightarrow \quad y = (x-3)(x^2 - x - 2)$

on x-axis, $y = 0$

$\Rightarrow \quad (x-3)(x^2 - x - 2) = 0$

$(x-3)(x-2)(x+1) = 0$

$\Rightarrow$ roots at $x = -1, 2, 3$

$\Rightarrow \quad A(2,0) \qquad (B(3,0))$

c) Shaded area $= \displaystyle\int_0^2 (x^3 - 4x^2 + x + 6)\, dx$

$= \left[\frac{x^4}{4} - \frac{4x^3}{3} + \frac{x^2}{2} + 6x \right]_0^2$

$= \left(\frac{(2)^4}{4} - \frac{4(2)^3}{3} + \frac{(2)^2}{2} + 6(2) \right) - 0$

$= \frac{16}{4} - \frac{32}{3} + \frac{4}{2} + 12$

$= 18 - \frac{32}{3}$

$= \frac{54}{3} - \frac{32}{3}$

$= \frac{22}{3}$ units^2

9. a) $f(x) = 3x - x^3$

on x-axis, $y = 0$

$\Rightarrow \quad 3x - x^3 = 0$

$x(3 - x^2) = 0$

$x = 0 \quad$ or $\quad 3 - x^2 = 0$

$x^2 = 3$

$x = \pm\sqrt{3}$

$(-\sqrt{3}, 0)$, $(0,0)$, $(\sqrt{3}, 0)$

on y-axis, $x = 0$

$\Rightarrow \quad y = 0$

$(0,0)$

b) $f'(x) = 3 - 3x^2$

for stat pts. $f'(x) = 0$

$3 - 3x^2 = 0$

$3x^2 = 3$

$x^2 = 1$

$x = \pm 1$

$f(1) = 3(1) - (1)^3$

$= 3 - 1$

$= 2$

$f(-1) = 3(-1) - (-1)^3$

$= -3 - (-1)$

$= -2$

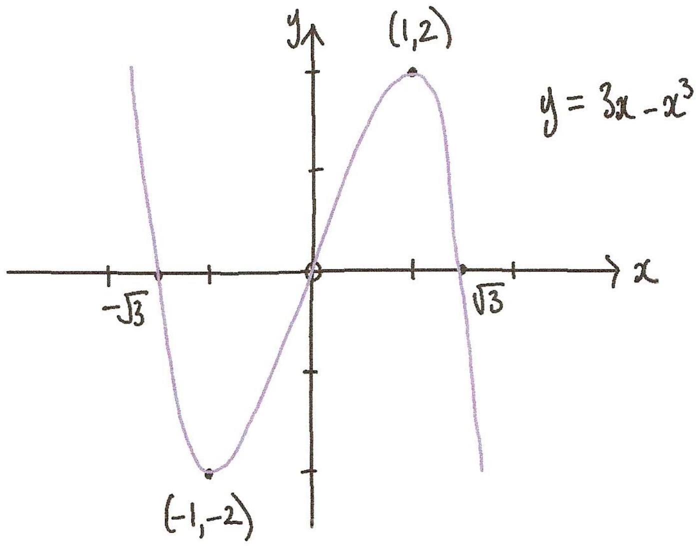
$\Rightarrow$ stat pts at $(-1, -2)$ and $(1, 2)$

x	$\rightarrow$	-1	$\rightarrow$	1	$\rightarrow$
$f'(x)$	$-$	0	$+$	0	$-$
$f(x)$ shape	\	—	/	—	\

Min TP at $(-1, -2)$

Max TP at $(1, 2)$

9. c)



10.

$$y = \sqrt{3x^2 + 2}$$

$$= (3x^2 + 2)^{1/2}$$

$$\frac{dy}{dx} = \frac{1}{2}(3x + 2)^{-1/2} \cdot 6x$$

$$= 3x(3x + 2)^{-1/2}$$

$$= \frac{3x}{\sqrt{3x^2 + 2}}$$

11. a) $f(x) = \sqrt{3}\cos x + \sin x$

$k\cos(x-\alpha) = k\cos x\cos\alpha + k\sin x\sin\alpha$

$k\cos\alpha = \sqrt{3}$

$k\sin\alpha = 1$

$k^2 = (\sqrt{3})^2 + (1)^2$

$k^2 = 4$

$k = 2$

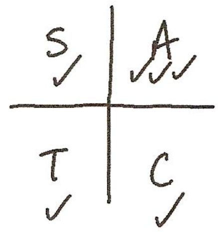
$\tan\alpha = \dfrac{k\sin\alpha}{k\cos\alpha}$

$\tan\alpha = \dfrac{1}{\sqrt{3}}$

acute angle $= \dfrac{\pi}{6}$

α is in 1st quadrant

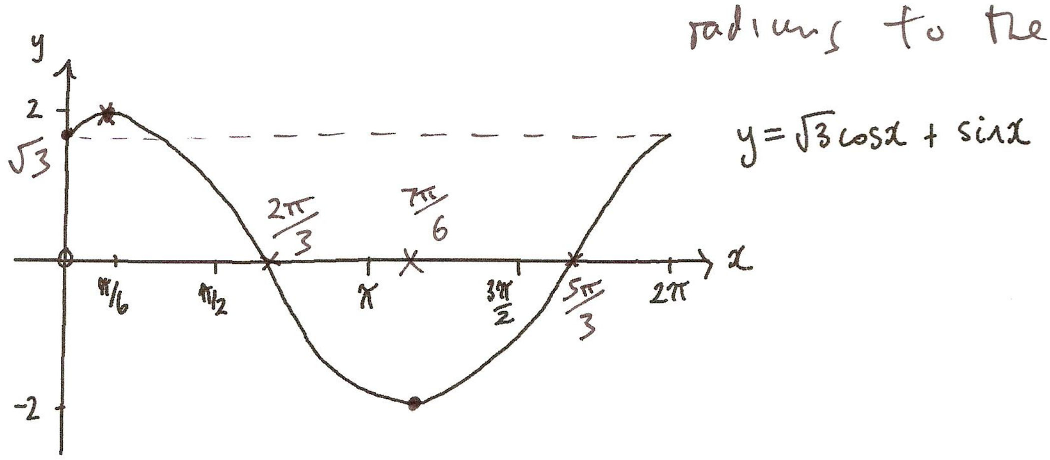
$\Rightarrow \alpha = \dfrac{\pi}{6}$



$\Rightarrow f(x) = 2\cos\left(x - \dfrac{\pi}{6}\right)$

From the $\cos x$ graph, we use a vertical stretch by factor 2 and horizontal shift $\frac{\pi}{6}$ radians to the right

b)



HIGHER 2007 PAPER 2 SOLUTIONS

1. a) $Q(0, 2, 2)$

b) $\underline{p} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$ $\underline{q} = \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$

c) $\underline{p} \cdot \underline{q} = 0.1 + 1.2 + 1.1$

$= 3$

$|\underline{p}| = \sqrt{0^2 + 1^2 + 1^2}$

$= \sqrt{2}$

$|\underline{q}| = \sqrt{1^2 + 2^2 + 1^2}$

$= \sqrt{6}$

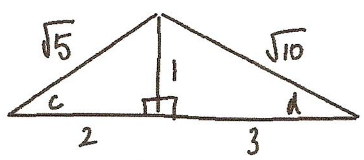
$\underline{p} \cdot \underline{q} = |\underline{p}||\underline{q}| \cos P\hat{O}Q$

$\cos P\hat{O}Q = \dfrac{3}{\sqrt{2}.\sqrt{6}}$

$P\hat{O}Q = \cos^{-1}\left(\dfrac{3}{\sqrt{2}.\sqrt{6}}\right)$

$P\hat{O}Q = 30°$

2. a)



$\sin(c + d) = \sin c \cos d + \cos c \sin d$

$= \dfrac{1}{\sqrt{5}} \cdot \dfrac{3}{\sqrt{10}} + \dfrac{2}{\sqrt{5}} \cdot \dfrac{1}{\sqrt{10}}$

$= \dfrac{3}{\sqrt{50}} + \dfrac{2}{\sqrt{50}}$

$= \dfrac{5}{\sqrt{50}}$

$= \dfrac{5}{5\sqrt{2}}$

$= \dfrac{1}{\sqrt{2}}$

b) (i) $\sin 2c = 2 \sin c \cos c$

$= 2 \cdot \dfrac{1}{\sqrt{5}} \cdot \dfrac{2}{\sqrt{5}}$

$= \dfrac{4}{5}$

(ii) $\cos 2d = 2\cos^2 d - 1$

$= 2\left(\dfrac{3}{\sqrt{10}}\right)^2 - 1$

$= 2 \cdot \dfrac{9}{10} - 1$

$= \dfrac{18}{10} - 1$

$= \dfrac{8}{10}$

$= \dfrac{4}{5}$

* any of 3 formulae will work

3. For points of intersection $y = y$,

$$x^2 + (6-2x)^2 + 6x - 4(6-2x) - 7 = 0$$

$$x^2 + 36 - 24x + 4x^2 + 6x - 24 + 8x - 7 = 0$$

$$5x^2 - 10x + 5 = 0$$

$$x^2 - 2x + 1 = 0$$ * or use discriminant

$$(x-1)(x-1) = 0$$

$$x = 1 \text{ twice}$$

$\Rightarrow$ line is tangent to circle

when $x = 1$,

$$y = 6 - 2(1)$$
$$= 4$$

Point of contact $(1, 4)$

4. a) $y = 2\sin 3x - 1$

$a = 2$, $b = 3$, $c = -1$

b) $2\sin 3x - 1 = 0$

$$\sin 3x = \frac{1}{2}$$

$$3x = 30^\circ, 150^\circ$$

$$x = 10^\circ, 50^\circ, \ldots$$

$\Rightarrow P(50^\circ, 0)$ by inspection

5. a) $y = \frac{1}{2}x^2 - 8x + 34$

$\frac{dy}{dx} = x - 8$

at Q, $m = 4$

$\Rightarrow \quad x - 8 = 4$

$x = 12$

$y = \frac{1}{2}(12)^2 - 8(12) + 34$

$= 72 - 96 + 34$

$= 10$

$\Rightarrow \quad Q(12, 10)$

b) P will have same y-coordinate as Q.

$\Rightarrow \quad \frac{1}{2}x^2 - 8x + 34 = 10$

$\frac{1}{2}x^2 - 8x + 24 = 0$

$x^2 - 16x + 48 = 0$

$(x-4)(x-12) = 0$

$\Rightarrow \quad x = 4, 12$

Hence $P(4, 10)$

c)* $C(8, y)$ by symmetry

gradient of curve at Q = 4

$\Rightarrow \quad m_{CQ} = -\frac{1}{4}$

$-\frac{1}{4} = \frac{10-y}{12-8}$

$-\frac{1}{4} = \frac{10-y}{4}$

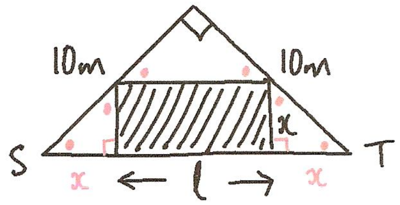
$-1 = 10 - y$

$y = 11$

$\Rightarrow \quad C(8, 11)$

* There are a number of alternative methods for part (c)

6. a)



$\bullet = 45^\circ$

i) $$ST = \sqrt{10^2 + 10^2}$$
$$= \sqrt{200}$$
$$= 10\sqrt{2} \text{ m}$$

ii) $$l = 10\sqrt{2} - 2x$$

$$\text{Area} = xl$$
$$= x(10\sqrt{2} - 2x)$$
$$= (10\sqrt{2})x - 2x^2$$

b) For maximum area, $A' = 0$

$$\Rightarrow \quad 10\sqrt{2} - 4x = 0$$
$$4x = 10\sqrt{2}$$
$$x = \frac{10\sqrt{2}}{4}$$
$$x = \frac{5\sqrt{2}}{2} \text{ m}$$

$$\Rightarrow \quad l = 10\sqrt{2} - 2\left(\frac{5\sqrt{2}}{2}\right)$$
$$= 10\sqrt{2} - 5\sqrt{2}$$
$$= 5\sqrt{2} \text{ m}$$

For maximum area, length $= 5\sqrt{2}$ m and breadth $= \frac{5\sqrt{2}}{2}$ m

7. $\displaystyle\int_0^2 \sin(4x+1)\,dx$

$$= \left[-\frac{\cos(4x+1)}{4}\right]_0^2$$

$$= \left(-\tfrac{1}{4}\cos(9)\right) - \left(-\tfrac{1}{4}\cos(1)\right) \qquad * \text{ radian mode!}$$

$$= 0.363 \quad \text{to } 3\text{ d.p.}$$

8. when $x = a$, $y = 0$

$$\log_3(a-1) - 2.2 = 0$$

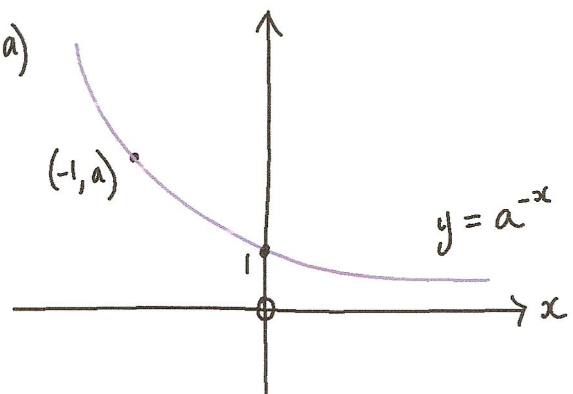
$$\log_3(a-1) = 2.2$$

$$a - 1 = 3^{2.2}$$

$$a = 3^{2.2} + 1$$

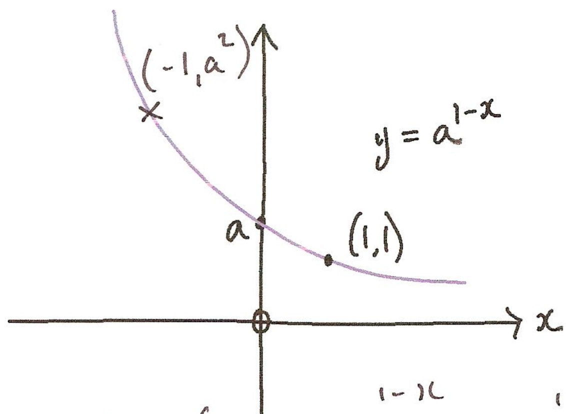
$$a = 12.2 \quad \text{to } 1\text{ d.p.}$$

9. a)



Reflect a^x graph in the y-axis

b)



Think of $y = a^{1-x}$ as $a^1 . a^{-x}$

So $y = a . a^{-x}$

So we get the a^{-x} graph given a "vertical stretch" by a factor of a.

OR use x values: $x = 0$; $a^{1-x} = a$ etc.

$x = 1$, $a^{1-x} = a^0 = 1$

10. a) (i) $f'(x) = k(x-2)(x-4)$

$a = 2, \; b = 4$

(ii) $f'(0) = 6$

$\Rightarrow \quad k(-2)(-4) = 6$

$8k = 6$

$k = \frac{6}{8}$

$k = \frac{3}{4}$

b) $f'(x) = \frac{3}{4}(x-2)(x-4)$

$= \frac{3}{4}(x^2 - 6x + 8)$

$f(x) = \int \frac{3}{4}(x^2 - 6x + 8)\, dx$

$= \frac{3}{4}\left(\frac{x^3}{3} - 3x^2 + 8x\right) + c$

$= \frac{3x^3}{12} - \frac{9x^2}{4} + 6x + c$

$f(0) = 6$

$\Rightarrow \quad c = 6$

$\Rightarrow \quad f(x) = \frac{x^3}{4} - \frac{9x^2}{4} + 6x + 6$

11. a) $y = 3 \times 4^x$

when $x = a$, $y = 6$

$\Rightarrow \quad 6 = 3 \times 4^a$

$2 = 4^a$

$a = \frac{1}{2}$

b) when $x = -\frac{1}{2}$, $y = b$

$\Rightarrow \quad b = 3 \times 4^{-1/2}$

$b = 3 . \frac{1}{\sqrt{4}}$

$b = \frac{3}{2}$

c) $y = 3 \times 4^x$

$\log_{10} y = \log_{10}(3 \times 4^x)$

$\log_{10} y = \log_{10} 3 + \log_{10} 4^x$

$\log_{10} y = x \log_{10} 4 + \log_{10} 3$

where $P = \log_{10} 4$ & $Q = \log_{10} 3$

$\Rightarrow \quad m = \log_{10} 4$

OR Use gradient formula for the gradient of the line joining the 2 points in (a) and (b) with the y-coordinates replaced by $\log_{10} y$.

So, (a) $(\frac{1}{2}, \log_{10} 6)$ (b) $(-\frac{1}{2}, \log_{10} \frac{3}{2})$

So, $m = \dfrac{\log_{10} 6 - \log_{10} \frac{3}{2}}{\frac{1}{2} - (-\frac{1}{2})}$

$= \dfrac{\log_{10}(6 \div \frac{3}{2})}{1} = \log_{10}(6 \times \frac{2}{3})$

$= \log_{10} 4$