# X100/301

NATIONAL QUALIFICATIONS 2008

TUESDAY, 20 MAY
9.00 AM – 10.30 AM

MATHEMATICS
HIGHER
Paper 1
(Non-calculator)

**Read carefully**

**Calculators may <u>NOT</u> be used in this paper.**

**Section A – Questions 1–20 (40 marks)**
Instructions for completion of **Section A** are given on page two.
For this section of the examination you must use an **HB pencil**.

**Section B (30 marks)**

1 Full credit will be given only where the solution contains appropriate working.

2 Answers obtained by readings from scale drawings will not receive any credit.

LI X100/301 6/27670

**Read carefully**

1 Check that the answer sheet provided is for **Mathematics Higher (Section A)**.

2 For this section of the examination you must use an **HB pencil** and, where necessary, an eraser.

3 Check that the answer sheet you have been given has **your name**, **date of birth**, **SCN** (Scottish Candidate Number) and **Centre Name** printed on it.

Do not change any of these details.

4 If any of this information is wrong, tell the Invigilator immediately.

5 If this information is correct, **print** your name and seat number in the boxes provided.

6 The answer to each question is **either** A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space provided (see sample question below).

7 There is **only one correct** answer to each question.

8 Rough working should **not** be done on your answer sheet.

9 At the end of the exam, put the **answer sheet for Section A inside the front cover of your answer book**.

**Sample Question**

A curve has equation $y = x^3 - 4x$.

What is the gradient at the point where $x = 2$?

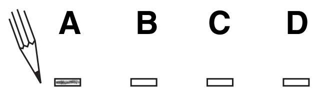
A 8

B 1

C 0

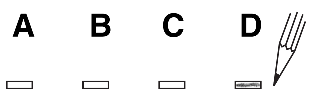
D $-4$

The correct answer is **A—**8. The answer **A** has been clearly marked in **pencil** with a horizontal line (see below).




**Changing an answer**

If you decide to change your answer, carefully erase your first answer and using your pencil, fill in the answer you want. The answer below has been changed to **D**.



**FORMULAE LIST**

**Circle:**

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre $(a, b)$ and radius $r$.

**Scalar Product:** $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos \theta$, where $\theta$ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

**Trigonometric formulae:**

$$\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2\sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

**Table of standard derivatives:**

| $f(x)$ | $f'(x)$ |
|---|---|
| $\sin ax$ | $a \cos ax$ |
| $\cos ax$ | $-a \sin ax$ |

**Table of standard integrals:**

| $f(x)$ | $\int f(x)\,dx$ |
|---|---|
| $\sin ax$ | $-\frac{1}{a}\cos ax + C$ |
| $\cos ax$ | $\frac{1}{a}\sin ax + C$ |

**[Turn over**

## SECTION A

**ALL questions should be attempted.**

**1.** A sequence is defined by the recurrence relation

$$u_{n+1} = 0 \cdot 3u_n + 6 \text{ with } u_{10} = 10.$$

What is the value of $u_{12}$?

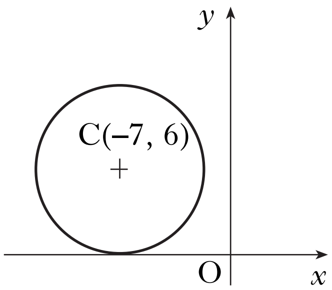
A 6·6

B 7·8

C 8·7

D 9·6

**2.** The $x$-axis is a tangent to a circle with centre (–7, 6) as shown in the diagram.




What is the equation of the circle?

A $(x + 7)^2 + (y - 6)^2 = 1$

B $(x + 7)^2 + (y - 6)^2 = 49$

C $(x - 7)^2 + (y + 6)^2 = 36$

D $(x + 7)^2 + (y - 6)^2 = 36$

**3.** The vectors $\boldsymbol{u} = \begin{pmatrix} k \\ -1 \\ 1 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 0 \\ 4 \\ k \end{pmatrix}$ are perpendicular.

What is the value of $k$?

A 0

B 3

C 4

D 5

**4.** A sequence is generated by the recurrence relation $u_{n+1} = 0{\cdot}4u_n - 240$.

What is the limit of this sequence as $n \to \infty$ ?

A $-800$

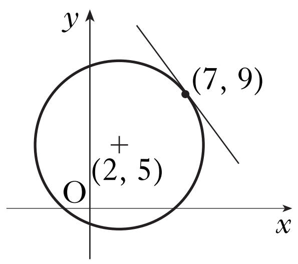
B $-400$

C $200$

D $400$

**5.** The diagram shows a circle, centre (2, 5) and a tangent drawn at the point (7, 9).

What is the equation of this tangent?




A $y - 9 = -\frac{5}{4}(x - 7)$

B $y + 9 = -\frac{4}{5}(x + 7)$

C $y - 7 = \frac{4}{5}(x - 9)$

D $y + 9 = \frac{5}{4}(x + 7)$

**[Turn over**

**6.** What is the solution of the equation $2\sin x - \sqrt{3} = 0$ where $\frac{\pi}{2} \leq x \leq \pi$?

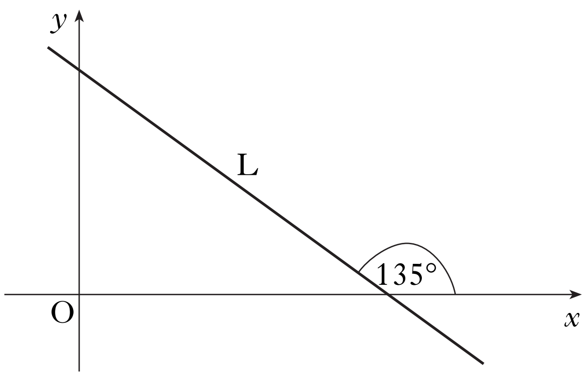
A $\frac{\pi}{6}$

B $\frac{2\pi}{3}$

C $\frac{3\pi}{4}$

D $\frac{5\pi}{6}$

**7.** The diagram shows a line L; the angle between L and the positive direction of the $x$-axis is 135°, as shown.




What is the gradient of line L?

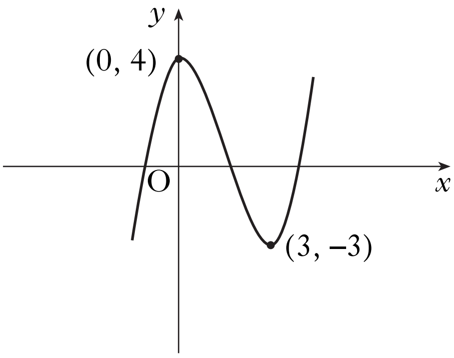
A $-\frac{1}{2}$

B $-\frac{\sqrt{3}}{2}$

C $-1$

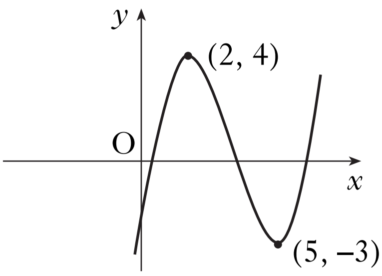
D $\frac{1}{2}$

**8.** The diagram shows part of the graph of a function with equation $y = f(x)$.

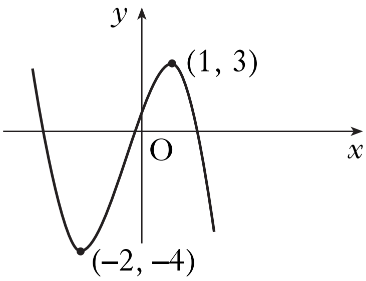



Which of the following diagrams shows the graph with equation $y = -f(x - 2)$?

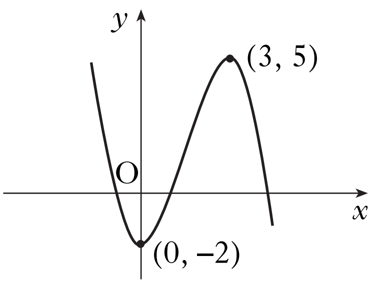
A



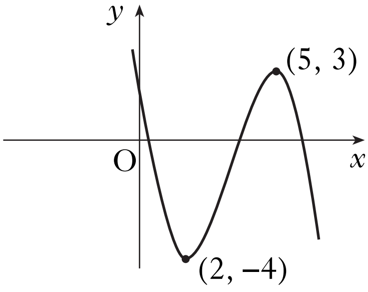

B




C




D



**9.** Given that $0 \le a \le \frac{\pi}{2}$ and $\sin a = \frac{3}{5}$, find an expression for $\sin(x + a)$.

A $\sin x + \frac{3}{5}$

B $\frac{4}{5}\sin x + \frac{3}{5}\cos x$

C $\frac{3}{5}\sin x - \frac{4}{5}\cos x$

D $\frac{2}{5}\sin x - \frac{3}{5}\cos x$

**10.** Here are two statements about the roots of the equation $x^2 + x + 1 = 0$:

(1) the roots are equal;

(2) the roots are real.

Which of the following is true?

A Neither statement is correct.

B Only statement (1) is correct.

C Only statement (2) is correct.

D Both statements are correct.

**11.** E(−2, −1, 4), P(1, 5, 7) and F(7, 17, 13) are three collinear points.

P lies between E and F.

What is the ratio in which P divides EF?

A 1:1

B 1:2

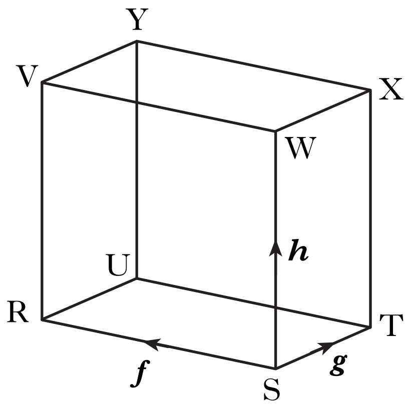
C 1:4

D 1:6

**12.** In the diagram RSTU, VWXY represents a cuboid.

$\overrightarrow{SR}$ represents vector $\boldsymbol{f}$, $\overrightarrow{ST}$ represents vector $\boldsymbol{g}$ and $\overrightarrow{SW}$ represents vector $\boldsymbol{h}$.

Express $\overrightarrow{VT}$ in terms of $\boldsymbol{f}$, $\boldsymbol{g}$ and $\boldsymbol{h}$.




A $\overrightarrow{VT} = \boldsymbol{f} + \boldsymbol{g} + \boldsymbol{h}$

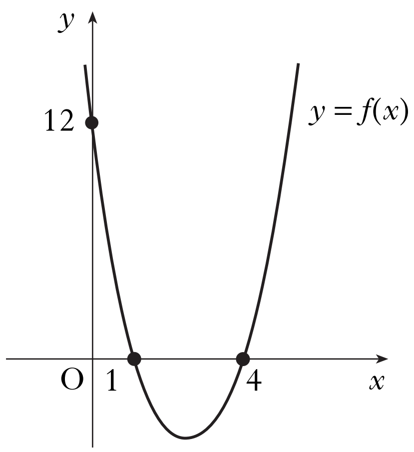
B $\overrightarrow{VT} = \boldsymbol{f} - \boldsymbol{g} + \boldsymbol{h}$

C $\overrightarrow{VT} = -\boldsymbol{f} + \boldsymbol{g} - \boldsymbol{h}$

D $\overrightarrow{VT} = -\boldsymbol{f} - \boldsymbol{g} + \boldsymbol{h}$

**13.** The diagram shows part of the graph of a quadratic function $y = f(x)$.

The graph has an equation of the form $y = k(x - a)(x - b)$.




What is the equation of the graph?

A $y = 3(x - 1)(x - 4)$

B $y = 3(x + 1)(x + 4)$

C $y = 12(x - 1)(x - 4)$

D $y = 12(x + 1)(x + 4)$

**14.** Find $\int 4\sin(2x+3)\,dx$.

A $-4\cos(2x+3)+c$

B $-2\cos(2x+3)+c$

C $4\cos(2x+3)+c$

D $8\cos(2x+3)+c$

**15.** What is the derivative of $(x^3+4)^2$?

A $(3x^2+4)^2$

B $\frac{1}{3}(x^3+4)^3$

C $6x^2(x^3+4)$

D $2(3x^2+4)^{-1}$

**16.** $2x^2+4x+7$ is expressed in the form $2(x+p)^2+q$.

What is the value of $q$?

A 5

B 7

C 9

D 11

**17.** A function $f$ is given by $f(x)=\sqrt{9-x^2}$.

What is a suitable domain of $f$?

A $x \geq 3$

B $x \leq 3$

C $-3 \leq x \leq 3$

D $-9 \leq x \leq 9$

**18.** Vectors $\boldsymbol{p}$ and $\boldsymbol{q}$ are such that $|\boldsymbol{p}| = 3$, $|\boldsymbol{q}| = 4$ and $\boldsymbol{p}.\boldsymbol{q} = 10$.

Find the value of $\boldsymbol{q}.(\boldsymbol{p} + \boldsymbol{q})$.

A 0

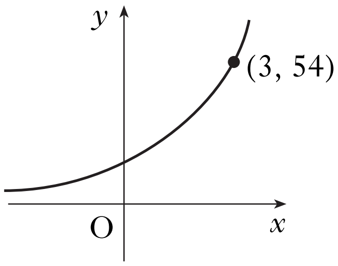
B 14

C 26

D 28

**19.** The diagram shows part of the graph whose equation is of the form $y = 2m^x$.

What is the value of $m$?




A 2

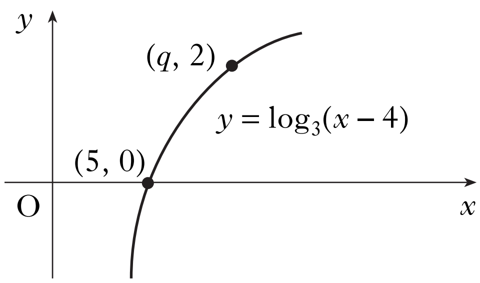
B 3

C 8

D 18

**20.** The diagram shows part of the graph of $y = \log_3(x - 4)$.

The point $(q, 2)$ lies on the graph.




What is the value of $q$?

A 6

B 7

C 8

D 13

[*END OF SECTION A*]

 **[Turn over for Section B on *Page twelve***

## SECTION B

**ALL questions should be attempted.**

*Marks*

**21.** A function $f$ is defined on the set of real numbers by $f(x) = x^3 - 3x + 2$.

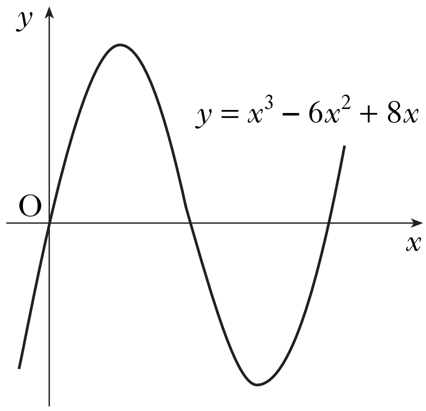
(*a*) Find the coordinates of the stationary points on the curve $y = f(x)$ and determine their nature. **6**

(*b*) (i) Show that $(x - 1)$ is a factor of $x^3 - 3x + 2$.

(ii) Hence or otherwise factorise $x^3 - 3x + 2$ fully. **5**

(*c*) State the coordinates of the points where the curve with equation $y = f(x)$ meets both the axes and hence sketch the curve. **4**

**22.** The diagram shows a sketch of the curve with equation $y = x^3 - 6x^2 + 8x$.




(*a*) Find the coordinates of the points on the curve where the gradient of the tangent is $-1$. **5**

(*b*) The line $y = 4 - x$ is a tangent to this curve at a point A. Find the coordinates of A. **2**

**23.** Functions $f$, $g$ and $h$ are defined on suitable domains by

$$f(x) = x^2 - x + 10,\ g(x) = 5 - x \text{ and } h(x) = \log_2 x.$$

(*a*) Find expressions for $h(f(x))$ and $h(g(x))$. **3**

(*b*) Hence solve $h(f(x)) - h(g(x)) = 3$. **5**

[*END OF SECTION B*]

[*END OF QUESTION PAPER*]

# X100/302

NATIONAL
QUALIFICATIONS
2008

TUESDAY, 20 MAY
10.50 AM – 12.00 NOON

# MATHEMATICS
## HIGHER
### Paper 2

**Read Carefully**

1 **Calculators may be used in this paper.**

2 Full credit will be given only where the solution contains appropriate working.

3 Answers obtained by readings from scale drawings will not receive any credit.

## FORMULAE LIST

**Circle:**

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre $(a, b)$ and radius $r$.

**Scalar Product:** $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos\theta$, where $\theta$ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

**Trigonometric formulae:**

$$\begin{aligned}
\sin(\mathrm{A} \pm \mathrm{B}) &= \sin\mathrm{A}\cos\mathrm{B} \pm \cos\mathrm{A}\sin\mathrm{B} \\
\cos(\mathrm{A} \pm \mathrm{B}) &= \cos\mathrm{A}\cos\mathrm{B} \mp \sin\mathrm{A}\sin\mathrm{B} \\
\sin 2\mathrm{A} &= 2\sin\mathrm{A}\cos\mathrm{A} \\
\cos 2\mathrm{A} &= \cos^2\mathrm{A} - \sin^2\mathrm{A} \\
&= 2\cos^2\mathrm{A} - 1 \\
&= 1 - 2\sin^2\mathrm{A}
\end{aligned}$$

**Table of standard derivatives:**

| $f(x)$ | $f'(x)$ |
|---|---|
| $\sin ax$ | $a\cos ax$ |
| $\cos ax$ | $-a\sin ax$ |

**Table of standard integrals:**

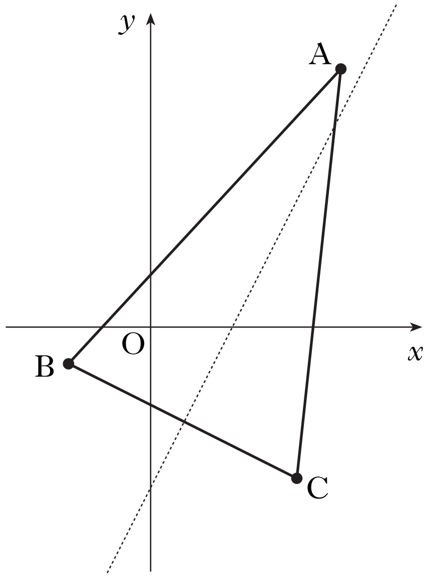
| $f(x)$ | $\int f(x)\,dx$ |
|---|---|
| $\sin ax$ | $-\frac{1}{a}\cos ax + C$ |
| $\cos ax$ | $\frac{1}{a}\sin ax + C$ |

**ALL questions should be attempted.**

*Marks*

**1.** The vertices of triangle ABC are A(7, 9), B(–3, –1) and C(5, –5) as shown in the diagram.

The broken line represents the perpendicular bisector of BC.




(*a*) Show that the equation of the perpendicular bisector of BC is $y = 2x - 5$. **4**

(*b*) Find the equation of the median from C. **3**

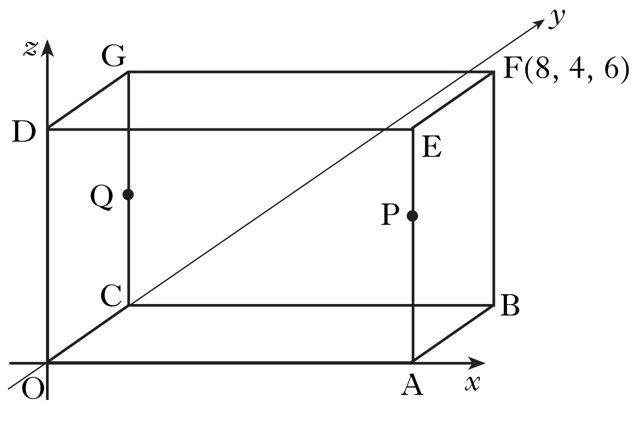
(*c*) Find the coordinates of the point of intersection of the perpendicular bisector of BC and the median from C. **3**

**2.** The diagram shows a cuboid OABC, DEFG.

F is the point (8, 4, 6).

P divides AE in the ratio 2:1.

Q is the midpoint of CG.




(*a*) State the coordinates of P and Q. **2**

(*b*) Write down the components of $\overrightarrow{PQ}$ and $\overrightarrow{PA}$. **2**

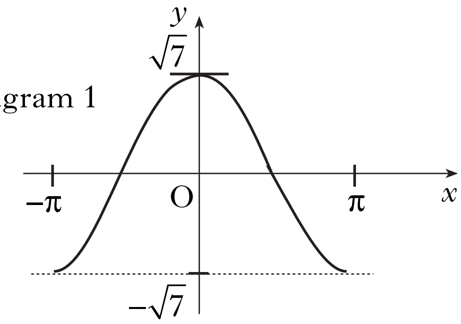
(*c*) Find the size of angle QPA. **5**

**[Turn over**

**3.** (*a*) (i) Diagram 1 shows part of the graph of $y = f(x)$, where $f(x) = p\cos x$.

Write down the value of $p$.

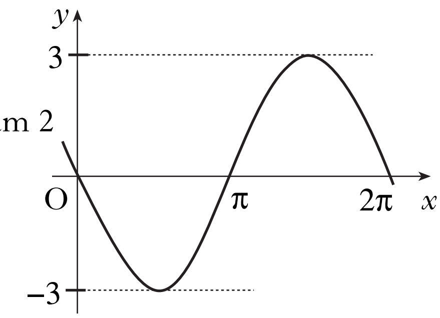
Diagram 1




(ii) Diagram 2 shows part of the graph of $y = g(x)$, where $g(x) = q\sin x$.

Write down the value of $q$. **2**

Diagram 2




(*b*) Write $f(x) + g(x)$ in the form $k\cos(x + a)$ where $k > 0$ and $0 < a < \frac{\pi}{2}$. **4**

(*c*) Hence find $f'(x) + g'(x)$ as a single trigonometric expression. **2**

**4.** (*a*) Write down the centre and calculate the radius of the circle with equation $x^2 + y^2 + 8x + 4y - 38 = 0$. **2**

(*b*) A second circle has equation $(x - 4)^2 + (y - 6)^2 = 26$.

Find the distance between the centres of these two circles and hence show that the circles intersect. **4**

(*c*) The line with equation $y = 4 - x$ is a common chord passing through the points of intersection of the two circles.

Find the coordinates of the points of intersection of the two circles. **5**

**5.** Solve the equation $\cos 2x° + 2\sin x° = \sin^2 x°$ in the interval $0 \leq x < 360$. **5**

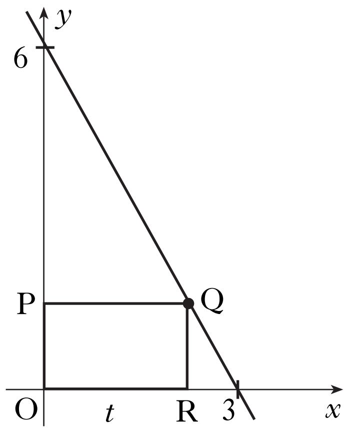
*Marks*

**6.** In the diagram, Q lies on the line joining (0, 6) and (3, 0).

OPQR is a rectangle, where P and R lie on the axes and OR = $t$.

($a$) Show that QR = $6 - 2t$. **3**

($b$) Find the coordinates of Q for which the rectangle has a maximum area. **6**

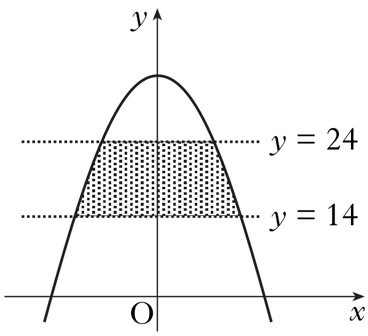



**7.** The parabola shown in the diagram has equation

$$y = 32 - 2x^2.$$

The shaded area lies between the lines $y = 14$ and $y = 24$.

Calculate the shaded area. **8**




[*END OF QUESTION PAPER*]

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# 2008 Mathematics

# Higher – Paper 1 and Paper 2

# Finalised Marking Instructions

1. Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than marks deducted for what is wrong.

2. Award one mark for each 'bullet' point. Each error should be underlined in RED at the point in the working where it first occurs, and not at any subsequent stage of the working.

3. The working subsequent to an error must be followed through by the marker with possible full marks for the subsequent working, provided that the difficulty involved is approximately similar. Where, subsequent to an error, the working is eased, a deduction(s) of mark(s) should be made.
   This may happen where a question is divided into parts. In fact, failure to even answer an earlier section does not preclude a candidate from assuming the result of that section and obtaining full marks for a later section.

4. Correct working should be ticked (√ ). This is essential for later stages of the SQA procedures. Where working subsequent to an error(s) is correct and scores marks, it should be marked with a crossed tick (  or X√ ). In appropriate cases attention may be directed to work which is not quite correct (e.g. bad form) but which has not been penalised, by underlining with a dotted or wavy line.
   Work which is correct but inadequate to score any marks should be corrected with a double cross tick ( ).

5. • The total mark for each section of a question should be entered in red in the **outer** right hand margin, opposite the end of the working concerned.
   • Only the mark should be written, **not** a fraction of the possible marks.
   • These marks should correspond to those on the question paper and these instructions.

6. It is of great importance that the utmost care should be exercised in adding up the marks. Where appropriate, all summations for totals and grand totals must be carefully checked. Where a candidate has scored zero marks for any question attempted, "0" should be shown against the answer.

7. As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Accept answers arrived at by inspection or mentally where it is possible for the answer so to have been obtained. Situations where you may accept such working will normally be indicated in the marking instructions.

8. Do not penalise:
   - working subsequent to a correct answer
   - legitimate variations in numerical answers
   - correct working in the "wrong" part of a question
   - omission of units
   - bad form

9. No piece of work should be scored through without careful checking - even where a fundamental misunderstanding is apparent early in the answer. Reference should always be made to the marking scheme - answers which are widely off-beam are unlikely to include anything of relevance but in the vast majority of cases candidates still have the opportunity of gaining the odd mark or two provided it satisfies the criteria for the mark(s).

10. If in doubt between two marks, give an intermediate mark, but without fractions. When in doubt between consecutive numbers, give the higher mark.

11. In cases of difficulty covered neither in detail nor in principle in the Instructions, attention may be directed to the assessment of particular answers by making a referal to the P.A. Please see the general instructions for P.A. referrals.

12. No marks should be deducted at this stage for careless or badly arranged work. In cases where the writing or arrangement is very bad, a note may be made on the upper left-hand corner of the front cover of the script.

13 Transcription errors: In general, as a consequence of a transcription error, candidates lose the opportunity of gaining either the first ic mark or the first pr mark.

14 Casual errors: In general, as a consequence of a casual error, candidates lose the opportunity of gaining the appropriate ic mark or pr mark.

15 **Do not write any comments on the scripts**. A **revised** summary of acceptable notation is given on page 4.

16 Throughout this paper, unless specifically mentioned, a correct answer with no working receives no credit.

**Summary**

Throughout the examination procedures many scripts are remarked. It is essential that markers follow common procedures:

1 **Tick** correct working.
2 Put a mark in the **outer right-hand margin to match the marks allocations on the question paper.**
3 Do **no**t write marks as fractions.
4 Put each mark **at the end** of the candidate's response to the question.
5 **Follow through** errors to see if candidates can score marks subsequent to the error.
6 Do **not** write any comments on the scripts.

## Higher Mathematics : A Guide to Standard Signs and Abbreviations

**Remember - No comments on the scripts. Please use the following and nothing else.**

### Signs

✓ The tick. You are not expected to tick every line but of course you must check through the whole of a response.

___ ✗ The cross and underline. Underline an error and place a cross at the end of the line.

The tick-cross. Use this to show correct work where you are **following through** subsequent to an error.

∧ The roof. Use this to show something is missing such as a crucial step in a proof or a 'condition' etc.

The tilde. Use this to indicate a minor transgression which is not being penalised (such as bad form).

The double cross-tick. Use this to show correct work but which is inadequate to score any marks. This may happen when working has been eased.

Bullets showing where marks are being allotted may be shown on scripts

margins

$\frac{dy}{dx} = 4x - 7$ ✓ •

$4x - 7 = \underline{0}$ ✗

$x = \frac{7}{4}$

$y = 3\frac{7}{8}$ • 2

$C = \underline{(1,-1)}$ ✗

$m = \frac{3-(-1)}{4-1}$

$m_{rad} = \frac{4}{3}$ •

$m_{tgt} = \frac{-1}{\frac{4}{3}}$

$m_{tgt} = -\frac{3}{4}$ •

$y - 3 = -\frac{3}{4}(x - 2)$ • 3

$x^2 - 3x = 28$ ✓ •

$x = 7$ ∧ 1

$\sin(x) = 0.75 = inv\sin(0.75) = 48.6°$ ✓ • 1

**Remember - No comments on the scripts. No abreviations. No new signs.**
**Please use the above and nothing else.**

All of these are to help us be more consistent and **accurate**.

Note: There is no such thing as a transcription error, a trivial error, a casual error or an insignificant error. These are all mistakes and as a consequence a mark is lost.

Page 5 lists the syllabus coding for each topic. This information is given in the legend underneath the question. The calculator classification is CN(calculator neutral), CR(calculator required) and NC(non-calculator).

Year

| 1 | 2 | | UNIT 1 |
|---|---|---|---|
| | | A1 | determine range/domain |
| | | A2 | recognise general features of graphs:poly,exp,log |
| | | A3 | sketch and annotate related functions |
| | | A4 | obtain a formula for composite function |
| | | A5 | complete the square |
| | | A6 | interpret equations and expressions |
| | | A7 | determine function(poly,exp,log) from graph & vv |
| | | A8 | sketch/annotate graph given critical features |
| | | A9 | interpret loci such as st.lines,para,poly,circle |
| | | A10 | use the notation $u_n$ for the nth term |
| | | A11 | evaluate successive terms of a RR |
| | | A12 | decide when RR has limit/interpret limit |
| | | A13 | evaluate limit |
| | | A14 | apply A10-A14 to problems |
| | | G1 | use the distance formula |
| | | G2 | find gradient from 2 pts,/angle/equ. of line |
| | | G3 | find equation of a line |
| | | G4 | interpret all equations of a line |
| | | G5 | use property of perpendicular lines |
| | | G6 | calculate mid-point |
| | | G7 | find equation of median, altitude,perp. bisector |
| | | G8 | apply G1-G7 to problems eg intersect.,concur.,collin. |
| | | C1 | differentiate sums, differences |
| | | C2 | differentiate negative & fractional powers |
| | | C3 | express in differentiable form and differentiate |
| | | C4 | find gradient at point on curve & vv |
| | | C5 | find equation of tangent to a polynomial/trig curve |
| | | C6 | find rate of change |
| | | C7 | find when curve strictly increasing etc |
| | | C8 | find stationary points/values |
| | | C9 | determinenature of stationary points |
| | | C10 | sketch curvegiven the equation |
| | | C11 | apply C1-C10 to problems eg optimise, greatest/least |
| | | T1 | use gen. features of graphs of $f(x)=k\sin(ax+b)$, |
| | | | $f(x)=k\cos(ax+b)$; identify period/amplitude |
| | | T2 | use radians inc conversion from degrees & vv |
| | | T3 | know and use exact values |
| | | T4 | recognise form of trig. function from graph |
| | | T5 | interpret trig. equations and expressions |
| | | T6 | apply T1-T5 to problems |

| 1 | 2 | | UNIT 2 |
|---|---|---|---|
| | | A15 | use the general equation of a parabola |
| | | A16 | solve a quadratic inequality |
| | | A17 | find nature of roots of a quadratic |
| | | A18 | given nature of roots, find a condition on coeffs |
| | | A19 | form an equation with given roots |
| | | A20 | apply A15-A19 to solve problems |
| | | A21 | use Rem Th. For values, factors, roots |
| | | A22 | solve cubic and quartic equations |
| | | A23 | find intersection of line and polynomial |
| | | A24 | find if line is tangent to polynomial |
| | | A25 | find intersection of two polynomials |
| | | A26 | confiirm and improve on approx roots |
| | | A27 | apply A21-A26 to problems |
| | | G9 | find C/R of a circle from its equation/other data |
| | | G10 | find the equation of a circle |
| | | G11 | find equation of a tangent to a circle |
| | | G12 | find intersection of line & circle |
| | | G13 | find if/when line is tangent to circle |
| | | G14 | find if two circles touch |
| | | G15 | apply G9-G14 to problems |
| | | C12 | find integrals of $px^n$ and sums/diffs |
| | | C13 | integrate with negative & fractional powers |
| | | C14 | express in integrable form and integrate |
| | | C15 | evaluate definite integrals |
| | | C16 | find area between curve and x-axis |
| | | C17 | find area between two curves |
| | | C18 | solve differential equations(variables separable) |
| | | C19 | apply C12-C18 to problems |
| | | T7 | solve linear & quadratic equations in radians |
| | | T8 | apply compound and double angle (c & da) formulae |
| | | | in numerical & literal cases |
| | | T9 | apply c & da formulae in geometrical cases |
| | | T10 | use c & da formulaewhen solving equations |
| | | T11 | apply T7-T10 to problems |

| 1 | 2 | | UNIT 3 |
|---|---|---|---|
| | | A28 | use the laws of logs to simplify/find equiv. expression |
| | | A29 | sketch associated graphs |
| | | A30 | solve equs of the form $A = Be^{kt}$ for A,B,k or t |
| | | A31 | solve equs of the form $\log_b(a) = c$ for a,b or c |
| | | A32 | solve equations involving logarithms |
| | | A33 | use relationships of the form $y = ax^n$ or $y = ab^x$ |
| | | A34 | apply A28-A33 to problems |
| | | G16 | calculate the length of a vector |
| | | G17 | calculate the 3rd given two from A,B and vector AB |
| | | G18 | use unit vectors |
| | | G19 | use: if $\mathbf{u}$, $\mathbf{v}$ are parallel then $\mathbf{v} = k\mathbf{u}$ |
| | | G20 | add, subtract, find scalar mult. of vectors |
| | | G21 | simplify vector pathways |
| | | G22 | interpret 2D sketches of 3D situations |
| | | G23 | find if 3 points in space are collinear |
| | | G24 | find ratio which one point divides two others |
| | | G25 | given a ratio, find/interpret 3rd point/vector |
| | | G26 | calculate the scalar product |
| | | G27 | use: if $\mathbf{u}$, $\mathbf{v}$ are perpendicular then $\mathbf{v.u=0}$ |
| | | G28 | calculate the angle between two vectors |
| | | G29 | use the distributive law |
| | | G30 | apply G16-G29 to problems eg geometry probs. |
| | | C20 | differentiate $p\sin(ax+b)$, $p\cos(ax+b)$ |
| | | C21 | differentiate using the chain rule |
| | | C22 | integrate $(ax + b)^n$ |
| | | C23 | integrate $p\sin(ax+b)$, $p\cos(ax+b)$ |
| | | C24 | apply C20-C23 to problems |
| | | T12 | solve sim.equs of form $k\cos(a)=p$, $k\sin(a)=q$ |
| | | T13 | express $p\cos(x)+q\sin(x)$ in form $k\cos(x\pm a)$ etc |
| | | T14 | find max/min/zeros of $p\cos(x)+q\sin(x)$ |
| | | T15 | sketch graph of $y=p\cos(x)+q\sin(x)$ |
| | | T16 | solve equ of the form $y=p\cos(rx)+q\sin(rx)$ |
| | | T17 | apply T12-T16 to problems |

# 2008 Higher Mathematics
# Paper 1 Section A

| | |
|---|---|
| 1 | C |
| 2 | D |
| 3 | C |
| 4 | B |
| 5 | A |
| 6 | B |
| 7 | C |
| 8 | D |
| 9 | B |
| 10 | A |
| 11 | B |
| 12 | C |
| 13 | A |
| 14 | B |
| 15 | C |
| 16 | A |
| 17 | C |
| 18 | C |
| 19 | B |
| 20 | D |

# 1.21

| QU | part | mk | code | calc | source | ss | pd | ic | C | B | A | | U1 | U2 | U3 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.21 | a | 6 | C8,C9 | NC | | 1 | 3 | 2 | 6 | | | | 6 | | |
| | b | 5 | A21,A22 | | | 1 | 3 | 1 | 5 | | | | | 5 | |
| | c | 4 | C10 | | | | | 4 | 2 | 2 | | | 4 | | |

A function $f$ is defined on the set of real numbers by $f(x) = x^3 - 3x + 2$.

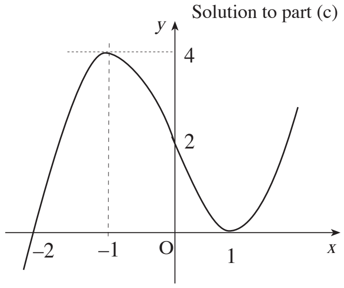
$(a)$ Find the coordinates of the stationary points on the curve $y = f(x)$ and determine their nature. 6

$(b)$ (i) Show that $(x-1)$ is a factor of $x^3 - 3x + 2$.

(ii) Hence or otherwise factorise $x^3 - 3x + 2$ fully. 5

$(c)$ State the coordinates of the points where the curve with equation $y = f(x)$ meets both the axes and hence sketch the curve. 4

Solution to part (c)




The primary method is based on this generic marking scheme which may be used as a guide for any method not shown in detail.

**Generic Marking Scheme**

| | | |
|---|---|---|
| •1 | ss | set derivative to zero |
| •2 | pd | differentiate |
| •3 | pd | solve |
| •4 | pd | evaluate $y$-coordinates |
| •5 | ic | justification |
| •6 | ic | state conclusions |
| •7 | ss | know to use $x = 1$ |
| •8 | pd | complete eval. & conclusion |
| •9 | ic | start to find quadratic factor |
| •10 | pd | complete quadratic factor |
| •11 | pd | factorise completely |
| •12 | ic | interpret $y$-intercept |
| •13 | ic | interpret $x$-intercepts |
| •14 | ic | sketch : showing turning points |
| •15 | ic | sketch : showing intercepts |

**Primary Method : Give 1 mark for each •**

•1 $f'(x) = 0$

•2 $3x^2 - 3$

•3 •4

$$\begin{array}{c|c|c|} & \bullet^3 & \bullet^4 \\ x & -1 & 1 \\ y & 4 & 0 \end{array}$$

•5 •6

$$\begin{array}{c|ccc|ccc|} & & \bullet^5 & & & \bullet^6 & \\ & \dots & -1 & \dots & \dots & 1 & \dots \\ f' & + & 0 & - & - & 0 & + \\ & \text{max} & at\, x = & -1 & \text{min} & at\, x = & 1 \end{array}$$

•7 know to use $x = 1$

•8 $1 - 3 + 2 = 0 \Rightarrow x - 1$ is a factor

•9 $(x-1)(x^2 ...)$

•10 $(x-1)(x^2 + x - 2)$

•11 $(x-1)(x-1)(x+2)$ ***stated explicitly***

•12 $(0,2)$

•13 $(-2,0)$, $(1,0)$

•14 Sketch with turning pts marked

•15 Sketch with $(0,2)$ or $(-2,0)$

**Notes**

1 The "=0" shown at •1 must appear at least once before the •3 stage.

2 An unsimplified $\sqrt{1}$ should be penalised at the first occurrence.

3 •3 is only available as a consequence of solving $f'(x) = 0$.

4 The nature table must reflect previous working from •3.

5 Candidates who introduce an extra solution at the •3 stage cannot earn •3.

6 The use of the 2nd derivative is an acceptable strategy for •5.

7 As shown in the Primary Method, (•3 and •4) and (•5 and •6) can be marked in series or in parallel.

8 The working for (b) may appear in (a) or vice versa. Full marks are available wherever the working occurs.

**Notes**

9 In Primary method •8 and alternative •9, candidates must show some acknowledgement of the resulting "0". Although a statement wrt the zero is preferable, accept something as simple as "underlining the zero".

**Alternative Method : •7 to •10**

•7
$$\begin{array}{c|cccc} 1 & 1 & 0 & -3 & 2 \\ \hline & & & & \end{array}$$

•8
$$\begin{array}{c|cccc} 1 & 1 & 0 & -3 & 2 \\ & & 1 & 1 & -2 \\ \hline & 1 & 1 & -2 & 0 \end{array}$$

•9 $f(1) = 0$ *so* $(x-1)$ *is a factor*

•10 $x^2 + x - 2$

**Notes**

10 Evidence for •12 and •13 may not appear until the sketch.

11 •14 and •15 are only available for the graph of a cubic.

**Nota Bene**

For candidates who omit the $x^2$ coeff. leading to

•7 $X$

•8 $\checkmark$
$$\begin{array}{c|ccc} 1 & 1 & -3 & 2 \\ & & 1 & -2 \\ \hline & 1 & -2 & 0 \end{array}$$

•9 $\checkmark$ $f(1) = 0$ *so* $(x-1)$.........

•10 $X$ $x^2 - 2x$

•11 $\checkmark$ $x(x-1)(x-2)$

***but***

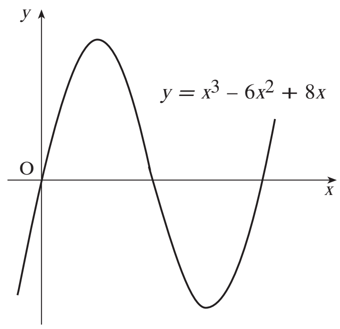
•10 $X$ $x - 2$

•11 $X$ $(x-1)(x-2)$

# 1.22

| qu | part | mk | code | calc | source | ss | pd | ic | C | B | A | | U1 | U2 | U3 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.22 | a | 5 | C4 | NC | | 2 | 3 | | 5 | | | | 5 | | |
| | b | 2 | C11 | | | 1 | | 1 | 2 | | | | | 2 | |

The diagram shows a sketch of the curve with equation $y = x^3 - 6x^2 + 8x$.




$(a)$ Find the coordinates of the points on the curve where the gradient of the tangent is $-1$. 5

$(b)$ The line $y = 4 - x$ is a tangent to this curve at a point A. Find the coordinates of A. 2

The primary method is based on this generic marking scheme which may be used as a guide for any method not shown in detail.

**Generic Marking Scheme**

| | | |
|---|---|---|
| $\bullet^1$ | ss | know to differentiate |
| $\bullet^2$ | pd | differentiate |
| $\bullet^3$ | ss | set derivative to $-1$ |
| $\bullet^4$ | pd | factorise and solve |
| $\bullet^5$ | pd | solve for $y$ |
| $\bullet^6$ | ss | use gradient |
| $\bullet^7$ | ic | interpret result |

**Primary Method : Give 1 mark for each •**

$\bullet^1$ $\frac{dy}{dx} = ..(1\ term\ correct)\quad s\,/\,i\ by\ \bullet^2$

$\bullet^2$ $3x^2 - 12x + 8 \quad s\,/\,i\ by\ \bullet^3$

$\bullet^3$ $3x^2 - 12x + 8 = -1$

| | | $\bullet^4$ | $\bullet^5$ |
|---|---|---|---|
| $\bullet^4$ | $x$ | 1 | 3 |
| $\bullet^5$ | $y$ | 3 | $-3$ |

$\bullet^6$ $y = 4 - x$ has gradient $= -1$

$\bullet^7$ *check* $(3,-3)$ and reject
*check* $(1,3)$ and accept

**Notes**

1 *in* $(a)$

$\bullet^1$ $\surd$ $\frac{dy}{dx} = ..(1\ term\ correct)$

$\bullet^2$ $\surd$ $3x^2 - 12x + 8$

For candidates who now guess $x = 1$ and check that $\frac{dy}{dx} = -1$, only one further mark ($\bullet^3$) can be awarded. Guessing and checking further answers gains no more credit.

2 An "=0" must appear at least once in the two lines shown in the alternative for $\bullet^6$ and $\bullet^7$.

**Common Error**

$\bullet^1$ $\surd$ $\frac{dy}{dx} = ..(1\ term\ correct)$

$\bullet^2$ $\surd$ $3x^2 - 12x + 8$

$\bullet^3$ $X$ $3x^2 - 12x + 8 = 0$

$\bullet^4$ $X$ irrespective of what is written.

$\bullet^5$ $X$

**Alternative for $\bullet^6$ and $\bullet^7$**

$\bullet^6$ $\begin{cases} x^3 - 6x^2 + 8x = 4 - x \\ x^3 - 6x^2 + 9x - 4 = 0 \\ (x-1)(x^2 - 5x + 4) \\ \quad (x-4)(x-1) \end{cases}$

$\bullet^7$ $\begin{cases} \text{repeated root implies} \\ \text{tangent at } (1,3). \end{cases}$

# 1.23

| qu | part | mk | A3 | calc | source | ss | pd | ic | C | B | A | | U1 | U2 | U3 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1.23 | a | 3 | A4 | NC | | | | 3 | 3 | | | | 3 | | |
| | b | 5 | A31 | | | 2 | 2 | 1 | | 1 | 4 | | | | 5 |

Functions $f, g$ and $h$ are defined on suitable domains by $f(x) = x^2 - x + 10$, $g(x) = 5 - x$ and $h(x) = \log_2 x$.

$(a)$ Find expressions for $h\big(f(x)\big)$ and $h\big(g(x)\big)$. 3

$(b)$ Hence solve $h\big(f(x)\big) - h\big(g(x)\big) = 3$ 5

The primary method is based on this generic marking scheme which may be used as a guide for any method not shown in detail.

**Generic Marking Scheme**

- $\bullet^1$ ic interpretation composition
- $\bullet^2$ ic interpretation composition
- $\bullet^3$ ic interpretation composition
- $\bullet^4$ ss use log laws
- $\bullet^5$ ss convert to exponential form
- $\bullet^6$ pd process conversion
- $\bullet^7$ pd express in standard form
- $\bullet^8$ ic find valid solutions

**Primary Method : Give 1 mark for each •**

- $\bullet^1$ $h\big(f(x)\big) = h\big(x^2 - x + 10\big)$ $s\,/\,i\ by\ \bullet^2$
- $\bullet^2$ $\log_2\left(x^2 - x + 10\right)$
- $\bullet^3$ $\log_2(5 - x)$
- $\bullet^4$ $\log_2\left(\dfrac{x^2 - x + 10}{5 - x}\right)$
- $\bullet^5$ $\dfrac{x^2 - x + 10}{5 - x} = 2^3$
- $\bullet^6$ $x^2 - x + 10 = 8(5 - x)$
- $\bullet^7$ $x^2 + 7x - 30 = 0$
- $\bullet^8$ $x = 3, \ -10$

**Notes**

1. In (a) 2 marks are available for finding one of $h(f(x))$ or $h(g(x))$ and the third mark is for the other.
2. Treat $\log_2 x^2 - x + 10$ *and* $\log_2 5 - x$ as bad form.
3. The omission of the base should not be penalised in $\bullet^2$ to $\bullet^4$.
4. $\bullet^7$ is only available for a quadratic equation and $\bullet^8$ must be the follow-through solutions.

**Common Error 1**

- $\bullet^4$ $X$ $\log_2(x^2 + 5) = 3$
- $\bullet^5$ $\checkmark$ $x^2 + 5 = 2^3$
- $\bullet^6$ $X$ $x^2 = 3$
- $\bullet^7$ $X$ $x = \pm\sqrt{3}$
- $\bullet^8$ $X$ not available

**Common Error 2**

- $\bullet^4$ $\checkmark$ $\log_2\left(\dfrac{x^2 - x + 10}{5 - x}\right)$

  $\log_2\left(\dfrac{x^2 \cancel{-} \cancel{x} + \cancel{10}}{\cancel{5} \cancel{-} \cancel{x}}\right)$

  $\log_2\left(x^2 + 2\right) = 3$
- $\bullet^5$ $X\checkmark$ $x^2 + 2 = 2^3$
- $\bullet^6$ $X$ $x = \pm\sqrt{6}$
- $\bullet^7$ $X$ *not available*
- $\bullet^8$ $X$ *not available*

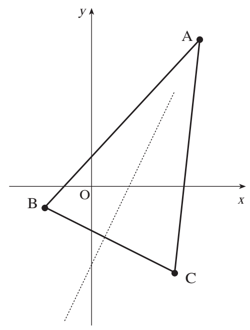
**Common Error 3**

- $\bullet^4$ $X$ not available
- $\bullet^5$ $\checkmark$ $\log_2(x^2 - x + 10) - \log_2(5 - x) = \log_2 8$
- $\bullet^6$ $X$ $x^2 - x + 10 - (5 - x) = 8$
- $\bullet^7$ $X$ not available
- $\bullet^8$ $X$ not available

# 2.01

| qu | part | mk | code | calc | source | ss | pd | ic | C | B | A | U1 | U2 | U3 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2.01 | a | 4 | G7 | CN | | 2 | | 2 | 4 | | | 4 | | |
| | b | 3 | G7 | CN | | 1 | 1 | 1 | 3 | | | 3 | | |
| | c | 3 | C8 | CN | | 1 | 2 | | 3 | | | 3 | | |

The vertices of triangle ABC are A(7, 9), B(–3, –1) and C(5, –5) as shown in the diagram.
The broken line represents the perpendicular bisector of BC.




$(a)$ Show that the equation of the perpendicular bisector of BC is $y = 2x - 5$. 4

$(b)$ Find the equation of the median from C. 3

(c) Find the coordinates of the point of intersection of the perpendicular bisector of BC and the median from C. 3

The primary method is based on this generic marking scheme which may be used as a guide for any method not shown in detail.

**Generic Marking Scheme**

| | | |
|---|---|---|
| $\bullet^1$ | ss | know and find gradient |
| $\bullet^2$ | ic | interpret perpendicular gradient |
| $\bullet^3$ | ss | know and find midpoint |
| $\bullet^4$ | ic | complete proof |
| $\bullet^5$ | ss | know and find midpoint |
| $\bullet^6$ | pd | calculate gradient |
| $\bullet^7$ | ic | state equation |
| $\bullet^8$ | ss | start to solve sim. equations |
| $\bullet^9$ | pd | find one variable |
| $\bullet^{10}$ | pd | find other variable |

**Primary Method : Give 1 mark for each •**

| | | |
|---|---|---|
| $\bullet^1$ | $m_{BC} = -\frac{1}{2}$ | ***stated explicitly*** |
| $\bullet^2$ | $m_{\perp} = 2$ | *stated / implied by* $\bullet^4$ |
| $\bullet^3$ | midpoint of BC = (1, –3) | |
| $\bullet^4$ | $y + 3 = 2(x - 1)$ and complete | |
| $\bullet^5$ | midpoint of AB = (2,4) | |
| $\bullet^6$ | $m_{\text{median}} = -3$ | |
| $\bullet^7$ | $y + 5 = -3(x - 5)$ *or* $y - 4 = -3(x - 2)$ | |
| $\bullet^8$ | use $y = 2x - 5$, $y = -3x + 10$ | |
| $\bullet^9$ | $x = 3$ | |
| $\bullet^{10}$ | $y = 1$ | |

**Notes**

In (a)

1 $\bullet^4$ is only available as a consequence of attempting to find and use both a perpendicular gradient and a midpoint.

2 To gain $\bullet^4$ some evidence of completion needs to be shown.
The minimum requirements for this evidence is as shown:

$$y + 3 = 2(x - 1)$$
$$y + 3 = 2x - 2$$
$$y = 2x - 5$$

3 $\bullet^4$ is only available for completion to $y = 2x - 5$ and nothing else.

4 Alternative for $\bullet^4$ :
$\bullet^4$ may be obtained by using $y = mx + c$

**Notes**

In (b)

5 $\bullet^7$ is only available as a consequence of finding the gradient via a midpoint.

6 For candidates who find the equation of the perpendicular bisector of AB, only $\bullet^5$ is available.

In (c)

7 $\bullet^8$ is a strategy mark for juxtaposing the two correctly rearranged equations.

**Follow - throughs**

Note that from an incorrect equation in (b), full marks are still available in (c). Please follow-through carefully.

**Cave**

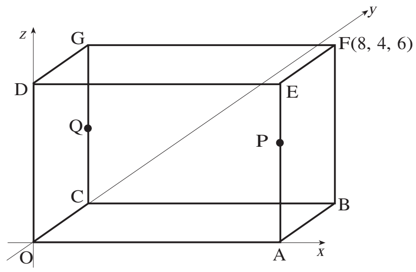
Candidates who find the median, angle bisector or altitude need to show the triangle is isosceles to gain full marks in (a).
For those candidates who do not justify the isosceles triangle, marks may be allocated as shown below:

| | Altitude | Median |
|---|---|---|
| $\bullet^1$ | √ | √ |
| $\bullet^2$ | √ | X |
| $\bullet^3$ | X | √ |
| $\bullet^4$ | X | X |

# 2.02

| qu | part | mk | code | calc | source | ss | pd | ic | C | B | A | | U1 | U2 | U3 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2.02 | a | 2 | G25 | CN | 8202 | | | 2 | 2 | | | | | | 2 |
| | b | 2 | G25 | CN | | | 1 | 1 | 2 | | | | | | 2 |
| | c | 5 | G28 | CR | | 1 | 4 | | 5 | | | | | | 5 |

The diagram shows a cuboid OABC,DEFG.
F is the point (8, 4, 6).
P divides AE in the ratio 2:1.
Q is the midpoint of CG.




(*a*) State the coordinates of P and Q. 2

(*b*) Write down the components of $\overrightarrow{PQ}$ and $\overrightarrow{PA}$. 2

(*c*) Find the size of angle QPA. 5

The primary method is based on this generic marking scheme which may be used as a guide for any method not shown in detail.

**Generic Marking Scheme**

| | | |
|---|---|---|
| $\bullet^1$ | ic | interpret ratio |
| $\bullet^2$ | ic | interpret ratio |
| $\bullet^3$ | pd | process vectors |
| $\bullet^4$ | ic | interpret diagram |
| $\bullet^5$ | ss | know to use scalar product |
| $\bullet^6$ | pd | find scalar product |
| $\bullet^7$ | pd | find magnitude of vector |
| $\bullet^8$ | pd | find magnitude of vector |
| $\bullet^9$ | pd | evaluate angle |

**Primary Method : Give 1 mark for each •**

$\bullet^1$ P = (8, 0, 4)

$\bullet^2$ Q = (0, 4, 3)

$\bullet^3$ $\overrightarrow{PQ} = \begin{pmatrix}-8\\4\\-1\end{pmatrix}$

$\bullet^4$ $\overrightarrow{PA} = \begin{pmatrix}0\\0\\-4\end{pmatrix}$

$\bullet^5$ $\cos QPA = \dfrac{\overrightarrow{PQ}.\overrightarrow{PA}}{|\overrightarrow{PQ}||\overrightarrow{PA}|}$ *stated / implied by* $\bullet^9$

$\bullet^6$ $\overrightarrow{PQ}.\overrightarrow{PA} = 4$

$\bullet^7$ $|\overrightarrow{PQ}| = \sqrt{81}$

$\bullet^8$ $|\overrightarrow{PA}| = \sqrt{16}$

$\bullet^9$ 83·6°, 1.459 *radians*, 92.9 *gradians*

**Notes**

1 Treat coordinates written as column vectors as bad form.

2 Treat column vectors written as coordinates as bad form.

3 For candidates who do not attempt $\bullet^9$, the formula quoted at $\bullet^5$ must relate to the labelling in order for $\bullet^5$ to be awarded.

4 Candidates who evaluate $P\hat{O}Q$ correctly gain 4/5 marks in (c) (74° or 75°)

**Exemplar 1**

$\bullet^3,\bullet^4$ $X, X$ $\overrightarrow{OA} = \begin{pmatrix}8\\0\\0\end{pmatrix}$ $\overrightarrow{OQ} = \begin{pmatrix}0\\4\\3\end{pmatrix}$

$\bullet^5$ $X$ $\cos AOQ = \dfrac{\overrightarrow{OA}.\overrightarrow{OQ}}{|\overrightarrow{OA}||\overrightarrow{OQ}|}$

$\bullet^6$ $\surd$ $\overrightarrow{OA}.\overrightarrow{OQ} = 0$

$\bullet^7$ $\surd$ $|\overrightarrow{OA}| = \sqrt{64}$

$\bullet^8$ $\surd$ $|\overrightarrow{OQ}| = \sqrt{25}$

$\bullet^9$ $\surd$ 90°

**Exemplar 2**

$\bullet^3,\bullet^4$ $X, X$ $\overrightarrow{OA} = \begin{pmatrix}8\\0\\0\end{pmatrix}$ $\overrightarrow{OQ} = \begin{pmatrix}0\\4\\3\end{pmatrix}$

$\bullet^6$ $\surd$ $\overrightarrow{OA}.\overrightarrow{OQ} = 0$

$\bullet^9$ $\surd$ 90°

**Alternative for $\bullet^5$ to $\bullet^8$**

$\bullet^5$ $\cos QPA = \dfrac{PA^2 + PQ^2 - QA^2}{2PA \times PQ}$

$\bullet^6$ $|\overrightarrow{PA}| = \sqrt{16}$

$\bullet^7$ $|\overrightarrow{PQ}| = \sqrt{81}$

$\bullet^8$ $|\overrightarrow{QA}| = \sqrt{89}$

# 2.03

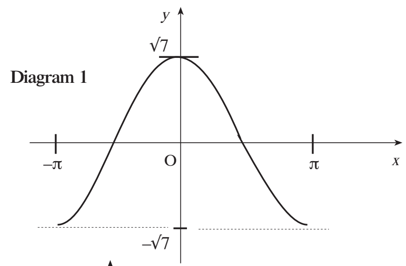
| qu | part | 2 | code | calc | source | ss | pd | ic | C | B | A | | U1 | U2 | U3 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2.03 | a | 2 | T4 | CN | 8203 | | | 2 | 2 | | | | 2 | | |
| | b | 4 | T13 | CR | | 1 | 2 | 1 | 4 | | | | | | 4 |
| | c | 2 | C20 | CN | | | 1 | 1 | 1 | 1 | | | | | 2 |

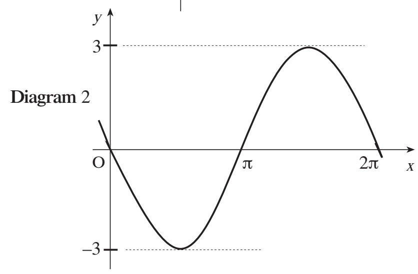
(*a*) (i) Diagram 1 shows part of the graph of $y = f(x)$, where $f(x) = p\cos x$.
Write down the value of $p$.

(ii) Diagram 2 shows part of the graph of $y = g(x)$, where $g(x) = q\sin x$.
Write down the value of $q$. 2

(*b*) Write $f(x) + g(x)$ in the form $k\cos(x + a)$ where $k > 0$ and $0 < a < \frac{\pi}{2}$. 4

(*c*) Hence find $f'(x) + g'(x)$ as a single trigonometric expression. 2




The primary method is based on this generic marking scheme which may be used as a guide for any method not shown in detail.

| Generic Marking Scheme | | | Primary Method : Give 1 mark for each • |
|---|---|---|---|
| •1 | ic | interpret graph | $p = \sqrt{7}$ |
| •2 | ic | interpret graph | $q = -3$ |
| •3 | ss | expand | $k\cos x\cos a - k\sin x\sin a$ ***stated explicitly*** |
| •4 | ic | compare coefficients | $k\cos a = \sqrt{7}$ ***and*** $k\sin a = 3$ ***stated explicitly*** |
| •5 | pd | process "$k$" | $k = 4$ |
| •6 | pd | process "$a$" | $a \approx 0{\cdot}848$ |
| •7 | ss | state equation | $4\cos(x + 0{\cdot}848)$ |
| •8 | pd | differentiate | $-4\sin(x + 0{\cdot}848)$ |

**Notes**

In (a)

1 For $\bullet^1$ accept $p = 2.6$ leading to $k = 4.0$, $a = 0.86$ in (b).

In (b)

2 $k(\cos x\cos a - \sin x\sin a)$ is acceptable for $\bullet^3$.

3 Treat $k\cos x\cos a - \sin x\sin a$ as bad form only if the equations at the $\bullet^4$ stage both contain $k$.

4 $4(\cos x\cos a - \sin x\sin a)$ is acceptable for $\bullet^3$ and $\bullet^5$.

5 $k = \sqrt{16}$ does not earn $\bullet^5$.

6 No justification is needed for $\bullet^5$.

7 Candidates may use any form of wave equation as long as their final answer is in the form $k\cos(x + a)$. If not, then $\bullet^6$ is not available.

**Notes**

8 Candidates who use degrees throughout this question lose $\bullet^6$, $\bullet^7$ and $\bullet^8$.

**Common Error 1**

**(*sic*)**

$q = 3 \quad \Rightarrow k = 4,\ \tan a = -\frac{3}{\sqrt{7}}$

$\Rightarrow a = 5.44 \ \ or\ -0.85$

$\bullet^2 X$, $\bullet^3 \checkmark$, $\bullet^4 \checkmark$, $\bullet^5 \checkmark$, $\bullet^6 \checkmark$

**Common Error 2**

**(*sic*)**

$q = 3 \quad \Rightarrow k = 4,\ \tan a = -\frac{3}{\sqrt{7}}$

$\Rightarrow a = 0.85$

$\bullet^2 X$, $\bullet^3 \checkmark$, $\bullet^4 \checkmark$, $\bullet^5 \checkmark$, $\bullet^6 X$

Note that $\bullet^6$ is not awarded as it is not consistent with previous working.

**Alternative Method (**for $\bullet^7$ and $\bullet^8$**)**

*If* :

$f'(x) + g'(x) = -\sqrt{7}\sin x - 3\cos x$ .........

then $\bullet^7$ is only available once the candidate has reached e.g.

"choose $k\sin(x + a)$

$\Rightarrow k\sin a = -3,\ k\cos a = -7$."

$\bullet^8$ is available for evaluating $k$ and $a$.

# 2.04

| qu | part | mk | code | calc | source | ss | | ic | C | B | A | | U1 | U2 | U3 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2.04 | a | 2 | G9 | CN | 8204 | | | 2 | 2 | | | | | 2 | |
| | b | 4 | G14 | CN | | 1 | 1 | 2 | 2 | 2 | | | | 4 | |
| | c | 5 | G12 | CN | | 1 | 4 | | | 5 | | | | 5 | |

$(a)$ Write down the centre and calculate the radius of the circle with equation $x^2 + y^2 + 8x + 4y - 38 = 0$. 2

$(b)$ A second circle has equation $(x-4)^2 + (y-6)^2 = 26$.
Find the distance between the centres of these two circles and hence show that the circles intersect. 4

$(c)$ The line with equation $y = 4 - x$ is a common chord passing through the points of intersection of the two circles.
Find the coordinates of the points of intersection of the two circles. 5

The primary method is based on this generic marking scheme which may be used as a guide for any method not shown in detail.

**Generic Marking Scheme**

| | | |
|---|---|---|
| $\bullet^1$ | ic | state centre of circle |
| $\bullet^2$ | ic | find radius of circle |
| $\bullet^3$ | ic | state centre and radius |
| $\bullet^4$ | pd | find distance between centres |
| $\bullet^5$ | ss | find sum of radii |
| $\bullet^6$ | ic | interpret result |
| $\bullet^7$ | ss | know to and substitute |
| $\bullet^8$ | pd | start process |
| $\bullet^9$ | pd | write in standard form |
| $\bullet^{10}$ | pd | solve for $x$ |
| $\bullet^{11}$ | pd | solve for $y$ |

**Primary Method : Give 1 mark for each •**

$\bullet^1$ $(-4,-2)$

$\bullet^2$ $\sqrt{58}\ (\approx 7.6)$

$\bullet^3$ $(4,6)$ ***and*** $\sqrt{26}\ (\approx 5.1)$ *s / i* $\bullet^4$ *and* $\bullet^5$

$\bullet^4$ $d_{centres} = \sqrt{128}$ *accept* 11.3

$\bullet^5$ $\sqrt{58} + \sqrt{26}$ *accept* 12.7

$\bullet^6$ compare 12.7 and 11.3

$\bullet^7$ $x^2 + (4-x)^2 + \ldots$

$\bullet^8$ $x^2 + 16 - 8x + x^2 + \ldots$

$\bullet^9$ $2x^2 - 4x - 6 = 0$

| | | $\bullet^{10}$ | $\bullet^{11}$ |
|---|---|---|---|
| $\bullet^{10}$ | $x$ | 3 | $-1$ |
| $\bullet^{11}$ | $y$ | 1 | 5 |

**Notes**

In (a)

1 If a linear equation is obtained at the $\bullet^9$ stage, then $\bullet^9$, $\bullet^{10}$ and $\bullet^{11}$ are not available.

2 Solving the circles simultaneously to obtain the equation of the common chord gains no marks.

3 The comment given at the $\bullet^6$ stage must be consistent with previous working.

**alt. for $\bullet^7$ to $\bullet^{11}$ :**

$\bullet^7$ $(4-y)^2 + \ldots$

$\bullet^8$ $y^2 - 8y + 16 + y^2 + \ldots$

$\bullet^9$ $y^2 - 6y + 5 = 0$

| | | $\bullet^{10}$ | $\bullet^{11}$ |
|---|---|---|---|
| $\bullet^{10}$ | $y$ | 1 | 5 |
| $\bullet^{11}$ | $x$ | 3 | $-1$ |

# 2.05

| qu | part | mk | code | calc | source | ss | pd | ic | C | B | A | U1 | U2 | U3 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2.05 | | 5 | T10 | CR | | 1 | 4 | | | 5 | | | 5 | |

Solve the equation $\cos 2x° + 2\sin x° = \sin^2 x°$ in the interval $0 \le x < 360$. 5

The primary method is based on this generic marking scheme which may be used as a guide for any method not shown in detail.

**Generic Marking Scheme**

| | | |
|---|---|---|
| $\bullet^1$ | ss | use double angle formula |
| $\bullet^2$ | pd | obtains standard form (i.e. " ......... = 0") |
| $\bullet^3$ | pd | factorise |
| $\bullet^4$ | pd | process factors |
| $\bullet^5$ | pd | completes solutions |

**Primary Method : Give 1 mark for each •**

| | | |
|---|---|---|
| $\bullet^1$ | $\cos 2x = 1 - 2\sin^2 x$ | |
| $\bullet^2$ | $3\sin^2 x - 2\sin x - 1 = 0$ | |
| $\bullet^3$ | $(3\sin x + 1)(\sin x - 1) = 0$ | |
| | $\bullet^4$ | $\bullet^5$ |
| $\bullet^4$ | $\sin x = -\frac{1}{3}$ | $\sin x = 1$ |
| $\bullet^5$ | $199.5°, 340.5°$ | $90°$ |

**Notes**

1 $\bullet^1$ is not available for $1 - 2\sin^2 A$ with no further working.

2 $\bullet^2$ is only available for the three terms shown written in any correct order.

3 The "=0" has to appear at least once "en route" to $\bullet^3$.

4 $\bullet^4$ and $\bullet^5$ are only available for solving a quadratic equation.

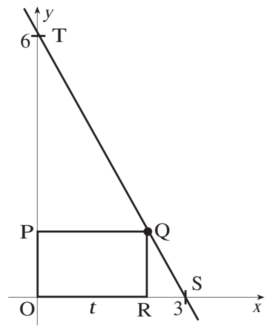
# 2.06

| qu | part | mk | code | calc | source | ss | pd | ic | C | B | A | | U1 | U2 | U3 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2.06 | | 3 | G3 | CN | 8206 | 1 | | 2 | | | 3 | | 3 | | |
| | | 6 | C11 | CN | | 2 | 2 | 2 | | 6 | | | 6 | | |

In the diagram Q lies on the line joining (0, 6) and (3, 0).
OPQR is a rectangle, where P and R lie on the axes and OR = $t$.

($a$) Show that QR $= 6 - 2t$. 3

($b$) Find the coordinates of Q for which the rectangle has a maximum area. 6




The primary method is based on this generic marking scheme which may be used as a guide for any method not shown in detail.

**Generic Marking Scheme**

| | | |
|---|---|---|
| $\bullet^1$ | ss | know and use e.g. similar triangles, trigonometry or gradient |
| $\bullet^2$ | ic | establish equation |
| $\bullet^3$ | ic | find a length |
| $\bullet^4$ | ss | know how and find area |
| $\bullet^5$ | ss | set derivative of the area function to zero |
| $\bullet^6$ | pd | differentiate |
| $\bullet^7$ | pd | solve |
| $\bullet^8$ | ic | justify stationary point |
| $\bullet^9$ | ic | state coordinates |

**Primary Method : Give 1 mark for each •**

| | |
|---|---|
| $\bullet^1$ | $\Delta$OST, RSQ are similar $s\,/\,i$ by $\bullet^2$ |
| $\bullet^2$ | $\frac{\text{QR}}{6} = \frac{3-t}{3}$ or equivalent |
| $\bullet^3$ | $\text{QR} = 6 - 2t$ |
| $\bullet^4$ | $A(t) = t(6-2t)$ |
| $\bullet^5$ | $A'(t) = 0$ |
| $\bullet^6$ | $6 - 4t$ |
| $\bullet^7$ | $t = \frac{3}{2}$ |
| $\bullet^8$ | *e.g. nature table* |
| $\bullet^9$ | $\text{Q} = \left(\frac{3}{2}, 3\right)$ |

**Notes**

1 "$y = 6 - 2x$" appearing *ex nihilo* can be awarded neither $\bullet^1$ nor $\bullet^2$.
$\bullet^3$ is still available with some justification *e.g.* OR $= t$ gives $y = 6 - 2t$.

2 The "=0" has to appear at least once before the $\bullet^7$ stage for $\bullet^5$ to be awarded.

3 Do not penalise the use of $\frac{dy}{dx}$ in lieu of $A'(t)$ for instance in the nature table.

4 The minimum requirements for the nature table are shown on the right.
Of course other methods may be used to justify the nature of the stationary point(s).

**Variation 1:**

$\bullet^1$ $\tan \text{'S'} = \frac{6}{3}$

$\bullet^2$ $\tan \text{'S'} = \frac{\text{QR}}{3-t}$ *and equate*

**Variation 2 :**

$\bullet^1$ $\checkmark m_{\text{line}} = -2$ $s\,/\,i$ *by* $\bullet^2$

$\bullet^2$ $\checkmark$equation of line : $y = -2x + 6$

**Variation 3**

$\bullet^1$ $\checkmark$ $m_{\text{line}} = -2$

$\bullet^2$ $\checkmark$equation of line : $y = 6 - 2x$

**Variation 4**

$\bullet^1$ $X$ *(nothing stated)*

$\bullet^2$ $X$equation of line : $y = 6 - 2x$

**Alternative Method : (for $\bullet^5$ *to* $\bullet^8$)**

$\bullet^5$ strategy to find roots $\Rightarrow$ t.p.s

$\bullet^6$ $t = 0,\ t = 3$

$\bullet^7$ max *t.p.* since coeff of "$t^2$" $< 0$

$\bullet^8$ turning pt at $t = \frac{3}{2}$

**Nature Table**

**minimum requirements for •$^8$**

$\bullet^8$

| | | $\frac{3}{2}$ | |
|---|---|---|---|
| $A'$ | $+$ | $0$ | $-$ |
| | $\nearrow$ | $\rightarrow$ | $\searrow$ |

# 2.07

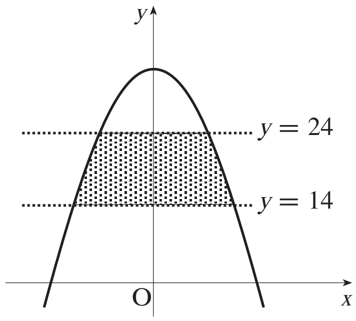
| qu | part | mk | code | calc | source | ss | pd | ic | C | B | A | | U1 | U2 | U3 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 2.07 | | 8 | C19 | CN | | 3 | 4 | 1 | | | 8 | | | 8 | |

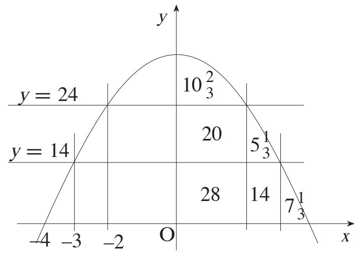
The parabola shown in the diagram has equation

$y = 32 - 2x^2$.

The shaded area lies between the lines $y = 14$ and $y = 24$.

Calculate the shaded area. 8




The primary method is based on this generic marking scheme which may be used as a guide for any method not shown in detail.

**Generic Marking Scheme**

| | | |
|---|---|---|
| $\bullet^1$ | ic | interpret limits |
| $\bullet^2$ | pd | find both $x$-values |
| $\bullet^3$ | ss | know to integrate |
| $\bullet^4$ | pd | integrate |
| $\bullet^5$ | ic | state limits |
| $\bullet^6$ | pd | evaluate limits |
| $\bullet^7$ | ss | select "what to add to what" |
| $\bullet^8$ | pd | completes a valid strategy |

**Primary Method : Give 1 mark for each •**

| | |
|---|---|
| $\bullet^1$ | $32 - 2x^2 = 24$ ***or*** $14$ |
| $\bullet^2$ | $x = 2$ ***and*** $3$ |
| $\bullet^3$ | $\int (32 - 2x^2)\ \ dx$ |
| $\bullet^4$ | $32x - \frac{2}{3}x^3$ |
| $\bullet^5$ | $\left[\ldots\right]_2^3$ |
| $\bullet^6$ | $19\frac{1}{3}$ |
| $\bullet^7$ | *e.g.* $19\frac{1}{3} - 14 + 20$ and then double $s\,/\,i$ by $\bullet^8$ |
| $\bullet^8$ | $50\frac{2}{3}$ |

**Notes**

**1** For $\int_{14}^{24} (32 \text{ - } 2x^2)dx \quad = \left[32x - \frac{2}{3}x^3\right]$

may be awarded $\bullet^3$ and $\bullet^4$ *ONLY*.

**2 For integrating "along the $y$ - axis"**

$\bullet^1$ strategy: choose to integrate along $y$-axis

$\bullet^2$ $x = \sqrt{\left(16 - \frac{1}{2}y\right)}$

$\bullet^3$ $\int \left(16 - \frac{1}{2}y\right)^{\frac{1}{2}}\ \ dy$

$\bullet^4$ $-2.\frac{2}{3}\left(16 - \frac{1}{2}y\right)^{\frac{3}{2}}$

$\bullet^5$ $\left[\ldots\right]_{14}^{24}$

$\bullet^6$ $-\frac{4}{3}\left(4^{\frac{3}{2}} - 9^{\frac{3}{2}}\right)$

$\bullet^7$ $2\times\ldots\ldots\ldots$

$\bullet^8$ $50\frac{2}{3}$

**Exemplar 1** ( $\bullet^3$ *to* $\bullet^8$)

$\bullet^3$ $\int (32 - 2x^2 - 14)\ \ dx$

$\bullet^4$ $18x - \frac{2}{3}x^3$

$\bullet^5$ $\left[\ldots\right]_{-3}^3$

$\bullet^6$ $72$

$\bullet^7$ *e.g.* $72 - \int_{-2}^{2} (32 - 2x^2 - 24)\ \ dx$

$\bullet^8$ $50\frac{2}{3}$

***or***

$\bullet^5$ $\left[\ldots\right]_0^3$

$\bullet^6$ $36$

$\bullet^7$ *e.g.* $2\times\left[36 - \int_0^2 (32 - 2x^2 - 24)\ \ dx\right]$

**Variations** ( $\bullet^3$ *to* $\bullet^6$)

The following are examples of sound opening integrals which will lead to the area after one more integral at most.

$\int_0^2 (32 - 2x^2)dx \quad = \ldots\ldots = 58\frac{2}{3}$

$\int_0^3 (32 - 2x^2)dx \quad = \ldots\ldots = 78$

$\int_2^3 (32 - 2x^2)dx \quad = \ldots\ldots = 19\frac{1}{3}$

$\int_0^2 (32 - 2x^2 - 24)dx \quad = \ldots\ldots = 10\frac{2}{3}$

$\int_0^3 (32 - 2x^2 - 14)dx \quad = \ldots\ldots = 36$

$\int_2^3 (32 - 2x^2 - 14)dx \quad = \ldots\ldots = 5\frac{1}{3}$