H

National
Qualifications
SPECIMEN ONLY

NEW FORMAT
FROM 2019

S847/76/11

Mathematics
Paper 1 (Non-calculator)

Date — Not applicable

Duration — 1 hour 30 minutes

Total marks — 70

Attempt ALL questions.

You may NOT use a calculator.

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate.

You will not earn marks for answers obtained by readings from scale drawings.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

Additional space for answers is provided at the end of the answer booklet. If you use this space you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not, you may lose all the marks for this paper.

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar product: $\mathbf{a.b} = |\mathbf{a}||\mathbf{b}|\cos\,\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(\mathrm{A} \pm \mathrm{B}) = \sin \mathrm{A} \cos \mathrm{B} \pm \cos \mathrm{A} \sin \mathrm{B}$$

$$\cos(\mathrm{A} \pm \mathrm{B}) = \cos \mathrm{A} \cos \mathrm{B} \mp \sin \mathrm{A} \sin \mathrm{B}$$

$$\sin 2\mathrm{A} = 2 \sin \mathrm{A} \cos \mathrm{A}$$

$$\begin{aligned} \cos 2\mathrm{A} &= \cos^2 \mathrm{A} - \sin^2 \mathrm{A} \\ &= 2\cos^2 \mathrm{A} - 1 \\ &= 1 - 2\sin^2 \mathrm{A} \end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

MARKS

Attempt ALL questions

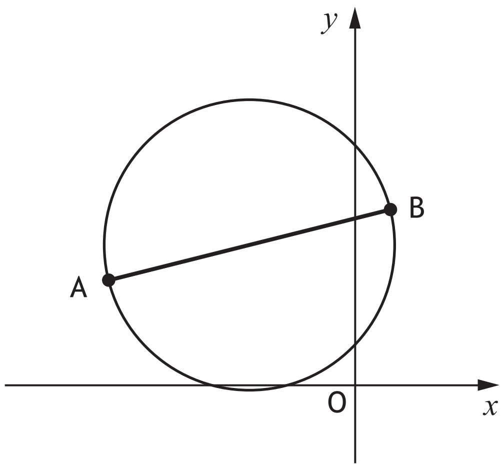
Total marks — 70

1. A curve has equation $y = x^2 - 4x + 7$.

Find the equation of the tangent to this curve at the point where $x = 5$. **4**

2. A and B are the points $(-7, 3)$ and $(1, 5)$.

AB is a diameter of a circle.



Find the equation of this circle. **3**

3. Line l_1 has equation $\sqrt{3}y - x = 0$.

(a) Line l_2 is perpendicular to l_1. Find the gradient of l_2. **2**

(b) Calculate the angle l_2 makes with the positive direction of the x-axis. **2**

MARKS

4. Evaluate $\int_1^2 \frac{1}{6}x^{-2}\,dx$. **3**

5. The points A(0, 9, 7), B(5, −1, 2), C(4, 1, 3) and D(x, −2, 2) are such that $\overrightarrow{AB}$ is perpendicular to $\overrightarrow{CD}$.

Determine the value of x. **4**

6. Determine the range of values of p such that the equation $x^2 + (p+1)x + 9 = 0$ has no real roots. **4**

7. Show that the line with equation $y = 3x - 5$ is a tangent to the circle with equation $x^2 + y^2 + 2x - 4y - 5 = 0$ and find the coordinates of the point of contact. **5**

MARKS

8. For the polynomial, $x^3 - 4x^2 + ax + b$

- $x - 1$ is a factor
- -12 is the remainder when it is divided by $x - 2$

(a) Determine the values of a and b. **5**

(b) Hence solve $x^3 - 4x^2 + ax + b = 0$. **3**

9. A sequence is generated by the recurrence relation $u_{n+1} = mu_n + 6$ where m is a constant.

(a) Given $u_1 = 28$ and $u_2 = 13$, find the value of m. **2**

(b) (i) Explain why this sequence approaches a limit as $n \to \infty$. **1**

(ii) Calculate this limit. **2**

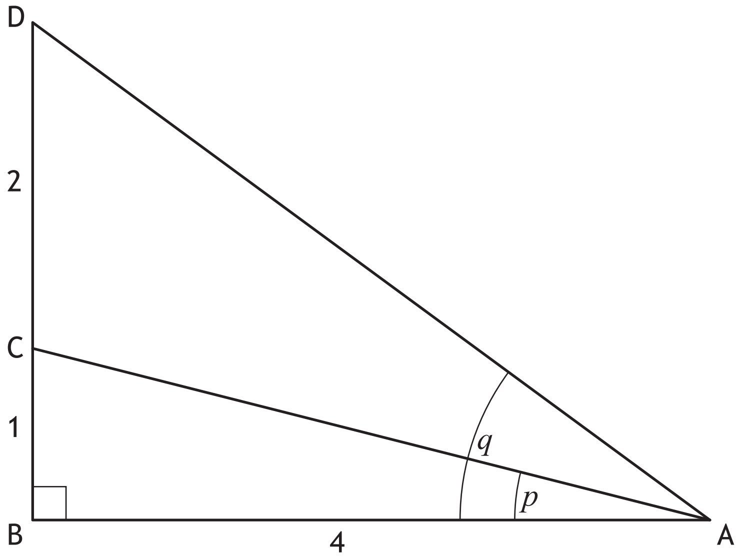
10. (a) Evaluate $\log_5 25$. **1**

(b) Hence solve $\log_4 x + \log_4 (x - 6) = \log_5 25$, where $x > 6$. **5**

11. Find the rate of change of the function $f(x) = 4\sin^3 x$ when $x = \frac{5\pi}{6}$. **3**

MARKS

12. Triangle ABD is right-angled at B with angles BAC $= p$ and BAD $= q$ and lengths as shown in the diagram below.



Show that the exact value of $\cos(q-p)$ is $\frac{19\sqrt{17}}{85}$. **5**

13. The curve $y = f(x)$ is such that $\frac{dy}{dx} = 4x - 6x^2$. The curve passes through the point $(-1, 9)$. Express y in terms of x. **4**

14. (a) Solve $\cos 2x° - 3\cos x° + 2 = 0$ for $0 \leq x < 360$. **5**

(b) Hence solve $\cos 4x° - 3\cos 2x° + 2 = 0$ for $0 \leq x < 360$. **2**

MARKS

15. Functions f and g are defined on suitable domains by $f(x) = x^3 - 1$ and $g(x) = 3x + 1$.

(a) Find an expression for $k(x)$, where $k(x) = g(f(x))$. **2**

(b) If $h(k(x)) = x$, find an expression for $h(x)$. **3**

[END OF SPECIMEN QUESTION PAPER]

H

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SPECIMEN ONLY

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Mathematics
Paper 1 (Non-calculator)

Marking Instructions

These marking instructions have been provided to show how SQA would mark this specimen question paper.

General marking principles for Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If a candidate response does not seem to be covered by either the principles or detailed marking instructions, and you are uncertain how to assess it, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example $6 \times 6 = 12$, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.

This is no longer a solution of a quadratic equation, so the mark is not awarded.

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$x = 1$$

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.

$$x^2 + 5x + 7 = 9x + 4$$
$$x - 4x + 3 = 0$$
$$(x-3)(x-1) = 0$$
$$x = 1 \text{ or } 3$$

(i) **Horizontal/vertical marking**

If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	•5	•6
•5	$x = 2$	$x = -4$
•6	$y = 5$	$y = -7$

Horizontal: •5 $x = 2$ and $x = -4$ Vertical: •5 $x = 2$ and $y = 5$
•6 $y = 5$ and $y = -7$ •6 $x = -4$ and $y = -7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0{\cdot}3}$ must be simplified to 50 $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 100 must be known.

(k) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

  $(x^3+2x^2+3x+2)(2x+1)$ written as

  $(x^3+2x^2+3x+2)\times 2x+1$

  $=2x^4+5x^3+8x^2+7x+2$

  gains full credit

- repeated error within a question, but not between questions or papers

(l) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(m) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(n) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(o) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

Marking instructions for each question

Question	Generic scheme	Illustrative scheme	Max mark
1.	•1 differentiate •2 calculate gradient •3 find the value of y •4 find equation of tangent	•1 $2x-4$ •2 6 •3 12 •4 $y=6x-18$	**4**
2.	•1 find the centre •2 calculate the radius •3 state equation of circle	•1 $(-3,4)$ •2 $\sqrt{17}$ •3 $(x+3)^2+(y-4)^2=17$ or equivalent	**3**
3. (a)	•1 find gradient l_1 •2 state gradient l_2	•1 $\frac{1}{\sqrt{3}}$ •2 $-\sqrt{3}$	**2**
3. (b)	•3 using $m=\tan\theta$ •4 calculating angle	•3 $\tan\theta=-\sqrt{3}$ •4 $\theta=\frac{2\pi}{3}$ or 120°	**2**
4.	•1 complete integration •2 substitute limits •3 evaluate	•1 $-\frac{1}{6}x^{-1}$ •2 $\left(-\frac{1}{6\times2}\right)-\left(-\frac{1}{6\times1}\right)$ •3 $\frac{1}{12}$	**3**

Question	Generic scheme	Illustrative scheme	Max mark
5.	$\bullet^1$ find $\overrightarrow{CD}$ $\bullet^2$ find $\overrightarrow{AB}$ $\bullet^3$ equate scalar product to zero $\bullet^4$ calculate value of x	$\bullet^1$ $\begin{pmatrix} x-4 \\ -3 \\ -1 \end{pmatrix}$ $\bullet^2$ $\begin{pmatrix} 5 \\ -10 \\ -5 \end{pmatrix}$ $\bullet^3$ $5(x-4)+(-10)(-3)+(-5)(-1)=0$ $\bullet^4$ $x=-3$	4
6.	$\bullet^1$ substitute into discriminant $\bullet^2$ apply condition for no real roots $\bullet^3$ determine zeroes of quadratic expression $\bullet^4$ state range with justification	$\bullet^1$ $(p+1)^2-4\times1\times9$ $\bullet^2$ $\ldots<0$ $\bullet^3$ $-7, 5$ $\bullet^4$ $-7<p<5$ with eg sketch or table of signs	4
7.	$\bullet^1$ substitute for y in equation of circle $\bullet^2$ express in standard quadratic form $\bullet^3$ demonstrate tangency $\bullet^4$ find x-coordinate $\bullet^5$ find y-coordinate	$\bullet^1$ $x^2+(3x-5)^2+2x-4(3x-5)-5=0$ $\bullet^2$ $10x^2-40x+40=0$ $\bullet^3$ $10(x-2)^2=0$ only one solution implies tangency $\bullet^4$ $x=2$ $\bullet^5$ $y=1$	5

Question	Generic scheme	Illustrative scheme	Max mark
8. (a)	•[1] use appropriate strategy •[2] obtain an expression for a and b •[3] obtain a second expression for a and b •[4] find the value of a or b •[5] find the second value	•[1] $(1)^3 - 4(1)^2 + a(1) + b = 0$ •[2] $a + b = 3$ •[3] $2a + b = -4$ •[4] $a = -7$ or $b = 10$ •[5] $b = 10$ or $a = -7$	**5**
8. (b)	•[6] obtain quadratic factor •[7] complete factorisation •[8] state solutions	•[6] $(x^2 - 3x - 10)$ •[7] $(x-1)(x-5)(x+2)$ •[8] $x = 1$, $x = 5$, $x = -2$	**3**
9. (a)	•[1] interpret information •[2] solve to find m	•[1] $13 = 28m + 6$ •[2] $m = \frac{1}{4}$	**2**
9. (b) (i)	•[3] state condition	•[3] a limit exists as $-1 < \frac{1}{4} < 1$	**1**
9. (b) (ii)	•[4] know how to calculate limit •[5] calculate limit	•[4] $L = \frac{1}{4}L + 6$ •[5] $L = 8$	**2**

Question	Generic scheme	Illustrative scheme	Max mark
10. (a)	•1 state value	•1 2	**1**
10. (b)	•1 use laws of logarithms •2 link to part (a) •3 use laws of logarithms •4 write in standard quadratic form •5 solve for x and identify appropriate solution	•1 $\log_4 x(x-6)$ •2 $\log_4 x(x-6)=2$ •3 $x(x-6)=4^2$ •4 $x^2-6x-16=0$ •5 8	**5**
11.	•1 start to differentiate •2 complete differentiation •3 evaluate derivative	•1 $3\times4\sin^2 x\ldots$ •2 $\ldots\times\cos x$ •3 $\dfrac{-3\sqrt{3}}{2}$	**3**
12.	•1 calculate lengths AC and AD •2 select appropriate formula and express in terms of p and q •3 calculate two of $\cos p,\ \cos q,\ \sin p,\ \sin q$ •4 calculate other two and substitute into formula •5 arrange into required form	•1 $\text{AC}=\sqrt{17}$ and $\text{AD}=5$ stated or implied by •3 •2 $\cos q\cos p+\sin q\sin p$ stated or implied by •4 •3 $\cos p=\dfrac{4}{\sqrt{17}}$, $\cos q=\dfrac{4}{5}$ $\sin p=\dfrac{1}{\sqrt{17}}$, $\sin q=\dfrac{3}{5}$ •4 $\dfrac{4}{5}\times\dfrac{4}{\sqrt{17}}+\dfrac{3}{5}\times\dfrac{1}{\sqrt{17}}$ •5 $\dfrac{19}{5\sqrt{17}}\times\dfrac{\sqrt{17}}{\sqrt{17}}=\dfrac{19\sqrt{17}}{85}$ or $\dfrac{19}{5\sqrt{17}}=\dfrac{19\sqrt{17}}{5\times17}=\dfrac{19\sqrt{17}}{85}$	**5**

Question	Generic scheme	Illustrative scheme	Max mark
13.	●[1] know to and start to integrate ●[2] complete integration ●[3] substitute for x and y ●[4] state expression for y	●[1] eg $y=\frac{4}{2}x^2...$ ●[2] $y=\frac{4}{2}x^2-\frac{6}{3}x^3+c$ ●[3] $9=2(-1)^2-2(-1)^3+c$ ●[4] $y=2x^2-2x^3+5$	4
14. (a)	●[1] use double angle formula ●[2] express as a quadratic in $\cos x°$ ●[3] start to solve ●[4] reduce to equations in $\cos x°$ only ●[5] process solutions in given domain	**Method 1:** Using factorisation ●[1] $2\cos^2 x°-1...$ **stated or implied by ●[2]** ●[2] $2\cos^2 x°-3\cos x°+1=0$ ●[3] $(2\cos x°-1)(\cos x°-1)$ $= 0$ must appear at either of these lines to gain ●[2] **Method 2:** Using quadratic formula ●[1] $2\cos^2 x°-1...$ **stated or implied by ●[2]** ●[2] $2\cos^2 x°-3\cos x°+1=0$ **stated explicitly** ●[3] $\dfrac{-(-3)\pm\sqrt{(-3)^2-4\times2\times1}}{2\times2}$ **In both methods:** ●[4] $\cos x°=\frac{1}{2}$ and $\cos x°=1$ ●[5] 0, 60, 300 Candidates who include 360 lose ●[5]. **or** ●[4] $\cos x=1$ and $x=0$ ●[5] $\cos x°=\frac{1}{2}$ and $x=60$ or 300 Candidates who include 360 lose ●[5].	5
14. (b)	●[6] interpret relationship with (a) ●[7] state valid values	●[6] $2x=0$ and 60 and 300 ●[7] 0, 30, 150, 180, 210 and 330	2

Question	Generic scheme	Illustrative scheme	Max mark
15. (a)	●[1] interpret notation ●[2] complete process	●[1] $g\left(x^3-1\right)$ ●[2] $3x^3-2$	**2**
15. (b)	●[3] start to rearrange for x ●[4] rearrange ●[5] state expression for $h(x)$	●[3] $3x^3=y+2$ ●[4] $x=\sqrt[3]{\dfrac{y+2}{3}}$ ●[5] $h(x)=\sqrt[3]{\dfrac{x+2}{3}}$	**3**

[END OF SPECIMEN MARKING INSTRUCTIONS]

H

National
Qualifications
SPECIMEN ONLY

NEW FORMAT FROM 2019

S847/76/12

Mathematics
Paper 2

Date — Not applicable

Duration — 1 hour 45 minutes

Total marks — 80

Attempt ALL questions.

You may use a calculator.

To earn full marks you must show your working in your answers.

State the units for your answer where appropriate.

You will not earn marks for answers obtained by readings from scale drawings.

Write your answers clearly in the spaces provided in the answer booklet. The size of the space provided for an answer is not an indication of how much to write. You do not need to use all the space.

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Use **blue** or **black** ink.

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FORMULAE LIST

Circle:

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The equation $(x-a)^2 + (y-b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar product: $\mathbf{a.b} = |\mathbf{a}||\mathbf{b}|\cos\,\theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin(\mathrm{A} \pm \mathrm{B}) = \sin \mathrm{A} \cos \mathrm{B} \pm \cos \mathrm{A} \sin \mathrm{B}$$

$$\cos(\mathrm{A} \pm \mathrm{B}) = \cos \mathrm{A} \cos \mathrm{B} \mp \sin \mathrm{A} \sin \mathrm{B}$$

$$\sin 2\mathrm{A} = 2 \sin \mathrm{A} \cos \mathrm{A}$$

$$\begin{aligned} \cos 2\mathrm{A} &= \cos^2 \mathrm{A} - \sin^2 \mathrm{A} \\ &= 2\cos^2 \mathrm{A} - 1 \\ &= 1 - 2\sin^2 \mathrm{A} \end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

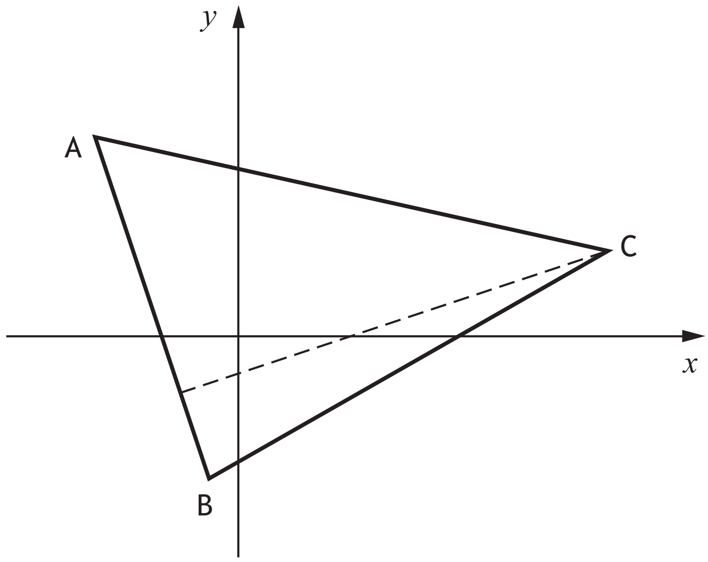
MARKS

Attempt ALL questions

Total marks — 80

1. The vertices of triangle ABC are A$(-5,7)$, B$(-1,-5)$ and C$(13,3)$ as shown in the diagram.

The broken line represents the altitude from C.



(a) Find the equation of the altitude from C. **3**

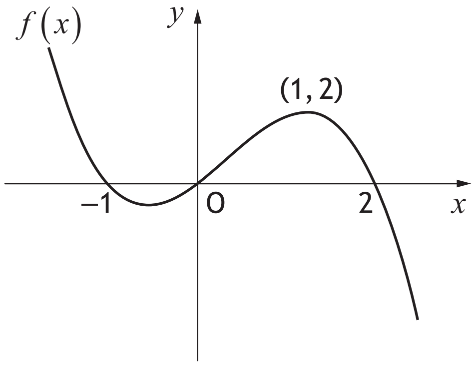
(b) Find the equation of the median from B. **3**

(c) Find the coordinates of the point of intersection of the altitude from C and the median from B. **2**

2. Find $\int \frac{4x^3+1}{x^2}\,dx,\ x \neq 0.$ **4**

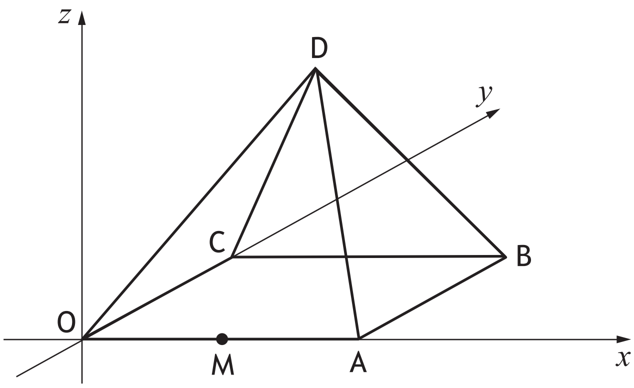
3. The diagram shows the curve with equation $y = f(x)$, where $f(x) = kx(x+a)(x+b)$.

The curve passes through $(-1, 0)$, $(0, 0)$, $(1, 2)$ and $(2, 0)$.



Find the values of a, b and k. **3**

4. D,OABC is a square-based pyramid as shown.



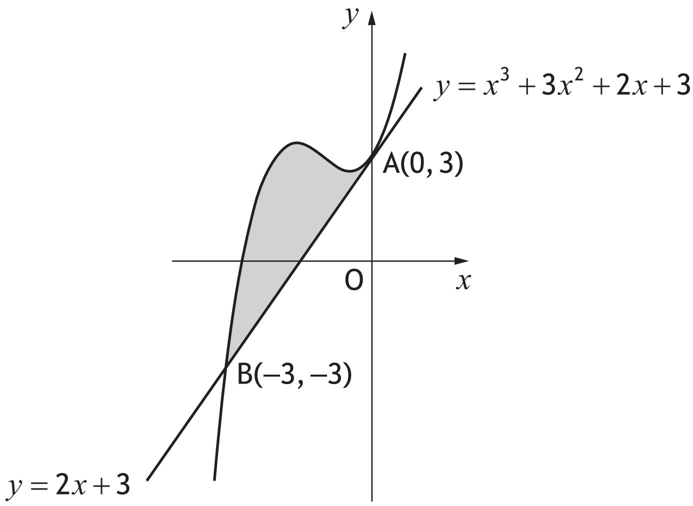
- O is the origin and OA = 4 units.
- M is the mid-point of OA.
- $\overrightarrow{OD} = 2\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$

(a) Express $\overrightarrow{DB}$ and $\overrightarrow{DM}$ in component form. **3**

(b) Find the size of angle BDM. **5**

MARKS

5. The line with equation $y = 2x + 3$ is a tangent to the curve with equation $y = x^3 + 3x^2 + 2x + 3$ at A(0, 3), as shown.



The line meets the curve again at B(–3, –3).

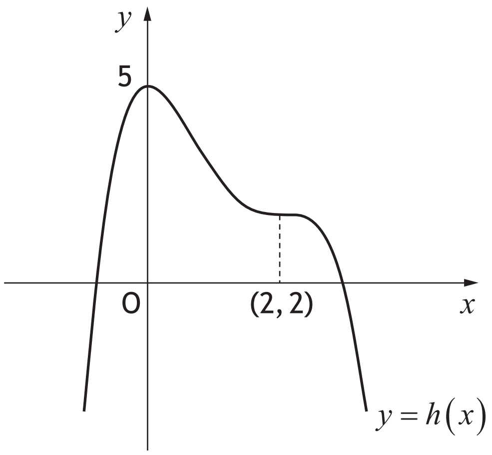
Find the area enclosed by the line and the curve. **5**

6. (a) Express $3x^2 + 24x + 50$ in the form $a(x+b)^2 + c$. **3**

(b) Given that $f(x) = x^3 + 12x^2 + 50x - 11$, find $f'(x)$. **2**

(c) Hence, or otherwise, explain why the curve with equation $y = f(x)$ is strictly increasing for all values of x. **2**

7. The diagram below shows the graph of a quartic $y = h(x)$, with stationary points at (0, 5) and (2, 2).



On separate diagrams sketch the graphs of:

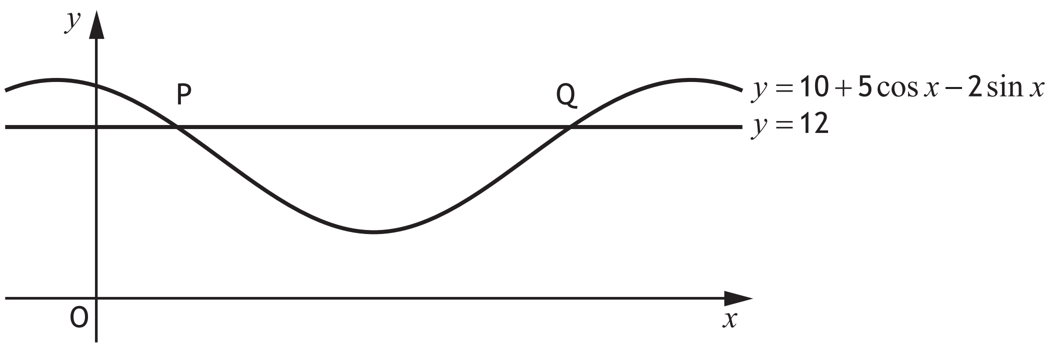
(a) $y = 2 - h(x)$. **2**

(b) $y = h'(x)$. **3**

8. (a) Express $5\cos x - 2\sin x$ in the form $k\cos(x + a)$, where $k > 0$ and $0 < a < 2\pi$. **4**

(b) The diagram shows a sketch of part of the graph of $y = 10 + 5\cos x - 2\sin x$ and the line with equation $y = 12$.

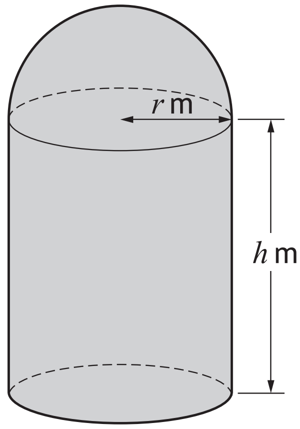
The line cuts the curve at the points P and Q.



Find the x-coordinates of P and Q. **4**

9. A design for a new grain container is in the shape of a cylinder with a hemispherical roof and a flat circular base. The radius of the cylinder is r metres, and the height is h metres.

The volume of the **cylindrical** part of the container needs to be 100 cubic metres.



(a) Given that the curved surface area of a hemisphere of radius r is $2\pi r^2$ show that the surface area of metal needed to build the grain container is given by:

$$A = \frac{200}{r} + 3\pi r^2 \text{ square metres}$$ **3**

(b) Determine the value of r which minimises the amount of metal needed to build the container. **6**

10. Given that

$$\int_{\frac{\pi}{8}}^{a} \sin\left(4x - \frac{\pi}{2}\right) dx = \frac{1}{2}, \quad 0 \le a < \frac{\pi}{2},$$

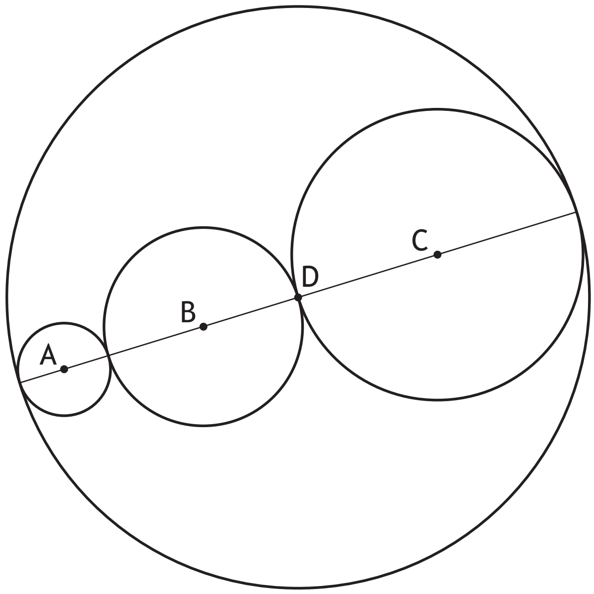
calculate the value of a. **6**

MARKS

11. Show that $\dfrac{\sin 2x}{2\cos x} - \sin x\cos^2 x = \sin^3 x$, where $0 < x < \dfrac{\pi}{2}$. **3**

12. (a) Show that the points A(–7, –2), B(2, 1) and C(17, 6) are collinear. **3**

Three circles with centres A, B and C are drawn inside a circle with centre D as shown.



The circles with centres A, B and C have radii r_A, r_B and r_C respectively.

- $r_A = \sqrt{10}$
- $r_B = 2r_A$
- $r_C = r_A + r_B$

(b) Determine the equation of the circle with centre D. **4**

MARKS

13. The concentration of a pesticide in soil can be modelled by the equation

$$P_t = P_0 e^{-kt}$$

where:

- P_0 is the initial concentration;
- P_t is the concentration at time t;
- t is the time, in days, after the application of the pesticide.

(a) It takes 25 days for the concentration of the pesticide to be reduced to one half of its initial concentration.

Calculate the value of k. **4**

(b) Eighty days after the initial application, what is the percentage decrease in concentration of the pesticide? **3**

[END OF SPECIMEN QUESTION PAPER]

National
Qualifications
SPECIMEN ONLY

S847/76/12

Mathematics
Paper 2

Marking Instructions

These marking instructions have been provided to show how SQA would mark this specimen question paper.

General marking principles for Higher Mathematics

Always apply these general principles. Use them in conjunction with the detailed marking instructions, which identify the key features required in candidates' responses.

For each question, the marking instructions are generally in two sections:

- *generic scheme — this indicates why each mark is awarded*
- *illustrative scheme — this covers methods which are commonly seen throughout the marking*

In general, you should use the illustrative scheme. Only use the generic scheme where a candidate has used a method not covered in the illustrative scheme.

(a) Always use positive marking. This means candidates accumulate marks for the demonstration of relevant skills, knowledge and understanding; marks are not deducted for errors or omissions.

(b) If a candidate response does not seem to be covered by either the principles or detailed marking instructions, and you are uncertain how to assess it, you must seek guidance from your team leader.

(c) One mark is available for each •. There are no half marks.

(d) If a candidate's response contains an error, all working subsequent to this error must still be marked. Only award marks if the level of difficulty in their working is similar to the level of difficulty in the illustrative scheme.

(e) Only award full marks where the solution contains appropriate working. A correct answer with no working receives no mark, unless specifically mentioned in the marking instructions.

(f) Candidates may use any mathematically correct method to answer questions, except in cases where a particular method is specified or excluded.

(g) If an error is trivial, casual or insignificant, for example 6 x 6 = 12, candidates lose the opportunity to gain a mark, except for instances such as the second example in point (h) below.

(h) If a candidate makes a transcription error (question paper to script or within script), they lose the opportunity to gain the next process mark, for example

This is a transcription error and so the mark is not awarded.	$x^2+5x+7=9x+4$
	$x-4x+3=0$
This is no longer a solution of a quadratic equation, so the mark is not awarded.	$x=1$

The following example is an exception to the above

This error is not treated as a transcription error, as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt and all marks awarded.	$x^2+5x+7=9x+4$
	$x-4x+3=0$
	$(x-3)(x-1)=0$
	$x=1$ or 3

(i) **Horizontal/vertical marking**
If a question results in two pairs of solutions, apply the following technique, but only if indicated in the detailed marking instructions for the question.

Example:

	•5	•6
•5	$x=2$	$x=-4$
•6	$y=5$	$y=-7$

Horizontal: •5 $x=2$ and $x=-4$ Vertical: •5 $x=2$ and $y=5$
•6 $y=5$ and $y=-7$ •6 $x=-4$ and $y=-7$

You must choose whichever method benefits the candidate, **not** a combination of both.

(j) In final answers, candidates should simplify numerical values as far as possible unless specifically mentioned in the detailed marking instruction. For example

$\frac{15}{12}$ must be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\frac{43}{1}$ must be simplified to 43

$\frac{15}{0\cdot3}$ must be simplified to 50 $\frac{4/5}{3}$ must be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8*

*The square root of perfect squares up to and including 100 must be known.

(k) Do not penalise candidates for any of the following, unless specifically mentioned in the detailed marking instructions:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in numerical answers/algebraic expressions, for example angles in degrees rounded to nearest degree
- omission of units
- bad form (bad form only becomes bad form if subsequent working is correct), for example

  $(x^3+2x^2+3x+2)(2x+1)$ written as

  $(x^3+2x^2+3x+2)\times 2x+1$

  $=2x^4+5x^3+8x^2+7x+2$

  gains full credit

- repeated error within a question, but not between questions or papers

(l) In any 'Show that…' question, where candidates have to arrive at a required result, the last mark is not awarded as a follow-through from a previous error, unless specified in the detailed marking instructions.

(m) You must check all working carefully, even where a fundamental misunderstanding is apparent early in a candidate's response. You may still be able to award marks later in the question so you must refer continually to the marking instructions. The appearance of the correct answer does not necessarily indicate that you can award all the available marks to a candidate.

(n) You should mark legible scored-out working that has not been replaced. However, if the scored-out working has been replaced, you must only mark the replacement working.

(o) If candidates make multiple attempts using the same strategy and do not identify their final answer, mark all attempts and award the lowest mark. If candidates try different valid strategies, apply the above rule to attempts within each strategy and then award the highest mark.

For example:

Strategy 1 attempt 1 is worth 3 marks.	Strategy 2 attempt 1 is worth 1 mark.
Strategy 1 attempt 2 is worth 4 marks.	Strategy 2 attempt 2 is worth 5 marks.
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

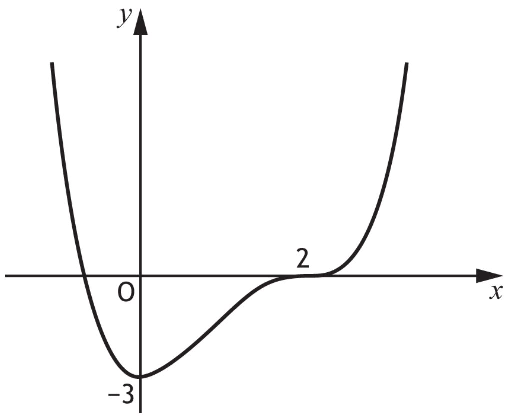
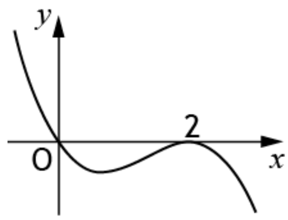
Marking instructions for each question

Question	Generic scheme	Illustrative scheme	Max mark
1. (a)	•[1] calculate gradient of AB •[2] use property of perpendicular lines •[3] determine equation of altitude	•[1] $m_{\text{AB}} = -3$ •[2] $m_{alt} = \frac{1}{3}$ •[3] $x - 3y = 4$	**3**
1. (b)	•[4] calculate midpoint of AC •[5] calculate gradient of median •[6] determine equation of median	•[4] $(4,5)$ •[5] $m_{\text{BM}} = 2$ •[6] $y = 2x - 3$	**3**
1. (c)	•[7] find x or y coordinate •[8] find remaining coordinate	•[7] $x = 1$ or $y = -1$ •[8] $y = -1$ or $x = 1$	**2**
2.	•[1] write in integrable form •[2] integrate one term •[3] integrate other term •[4] complete integration and simplify	•[1] $4x + x^{-2}$ •[2] eg $\frac{4}{2}x^2 + \ldots$ •[3] $\ldots\frac{x^{-1}}{-1}$ •[4] $2x^2 - x^{-1} + c$	**4**
3.	•[1] value of a •[2] value of b •[3] calculate k	•[1] 1 •[2] -2 •[3] -1	**3**

Question		Generic scheme	Illustrative scheme	Max mark
4.	(a)	$\bullet^1$ state components of $\overrightarrow{DB}$ $\bullet^2$ state coordinates of M $\bullet^3$ state components of $\overrightarrow{DM}$	$\bullet^1$ $\begin{pmatrix} 2 \\ 2 \\ -6 \end{pmatrix}$ $\bullet^2$ $(2,0,0)$ stated or implied by $\bullet^3$ $\bullet^3$ $\begin{pmatrix} 0 \\ -2 \\ -6 \end{pmatrix}$	**3**
4.	(b)	$\bullet^4$ evaluate $\overrightarrow{DB}.\overrightarrow{DM}$ $\bullet^5$ evaluate $\left\lvert\overrightarrow{DB}\right\rvert$ $\bullet^6$ evaluate $\left\lvert\overrightarrow{DM}\right\rvert$ $\bullet^7$ use scalar product $\bullet^8$ calculate angle	$\bullet^4$ 32 $\bullet^5$ $\sqrt{44}$ $\bullet^6$ $\sqrt{40}$ $\bullet^7$ $\cos \text{BDM} = \dfrac{32}{\sqrt{44}\sqrt{40}}$ $\bullet^8$ $40\cdot3^\circ$ or 0.703 rads	**5**

Question	Generic scheme	Illustrative scheme	Max mark
5.	•¹ know to integrate and interpret limits •² use ‘upper — lower’ •³ integrate •⁴ substitute limits •⁵ evaluate area	•¹ $\int_{-3}^{0} \dots dx$ •² $\int_{-3}^{0} \left(x^3+3x^2+2x+3\right)-\left(2x+3\right)dx$ •³ $\frac{1}{4}x^4+x^3$ •⁴ $0-\left(\frac{1}{4}(-3)^4+(-3)^3\right)$ •⁵ $\frac{27}{4}$ units^2	**5**

Question	Generic scheme	Illustrative scheme	Max mark
6. (a)	**Method 1**	**Method 1**	**3**
	•[1] identify common factor	•[1] $3(x^2+8x......$ stated or implied by •[2]	
	•[2] complete the square	•[2] $3(x+4)^2......$	
	•[3] process for c and write in required form	•[3] $3(x+4)^2+2$	
	Method 2	**Method 2**	**3**
	•[1] expand completed square form	•[1] $ax^2+2abx+ab^2+c$	
	•[2] equate coefficients	•[2] $a=3,\ \ 2ab=24,\ \ ab^2+c=50$	
	•[3] process for b and c and write in required form	•[3] $3(x+4)^2+2$	
6. (b)	•[4] differentiate two terms	•[4] $3x^2+24x....$	**2**
	•[5] complete differentiation	•[5] $....+50$	
6. (c)	**Method 1**	**Method 1**	**2**
	•[6] link with (a) and identify sign of $(x+4)^2$	•[6] $f'(x)=3(x+4)^2+2$ and $(x+4)^2\geq 0\ \ \forall\ x$	
	•[7] communicate reason	•[7] $\therefore 3(x+4)^2+2>0\ \Rightarrow$ always strictly increasing	
	Method 2	**Method 2**	**2**
	•[6] identify minimum value of $f'(x)$	•[6] eg minimum value $=2$ or annotated sketch	
	•[7] communicate reason	•[7] $2>0\ \therefore\left(f'(x)>0\right)\Rightarrow$ always strictly increasing	

Question		Generic scheme	Illustrative scheme	Max mark
7.	(a)	•¹ evidence of reflecting in x-axis •² vertical translation of 2 units identifiable from graph	•¹ reflection of graph in x-axis •² graph moves parallel to y-axis by 2 units upwards  	**2**
7.	(b)	•³ identify roots •⁴ interpret point of inflexion •⁵ complete cubic curve	•³ 0 **and** 2 only •⁴ turning point at $(2,0)$ •⁵ cubic passing through origin with negative gradient  	**3**

Question		Generic scheme	Illustrative scheme	Max mark
8.	(a)	●[1] use compound angle formula ●[2] compare coefficients ●[3] process for k ●[4] process for a and express in required form	●[1] $k\cos x\cos a - k\sin x\sin a$ **stated explicitly** ●[2] $k\cos a = 5, k\sin a = 2$ **stated explicitly** ●[3] $k = \sqrt{29}$ ●[4] $\sqrt{29}\cos(x + 0\cdot38)$	**4**
8.	(b)	●[5] equate to 12 and simplify constant terms ●[6] use result of part (a) and rearrange ●[7] solve for $x + a$ ●[8] solve for x	●[5] $5\cos x - 2\sin x = 2$ or $5\cos x - 2\sin x - 2 = 0$ ●[6] $\cos(x + 0\cdot3805\ldots) = \frac{2}{\sqrt{29}}$ ●[7] $1\cdot1902\ldots,$ (●[7]) | $5\cdot0928\ldots$ (●[8]) ●[8] $0\cdot8097\ldots,$ | $4\cdot712\ldots$	**4**

Question	Generic scheme	Illustrative scheme	Max mark
9. (a)	•[1] equate volume to 100 •[2] obtain an expression for h •[3] demonstrate result	•[1] $V = \pi r^2 h = 100$ •[2] $h = \dfrac{100}{\pi r^2}$ •[3] $A = \pi r^2 + 2\pi r^2 + 2\pi r \times \dfrac{100}{\pi r^2}$ leading to $A = \dfrac{200}{r} + 3\pi r^2$	**3**
9. (b)	•[4] start to differentiate •[5] complete differentiation •[6] set derivative to zero •[7] obtain r •[8] verify nature of stationary point •[9] interpret and communicate result	•[4] $A'(r) = 6\pi r \ldots$ •[5] $A'(r) = 6\pi r - \dfrac{200}{r^2}$ •[6] $6\pi r - \dfrac{200}{r^2} = 0$ •[7] $r = \sqrt[3]{\dfrac{100}{3\pi}}$ $(\approx 2\cdot 20)$ metres •[8] table of signs for a derivative when $r = 2\cdot 1974\ldots$ •[9] minimum when $r \approx 2\cdot 20$ (m) or •[8] $A''(r) = 6\pi + \dfrac{400}{r^3}$ •[9] $A''(2\cdot 1974\ldots) > 0$ $\therefore$ minimum when $r \approx 2\cdot 20$ (m)	**6**

Question	Generic scheme	Illustrative scheme	Max mark
10.	•¹ start to integrate •² complete integration •³ process limits •⁴ simplify numeric term and equate to $\frac{1}{2}$ •⁵ start to solve equation •⁶ solve for a	•¹ $-\frac{1}{4}\cos\ldots$ •² $-\frac{1}{4}\cos\left(4x-\frac{\pi}{2}\right)$ •³ $-\frac{1}{4}\cos\left(4a-\frac{\pi}{2}\right)+\frac{1}{4}\cos\left(\frac{4\pi}{8}-\frac{\pi}{2}\right)$ •⁴ $-\frac{1}{4}\cos\left(4a-\frac{\pi}{2}\right)+\frac{1}{4}=\frac{1}{2}$ •⁵ $\cos\left(4a-\frac{\pi}{2}\right)=-1$ •⁶ $a=\frac{3\pi}{8}$	**6**
11.	**Method 1** •¹ substitute for $\sin 2x$ •² simplify and factorise •³ substitute for $1-\cos^2 x$ and simplify	**Method 1** •¹ $\frac{2\sin x\cos x}{2\cos x}-\sin x\cos^2 x$ stated explicitly as above or in a simplified form of the above •² $\sin x\left(1-\cos^2 x\right)$ •³ $\sin x\times\sin^2 x$ leading to $\sin^3 x$	**3**
	Method 2 •¹ substitute for $\sin 2x$ •² simplify and substitute for $\cos^2 x$ •³ expand and simplify	**Method 2** •¹ $\frac{2\sin x\cos x}{2\cos x}-\sin x\cos^2 x$ stated explicitly as above or in a simplified form of the above •² $\sin x-\sin x\left(1-\sin^2 x\right)$ •³ $\sin x-\sin x+\sin^3 x$ leading to $\sin^3 x$	**3**

Question	Generic scheme	Illustrative scheme	Max mark
12. (a)	**Method 1**	**Method 1**	**3**
	•1 calculate m_{AB}	•1 eg $m_{\text{AB}} = \frac{3}{9} = \frac{1}{3}$	
	•2 calculate m_{BC}	•2 eg $m_{\text{BC}} = \frac{5}{15} = \frac{1}{3}$	
	•3 interpret result and state conclusion	•3 $\ldots \Rightarrow$ AB and BC are parallel (common direction), B is a common point, hence A, B and C are collinear.	
	Method 2	**Method 2**	**3**
	•1 calculate an appropriate vector, eg $\overrightarrow{AB}$	•1 eg $\overrightarrow{AB} = \begin{pmatrix} 9 \\ 3 \end{pmatrix}$	
	•2 calculate a second vector, eg $\overrightarrow{BC}$ and compare	•2 eg $\overrightarrow{BC} = \begin{pmatrix} 15 \\ 5 \end{pmatrix} \therefore \ \overrightarrow{AB} = \frac{3}{5}\overrightarrow{BC}$	
	•3 interpret result and state conclusion	•3 $\ldots \Rightarrow$ AB and BC are parallel (common direction), B is a common point, hence A, B and C are collinear.	
	Method 3	**Method 3**	**3**
	•1 calculate m_{AB}	•1 $m_{\text{AB}} = \frac{3}{9} = \frac{1}{3}$	
	•2 find equation of line and substitute point	•2 eg, $y - 1 = \frac{1}{3}(x - 2)$ leading to $6 - 1 = \frac{1}{3}(17 - 2)$	
	•3 communication	•3 since C lies on line A, B and C are collinear	
12. (b)	•4 find radius	•4 $6\sqrt{10}$	**4**
	•5 determine an appropriate ratio	•5 eg $2:3$ or $\frac{2}{5}$ (using B and C) or $3:5$ or $\frac{8}{5}$ (using A and C)	
	•6 find centre	•6 $(8,3)$	
	•7 state equation of circle	•7 $(x-8)^2 + (y-3)^2 = 360$	

Question	Generic scheme	Illustrative scheme	Max mark
13. (a)	•[1] interpret half-life •[2] process equation •[3] write in logarithmic form •[4] process for k	•[1] $\frac{1}{2}P_0 = P_0e^{-25k}$ **stated or implied by •[2]** •[2] $e^{-25k} = \frac{1}{2}$ •[3] $\log_e \frac{1}{2} = -25k$ •[4] $k \approx 0 \cdot 028$	**4**
13. (b)	•[5] interpret equation •[6] process •[7] state percentage decrease	•[5] $P_t = P_0e^{-80\times 0\cdot 028}$ •[6] $P_t \approx 0\cdot 1065P_0$ •[7] 89%	**3**

[END OF SPECIMEN MARKING INSTRUCTIONS]