H

National
Qualifications
EXEMPLAR PAPER ONLY

EP30/H/01

Mathematics
Paper 1
(Non-Calculator)

Date — Not applicable

Duration — 1 hour and 10 minutes

Total marks — 60

Attempt ALL questions.

You may NOT use a calculator.

Full credit will be given only to solutions which contain appropriate working.

State the units for your answer where appropriate.

Write your answers clearly in the answer booklet provided. In the answer booklet you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not you may lose all the marks for this paper.

EP30H01

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\mathbf{a.b} = |\mathbf{a}||\mathbf{b}| \cos \theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$
\begin{aligned}
\sin (A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos (A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2\sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}
$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

MARKS

Total marks — 60

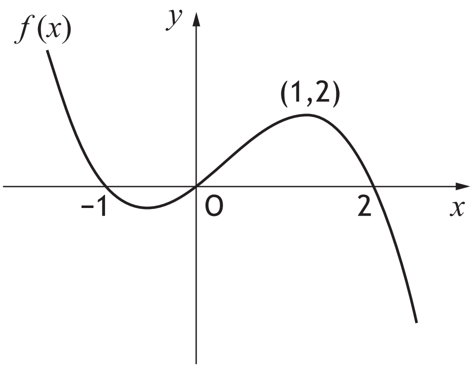
Attempt ALL questions

1. The point P (5,12) lies on the curve with equation $y = x^2 - 4x + 7$.

Find the equation of the tangent to this curve at P. **3**

2. The diagram shows the curve with equation $y = f(x)$, where $f(x) = kx(x+a)(x+b)$.

The curve passes through (-1,0), (0,0), (1,2) and (2,0).



Find the values of a, b and k. **3**

3. Evaluate $\int_1^2 \frac{1}{6}x^{-2}\,dx$. **3**

4. For the function $f(x) = 2 - 3\sin\left(x - \frac{\pi}{3}\right)$ in the interval $0 \leq x < 2\pi$, determine which two of the following statements are true **and justify your answer**. **3**

Statement A The maximum value of $f(x)$ is -1.

Statement B The maximum value of $f(x)$ is 5.

Statement C The maximum value occurs when $x = \frac{5\pi}{6}$.

Statement D The maximum value occurs when $x = \frac{11\pi}{6}$.

5. For the polynomial, $x^3 - 4x^2 + ax + b$

- $x - 1$ is a factor
- -12 is the remainder when it is divided by $x - 2$

(a) Determine the values of a and b. **4**

(b) Hence solve $x^3 - 4x^2 + ax + b = 0$. **4**

6. (a) Find the equation of l_1, the perpendicular bisector of the line joining P (3,–3) and Q (–1,9). **4**

(b) Find the equation of l_2 which is parallel to PQ and passes through R (1,–2). **2**

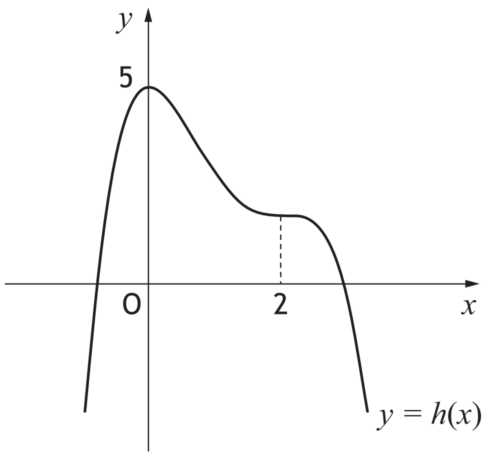
(c) Find the point of intersection of l_1 and l_2. **3**

(d) Hence find the shortest distance between PQ and l_2. **2**

7. (a) Solve $\cos 2x° - 3\cos x° + 2 = 0$ for $0 \leq x < 360$. **5**

(b) Hence solve $\cos 4x° - 3\cos 2x° + 2 = 0$ for $0 \leq x < 360$. **2**

8. The diagram below shows the graph of a quartic $y = h(x)$, with stationary points at $x = 0$ and $x = 2$.



On separate diagrams sketch the graphs of:

(a) $y = 2 - h(x)$. **3**

(b) $y = h'(x)$. **3**

MARKS

9. The expression $\cos 4x - \sqrt{3}\sin 4x$ can be written in the form $k\cos(4x + a)$ where $k > 0$ and $0 \leq a \leq 2\pi$.

(a) Calculate the values of k and a. **4**

(b) Find the points of intersection of the graph of $y = \cos 4x - \sqrt{3}\sin 4x$ with the x axis, in the interval $0 \leq x \leq \frac{\pi}{2}$. **3**

10. The gradient of a tangent to a curve is given by $\frac{dy}{dx} = 3\cos 2x$.

The curve passes through the point $\left(\frac{7\pi}{6}, \sqrt{3}\right)$.

Find y in terms of x. **4**

11. Functions f and g are defined on suitable domains by $f(x) = x^3 - 1$ and $g(x) = 3x + 1$.

(a) Find an expression for $k(x)$, where $k(x) = g(f(x))$. **2**

(b) If $h(k(x)) = x$, find an expression for $h(x)$. **3**

[END OF EXEMPLAR QUESTION PAPER]

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Marking Instructions

These Marking Instructions have been provided to show how SQA would mark this Exemplar Question Paper.

SQA

General Marking Principles for Higher Mathematics

This information is provided to help you understand the general principles you must apply when marking candidate responses to questions in this Paper. These principles must be read in conjunction with the Detailed Marking Instructions, which identify the key features required in candidate responses.

(a) Marks for each candidate response must always be assigned in line with these General Marking Principles and the Detailed Marking Instructions for this assessment.

(b) Marking should always be positive. This means that, for each candidate response, marks are accumulated for the demonstration of relevant skills, knowledge and understanding: they are not deducted from a maximum on the basis of errors or omissions.

(c) Credit must be assigned in accordance with the specific assessment guidelines.

(d) Candidates may use any mathematically correct method to answer questions except in cases where a particular method is specified or excluded.

(e) Working subsequent to an error must be followed through, with possible credit for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working is easier, candidates lose the opportunity to gain credit.

(f) Where transcription errors occur, candidates would normally lose the opportunity to gain a processing mark.

(g) Scored-out or erased working which has not been replaced should be marked where still legible. However, if the scored-out or erased working has been replaced, only the work which has not been scored out should be judged.

(h) Unless specifically mentioned in the specific assessment guidelines, do not penalise:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in solutions
- a repeated error within a question

Definitions of Mathematics-specific command words used in this Paper are:

Determine: obtain an answer from given facts, figures or information;

Expand: multiply out an algebraic expression by making use of the distributive law or a compound trigonometric expression by making use of one of the addition formulae for $\sin(A \pm B)$ or $\cos(A \pm B)$;

Express: use given information to rewrite an expression in a specified form;

Find: obtain an answer showing relevant stages of working;

Hence: use the previous answer to proceed;

Hence, or otherwise: use the previous answer to proceed; however, another method may alternatively be used;

Identify: provide an answer from a number of possibilities;

Justify: show good reason(s) for the conclusion(s) reached;

Show that: use mathematics to prove something, eg that a statement or given value is correct — all steps, including the required conclusion, must be shown;

Sketch: give a general idea of the required shape or relationship and annotate with all relevant points and features;

Solve: obtain the answer(s) using algebraic and/or numerical and/or graphical methods.

Detailed Marking Instructions for each question

Question		Expected response (Give one mark for each •)	Max mark	Additional guidance (Illustration of evidence for awarding a mark at each •)
1		$y-12=6(x-5)$	3	
		•¹ know to differentiate •² calculate gradient •³ state equation of tangent		•¹ $2x-4$ •² 6 •³ $y-12=6(x-5)$
2		$a=1$, $b=-2$ and $k=-1$	3	
		•¹ interpret a and b •² know to substitute (1, 2) •³ state the value of k		•¹ $a=1$, $b=-2$ or $a=-2$, $b=1$ •² $2=k\times1\times(1+1)\times(1-2)$ •³ -1
3		$\frac{1}{12}$	3	
		•¹ complete integration •² substitute limits •³ evaluate		•¹ $-\frac{1}{6}x^{-1}$ •² $\left(-\frac{1}{6\times2}\right)-\left(-\frac{1}{6\times1}\right)$ •³ $\frac{1}{12}$
4		Statements B and D are true.	3	
		•¹ statements B and D correct •² calculate maximum value •³ calculate value of x		•¹ B **and** D •² max is $2-3\times-1$ or $f\left(\frac{11\pi}{6}\right)=2-3\sin\left(\frac{11\pi}{6}-\frac{\pi}{3}\right)=2-3\sin\left(\frac{3\pi}{2}\right)=5$ •³ $x-\frac{\pi}{3}=\frac{3\pi}{2}\Rightarrow x=\frac{3\pi}{2}+\frac{\pi}{3}\Rightarrow x=\frac{11\pi}{6}$

<table>
<tr><td>5</td><td>(a)</td><td>$a=-7$ and $b=10$</td><td rowspan="2">4</td><td></td></tr>
<tr><td></td><td></td><td>●¹ know to use $x=1$ and obtain an equation

●² know to use $x=2$ and obtain an equation

●³ process equations to find one value

●⁴ find the other value</td><td>●¹ $(1)^3-4(1)^2+a(1)+b=0$

●² $(2)^3-4(2)^2+a(2)+b=-12$

●³ $a=-7$ and $b=10$

●⁴ $b=10$ and $a=-7$</td></tr>
<tr><td colspan="2">Notes</td><td colspan="3">1 An incorrect value at ●³ should be followed through for the possible award of ●⁴. However, if the equations are such that no solution exists, then ●³ and ●⁴ are not available.
2 Synthetic Division is an acceptable alternative method.</td></tr>
<tr><td>5</td><td>(b)</td><td>$x=1$, $x=5$, $x=-2$</td><td rowspan="2">4</td><td></td></tr>
<tr><td></td><td></td><td>●⁵ substitute for a and b and know to divide by $x-1$

●⁶ obtain quadratic factor

●⁷ complete factorisation

●⁸ state solution</td><td>●⁵ $\left(x^3-4x^2-7x+10\right)\div(x-1)$
stated or implied by ●⁶

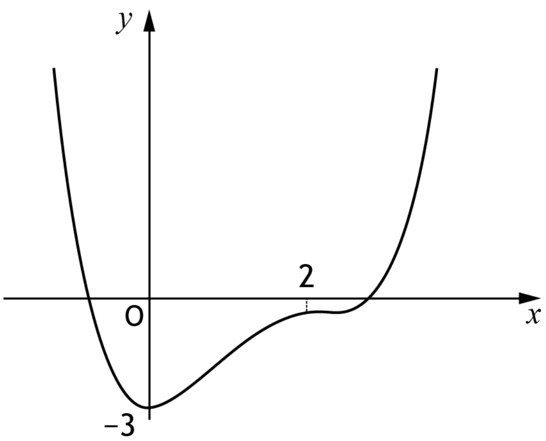
●⁶ $(x-1)\left(x^2-3x-10\right)$

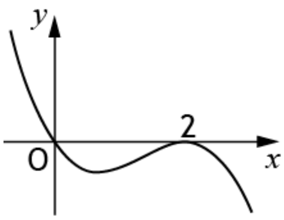
●⁷ $(x-1)(x-5)(x+2)$

●⁸ $x=1$, $x=5$, $x=-2$</td></tr>
<tr><td colspan="2">Notes</td><td colspan="3">3 For candidates who substitute $a=-7$ into the correct quotient from part (a), ●⁵, ●⁶ and ●⁷ are available.
4 Candidates who use incorrect values obtained in part (a) may gain ●⁵, ●⁶ and ●⁷.
5 Where the quadratic factor obtained is irreducible, candidates must clearly demonstrate that $b^2-4ac<0$ to gain mark ●⁷.
6 Do not penalise the inclusion of "$=0$" or for solving for x.
7 Candidates who use values, ex nihilo, for a and b can gain ●⁵, if division is correct.</td></tr>
</table>

6	**(a)**	$y-3=\frac{1}{3}(x-1)$	4	
		●[1] find midpoint of PQ ●[2] find gradient of PQ ●[3] interpret perpendicular gradient ●[4] state equation of perpendicular bisector		●[1] (1, 3) ●[2] -3 ●[3] $\frac{1}{3}$ ●[4] $y-3=\frac{1}{3}(x-1)$
Notes		1 ●[4] is only available if a midpoint **and** a perpendicular gradient are used. 2 Candidates who use $y=mx+c$ must obtain a numerical value for c before ●[4] is available.		
6	**(b)**	$y-(-2)=-3(x-1)$	2	
		●[5] use parallel gradients ●[6] state equation of line		●[5] -3 ●[6] $y-(-2)=-3(x-1)$
Notes		3 ●[6] is only available to candidates who use R and their gradient of PQ from (a).		
6	**(c)**	$x=-\frac{1}{2},\ y=\frac{5}{2}$	3	
		●[7] use valid approach ●[8] solve for one variable ●[9] solve for other variable		●[7] $x-3y=-8$ and $9x+3y=3$ **or** $-3x+1=\frac{1}{3}x+\frac{8}{3}$ **or** $3(3y-8)+y=1$ ●[8] $x=-\frac{1}{2}$ ●[9] $y=\frac{5}{2}$
Notes		4 **Neither** $x-3y=-8$ and $3x+y=1$ **nor** $y=-3x+1$ and $3y=x+8$ are sufficient to gain ●[7]. 5 ●[7], ●[8] and ●[9] are not available to candidates who: — equate zeros — give answers only, without working — use R for equations in both (a) and (b) — use the same gradient for the lines in (a) and (b)		

6	**(d)**	$\sqrt{\frac{5}{2}}$	2	
		●[10] identify appropriate points ●[11] calculate distance		●[10] (1, 3) and $\left(-\frac{1}{2},\frac{5}{2}\right)$ ●[11] $\sqrt{\frac{5}{2}}$ accept $\frac{\sqrt{10}}{2}$ or $\sqrt{2\cdot5}$
Notes		6 ●[10] and ●[11] are only available for considering the distance between the midpoint of PQ and the candidate's answer from (c) **or** for considering the perpendicular distance from P or Q to l_2. 7 At least one coordinate at ●[10] stage must be a fraction for ●[11] to be available. 8 There should only be one calculation of a distance to gain ●[11].		
7	**(a)**	$0, 60, 300$	5	
		●[1] know to use double angle formula ●[2] express as a quadratic in $\cos x°$ ●[3] start to solve ●[4] reduce to equations in $\cos x°$ only ●[5] process solutions in given domain		**Method 1:** Using factorisation ●[1] $2\cos^2 x° - 1...$ **stated or implied by ●[2]** ●[2] $2\cos^2 x° - 3\cos x° + 1 = 0$ ●[3] $(2\cos x° - 1)(\cos x° - 1)$ $= 0$ must appear at either of these lines to gain ●[2] **Method 2:** Using quadratic formula ●[1] $2\cos^2 x° - 1...$ **stated or implied by ●[2]** ●[2] $2\cos^2 x° - 3\cos x° + 1 = 0$ **stated explicitly** ●[3] $\frac{-(-3)\pm\sqrt{(-3)^2-4\times2\times1}}{2\times2}$ **In both methods:** ●[4] $\cos x° = \frac{1}{2}$ and $\cos x° = 1$ ●[5] $0, 60, 300$ Candidates who include 360 lose ●[5]. **or** ●[4] $\cos x = 1$ and $x = 0$ ●[5] $\cos x° = \frac{1}{2}$ and $x = 60$ or 300 Candidates who include 360 lose ●[5].
Notes		1 ●[1] is not available for simply stating that $\cos 2\mathrm{A} = 2\cos^2\mathrm{A} - 1$ with no further working. 2 In the event of $\cos^2 x - \sin^2 x$ or $1 - 2\sin^2 x$ being substituted for $\cos 2x$, ●[1] cannot		

		be awarded until the equation reduces to a quadratic in $\cos x$. 3 Substituting $\cos 2\text{A} = 2\cos^2 \text{A} - 1$ or $\cos 2a = 2\cos^2 a - 1$ etc should be treated as bad form throughout. 4 Candidates may express the quadratic equation obtained at the ●[2] stage in the form $2c^2 - 3c + 1$ or $2x^2 - 3x + 1$ etc. For candidates who do not solve a trigonometric quadratic equation at ●[5], $\cos x$ must appear explicitly to gain ●[4]. 5 ●[4] and ●[5] are only available as a consequence of solving a quadratic equation. 6 Any attempt to solve $ax^2 + bx = c$ loses ●[3], ●[4] and ●[5]. 7 ●[5] is not available to candidates who work in radian measure and do not convert their answers into degree measure.		
7	**(b)**	0, 30, 150, 180, 210 and 330	2	
		●[6] interpret relationship with (a) ●[7] state valid values		●[6] $2x = 0$ and 60 and 300 ●[7] 0, 30, 150, 180, 210 and 330
Notes		8 Do not penalise the inclusion of 360 in (b). 9 Ignore extra answers, correct or incorrect, outside the given interval, but penalise incorrect answers within the interval. 10 Do not penalise candidates who use radians in (b) if they have already been penalised in (a). 11 Candidates who go back to "first principles" for (b) can only gain ●[6] and ●[7] for a correct method leading to valid solutions.		
8	**(a)**		3	
		●[1] reflection in x-axis ●[2] translation $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$ ●[3] annotation of "transformed" graph		●[1] reflection of graph in x-axis ●[2] graph moves parallel to y-axis by 2 units upwards ●[3] two "transformed" points appropriately annotated

Notes		1 All graphs must include both the x and y axes (labelled or unlabelled), however the origin need not be labelled. 2 No marks are available unless a graph is attempted. 3 No marks are available to a candidate who makes several attempts at a graph on the same diagram, unless it is clear which is the final graph. 4 A linear graph gains no marks in both (a) and (b). 5 For $\bullet^3$ "transformed" means a reflection followed by a translation. 6 $\bullet^1$ and $\bullet^2$ apply to the entire curve. 7 A reflection in any line parallel to the y-axis does not gain $\bullet^1$ or $\bullet^3$. 8 A translation other than $\begin{bmatrix} 0 \\ 2 \end{bmatrix}$ does not gain $\bullet^2$ or $\bullet^3$.		
8	**(b)**		3	
		$\bullet^4$ identify roots $\bullet^5$ interpret point of inflection $\bullet^6$ complete cubic curve		$\bullet^4$ 0 **and** 2 only $\bullet^5$ turning point at (2, 0) $\bullet^6$ cubic passing through origin with negative gradient
9	**(a)**	$k = 2$ and $a = \frac{\pi}{3}$	4	
		$\bullet^1$ use appropriate compound angle formula $\bullet^2$ compare coefficients $\bullet^3$ process k $\bullet^4$ process a		$\bullet^1$ $k\cos A\cos B - k\sin A\sin B$ **stated explicitly** $\bullet^2$ $k\cos a = 1$ and $k\sin a = \sqrt{3}$ **stated explicitly** $\bullet^3$ 2 (do not accept $\sqrt{4}$) $\bullet^4$ $\frac{\pi}{3}$ **but** must be consistent with $\bullet^2$
Notes		1 Treat $k\cos A\cos B - \sin A\sin B$ as bad form only if the equations at the $\bullet^2$ stage both contain k. 2 $2\cos A\cos B - 2\sin A\sin B$ or $2(\cos A\cos B - \sin A\sin B)$ is acceptable for $\bullet^1$ and $\bullet^3$. 3 Accept $k\cos a = 1$ and $-k\sin a = -\sqrt{3}$ for $\bullet^2$. 4 $\bullet^2$ is not available for $k\cos 4x = 1$ and $k\sin 4x = \sqrt{3}$, however, $\bullet^4$ is still available. 5 $\bullet^4$ is only available for a single value of a. 6 Candidates who work in degrees and do not convert to radian measure in (a) do not gain $\bullet^4$.		

		7 Candidates may use any form of the wave equation for ●1, ●2 and ●3, however, ●4 is only available if the value of a is interpreted for the form $k\cos(4x+a)$.		
9	(b)	$\left(\frac{\pi}{24},0\right)$ $\left(\frac{7\pi}{24},0\right)$	3	
		●5 strategy for finding roots		●5 $2\cos\left(4x+\frac{\pi}{3}\right)=0$ or $\sqrt{3}\sin 4x=\cos 4x$
		●6 start to solve for multiple angles		●6 $4x=\left(\frac{\pi}{2}-\frac{\pi}{3}\right),\left(\frac{3\pi}{2}-\frac{\pi}{3}\right)...$
		●7 state both roots in given domain		●7 $\frac{\pi}{24},\frac{7\pi}{24}$
Notes		8 Candidates should only be penalised once for leaving their answer in degrees in (a) and (b). 9 If the expression used in (b) is not consistent with (a) then only ●6 and ●7 are available. 10 Correct roots without working cannot gain ●6 but will gain ●7. 11 Candidates should only be penalised once for not simplifying $\sqrt{4}$ in (a) and (b).		
10		$y=\frac{3}{2}\sin 2x+\frac{\sqrt{3}}{4}$	4	
		●1 know to integrate		●1 $\frac{3}{2}\sin 2x+...$
		●2 substitute $\left(\frac{7\pi}{6},\sqrt{3}\right)$		●2 $\sqrt{3}=\frac{3}{2}\sin\left(2\times\frac{7\pi}{6}\right)+c$
		●3 use exact values		●3 $\sqrt{3}=\frac{3}{2}\times\left(\frac{\sqrt{3}}{2}\right)+c$
		●4 express y in terms of x		●4 $y=\frac{3}{2}\sin 2x+\frac{\sqrt{3}}{4}$
11	(a)	$3\left(x^3-1\right)+1$	2	
		●1 interpret notation		●1 $g\left(x^3-1\right)$
		●2 complete process		●2 $3\left(x^3-1\right)+1$

11	**(b)**	$h(x)=\sqrt[3]{\frac{x+2}{3}}$	3	
		•[3] start to rearrange for $x=$ •[4] rearrange •[5] write in functional form: $h(x)=$ or $y=$		•[3] $3x^3=y+2$ •[4] $x=\sqrt[3]{\frac{y+2}{3}}$ •[5] $h(x)=\sqrt[3]{\frac{x+2}{3}}$

[END OF EXEMPLAR MARKING INSTRUCTIONS]

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Mathematics
Paper 2

Date — Not applicable

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Use **blue** or **black** ink.

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EP30H02

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\mathbf{a.b} = |\mathbf{a}||\mathbf{b}| \cos \theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\begin{aligned}
\sin(\mathrm{A} \pm \mathrm{B}) &= \sin \mathrm{A} \cos \mathrm{B} \pm \cos \mathrm{A} \sin \mathrm{B} \\
\cos(\mathrm{A} \pm \mathrm{B}) &= \cos \mathrm{A} \cos \mathrm{B} \mp \sin \mathrm{A} \sin \mathrm{B} \\
\sin 2\mathrm{A} &= 2\sin \mathrm{A} \cos \mathrm{A} \\
\cos 2\mathrm{A} &= \cos^2 \mathrm{A} - \sin^2 \mathrm{A} \\
&= 2\cos^2 \mathrm{A} - 1 \\
&= 1 - 2\sin^2 \mathrm{A}
\end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$ $\cos ax$	$a \cos ax$ $-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

Total marks — 70 **MARKS**

Attempt ALL questions

1. A sequence is defined by $u_{n+1} = -\frac{1}{2}u_n$ with $u_0 = -16$.

(a) Determine the values of u_1 and u_2. **1**

(b) A second sequence is given by 4, 5, 7, 11, ...

It is generated by the recurrence relation $v_{n+1} = pv_n + q$ with $v_1 = 4$.

Find the values of p and q. **3**

(c) Either the sequence in (a) or the sequence in (b) has a limit.

(i) Calculate this limit.

(ii) Why does this other sequence not have a limit? **3**

2. (a) Relative to a suitable set of coordinate axes, Diagram 1 shows the line $2x - y + 5 = 0$ intersecting the circle $x^2 + y^2 - 6x - 2y - 30 = 0$ at the points P and Q.

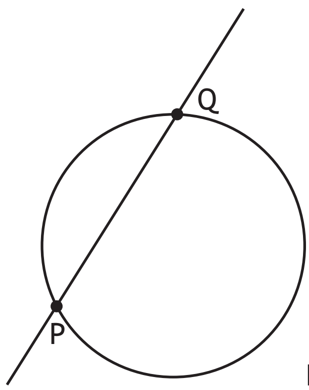


Diagram 1

Find the coordinates of P and Q. **6**

(b) Diagram 2 shows the circle from (a) and a second congruent circle, which also passes through P and Q.

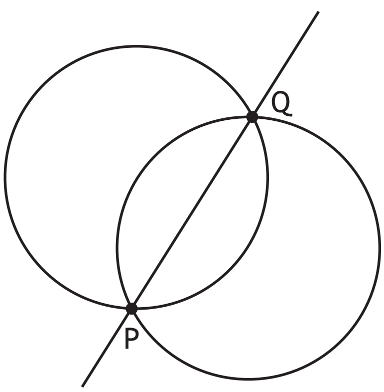


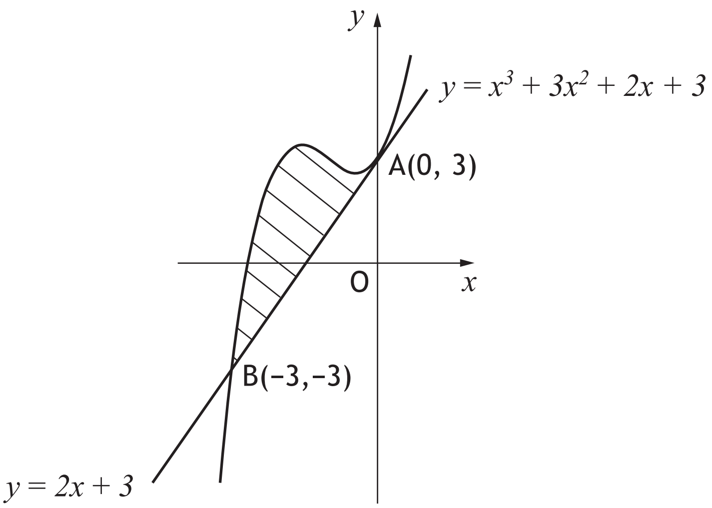
Diagram 2

Determine the equation of this second circle. **6**

MARKS

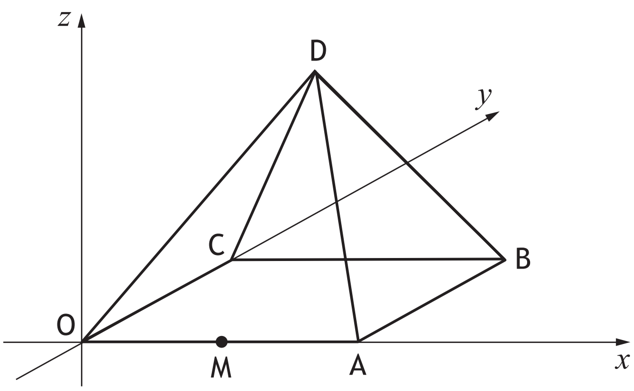
3. Find the value of p such that the equation $x^2+(p+1)x+9=0$ has no real roots. **4**

4. The line with equation $y=2x+3$ is a tangent to the curve with equation $y=x^3+3x^2+2x+3$ at A (0, 3), as shown.



The line meets the curve again at B (−3,−3). Find the area enclosed by the line and the curve. **5**

5. D,OABC is a square-based pyramid as shown.



O is the origin and OA = 4 units.

M is the mid-point of OA.

$\overrightarrow{OD} = 2\mathbf{i} + 2\mathbf{j} + 6\mathbf{k}$

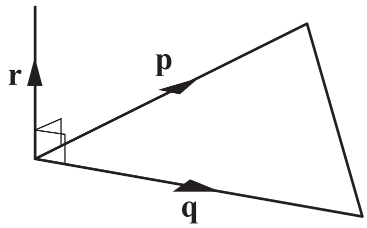
(a) Express $\overrightarrow{OB}$ in terms of $\mathbf{i}$ and $\mathbf{j}$ and $\mathbf{k}$. **1**

(b) Express $\overrightarrow{DB}$ and $\overrightarrow{DM}$ in component form. **3**

(c) Find the size of angle BDM. **5**

MARKS

6. An equilateral triangle with sides of length 3 units is shown.



Vector $\mathbf{r}$ is 2 units long and is perpendicular to both vectors $\mathbf{p}$ and $\mathbf{q}$.

Calculate the value of the scalar product $\mathbf{p}.(\mathbf{p}+\mathbf{q}+\mathbf{r})$. **4**

7. The concentration of the pesticide, *Xpesto*, in soil can be modelled by the equation.

$$P_t = P_0 e^{-kt}$$

where:

- P_0 is the initial concentration;
- P_t is the concentration at time t;
- t is the time, in days, after the application of the pesticide.

Once in the soil, the half-life of a pesticide is the time taken for its concentration to be reduced to one half of its initial value.

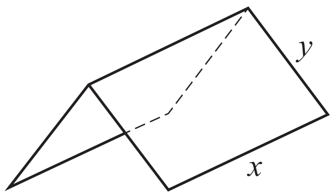
(a) If the half-life of *Xpesto* is 25 days, find the value of k to 2 significant figures. **4**

On all *Xpesto* packaging, the manufacturer states that 80 days after application the concentration of *Xpesto* in the soil will have decreased by over 90%.

(b) Is this statement correct? Justify your answer. **4**

8. Given that $\int_{\frac{\pi}{8}}^{a} 5\sin(4x-\frac{\pi}{2})dx = \frac{10}{4}$, $0 \le a < \frac{\pi}{2}$, calculate the value of a. **6**

9. A manufacturer is asked to design an open-ended shelter, as shown:



The frame of the shelter is to be made of rods of two different lengths:

- x metres for top and bottom edges;
- y metres for each sloping edge.

The total length, L metres, of the rods used in a shelter is given by:

$$L = 3x + \frac{48}{x}$$

To minimise production costs, the total length of rods used for a frame should be as small as possible.

(a) Find the value of x for which L is a minimum. **5**

The rods used for the frame cost £8·25 per metre.

The manufacturer claims that the minimum cost of a frame is less than £195.

(b) Is this claim correct? Justify your answer. **2**

MARKS

10. Acceleration is defined as the rate of change of velocity.

An object is travelling in a straight line. The velocity, v m/s, of this object, t seconds after the start of the motion, is given by $v(t) = 8\cos(2t - \frac{\pi}{2})$.

(a) Find a formula for $a(t)$, the acceleration of this object, t seconds after the start of the motion. **3**

(b) Determine whether the velocity of the object is increasing or decreasing when $t = 10$. **2**

(c) Velocity is defined as the rate of change of displacement.

Determine a formula for $s(t)$, the displacement of the object, given that $s(t) = 4$ when $t = 0$. **3**

[END OF EXEMPLAR QUESTION PAPER]

H

National
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EP30/H/02

Mathematics
Paper 2

Marking Instructions

These Marking Instructions have been provided to show how SQA would mark this Exemplar Question Paper.

SQA
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General Marking Principles for Higher Mathematics

This information is provided to help you understand the general principles you must apply when marking candidate responses to questions in this Paper. These principles must be read in conjunction with the Detailed Marking Instructions, which identify the key features required in candidate responses.

(a) Marks for each candidate response must always be assigned in line with these General Marking Principles and the Detailed Marking Instructions for this assessment.

(b) Marking should always be positive. This means that, for each candidate response, marks are accumulated for the demonstration of relevant skills, knowledge and understanding: they are not deducted from a maximum on the basis of errors or omissions.

(c) Credit must be assigned in accordance with the specific assessment guidelines.

(d) Candidates may use any mathematically correct method to answer questions except in cases where a particular method is specified or excluded.

(e) Working subsequent to an error must be followed through, with possible credit for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working is easier, candidates lose the opportunity to gain credit.

(f) Where transcription errors occur, candidates would normally lose the opportunity to gain a processing mark.

(g) Scored-out or erased working which has not been replaced should be marked where still legible. However, if the scored-out or erased working has been replaced, only the work which has not been scored out should be judged.

(h) Unless specifically mentioned in the specific assessment guidelines, do not penalise:

- working subsequent to a correct answer
- correct working in the wrong part of a question
- legitimate variations in solutions
- a repeated error within a question

Definitions of Mathematics-specific command words used in this Paper are:

Determine: obtain an answer from given facts, figures or information;

Expand: multiply out an algebraic expression by making use of the distributive law or a compound trigonometric expression by making use of one of the addition formulae for $\sin(A \pm B)$ or $\cos(A \pm B)$;

Express: use given information to rewrite an expression in a specified form;

Find: obtain an answer showing relevant stages of working;

Hence: use the previous answer to proceed;

Hence, or otherwise: use the previous answer to proceed; however, another method may alternatively be used;

Identify: provide an answer from a number of possibilities;

Justify: show good reason(s) for the conclusion(s) reached;

Show that: use mathematics to prove something, eg that a statement or given value is correct — all steps, including the required conclusion, must be shown;

Sketch: give a general idea of the required shape or relationship and annotate with all relevant points and features;

Solve: obtain the answer(s) using algebraic and/or numerical and/or graphical methods.

Detailed Marking Instructions for each question

Question			Expected Response (Give one mark for each •)	Max mark	Additional Guidance (Illustration of evidence for awarding a mark at each •)
1	(a)		$u_1 = 8$ and $u_2 = -4$	1	
			•[1] find terms of sequence		•[1] $u_1 = 8$ and $u_2 = -4$
1	(b)		$p = 2$ or $q = -3$	3	
			•[2] interpret sequence		•[2] eg $4p + q = 5$ and $5p + q = 7$
			•[3] solve for one variable		•[3] $p = 2$ or $q = -3$
			•[4] state second variable		•[4] $q = -3$ or $p = 2$
Notes			1 Candidates may use $7p + q = 11$ as one of their equations at •[2]. 2 Treat equations like $p4 + q = 5$ or $p(4) + q = 5$ as bad form. 3 Candidates should not be penalised for using $u_{n+1} = pu_n + q$.		
1	(c)	(i)	$l = 0$, $-1 < p < 1$	3	
			•[5] know how to find a valid limit		•[5] $l = -\frac{1}{2}l$ or $l = \dfrac{0}{1-\left(-\frac{1}{2}\right)}$
			•[6] calculate a valid limit only		•[6] $l = 0$
		(ii)	•[7] state reason		•[7] outside interval $-1 < p < 1$
Notes			4 Just stating that $l = al + b$ or $l = \dfrac{b}{1-a}$ is not sufficient for •[5]. 5 Any calculations based on formulae masquerading as a limit rule cannot gain •[5] and •[6]. 6 For candidates who use "$b = 0$", •[6] is only available to those who simplify $\dfrac{0}{\ldots}$ to 0. 7 Accept $2 > 1$ or $p > 1$ for •[7]. This may be expressed in words. 8 Candidates who use a without reference to p or 2 cannot gain •[7].		

2	**(a)**	P $(-3, -1)$ Q $(1, 7)$	6	
				Substituting for y
		$\bullet^1$ rearrange linear equation		$\bullet^1$ $y = 2x + 5$ **stated or implied by** $\bullet^2$
		$\bullet^2$ substitute into circle		$\bullet^2$ $\ldots(2x+5)^2 \ldots -2(2x+5)\ldots$
		$\bullet^3$ express in standard form		$\bullet^3$ $5x^2 + 10x - 15 = 0$
		$\bullet^4$ start to solve		$\bullet^4$eg $5(x+3)(x-1)$ — $= 0$ must appear at the $\bullet^3$ or $\bullet^4$ stage to gain $\bullet^3$
		$\bullet^5$ state roots		$\bullet^5$ $x = -3$ and $x = 1$
		$\bullet^6$ determine corresponding y coordinates		$\bullet^6$ $y = -1$ and $y = 7$
				Substituting for x
				$\bullet^1$ $x = \dfrac{y-5}{2}$ **stated or implied by** $\bullet^2$
				$\bullet^2$ $\left(\dfrac{y-5}{2}\right)^2 \ldots -6\left(\dfrac{y-5}{2}\right)\ldots$
				$\bullet^3$ $5y^2 - 30y - 35 = 0$
				$\bullet^4$eg $5(y+1)(y-7)$ — $= 0$ must appear at the $\bullet^3$ or $\bullet^4$ stage to gain $\bullet^3$
				$\bullet^5$ $y = -1$ and $y = 7$
				$\bullet^6$ $x = -3$ and $x = 1$

Notes

1. At $\bullet^4$ the quadratic must lead to two real distinct roots for $\bullet^5$ and $\bullet^6$ to be available.
2. Cross marking is available here for $\bullet^5$ and $\bullet^6$.
3. Candidates do not need to distinguish between points P and Q.

2	(b)	$(x+5)^2+(y-5)^2=40$	6	
		●[7] centre of original circle		●[7] (3, 1)
		●[8] radius of original circle		●[8] $\sqrt{40}$ accept $r^2=40$
		Method 1: Using midpoint		**Method 1: Using midpoint**
		●[9] midpoint of chord		●[9] (-1, 3)
		●[10] evidence for finding new centre		●[10] eg stepping out or midpoint formula
		●[11] centre of new circle		●[11] (-5, 5)
		●[12] equation of new circle		●[12] $(x+5)^2+(y-5)^2=40$
		Method 2: Stepping out using P and Q		**Method 2: Stepping out using P and Q**
		●[9] evidence of C_1 to P **or** C_1 to Q		●[9] eg stepping out or vector approach
		●[10] evidence of Q to C_2 **or** P to C_2		●[10] eg stepping out or vector approach
		●[11] centre of new circle		●[11] (-5, 5)
		●[12] equation of new circle		●[12] $(x+5)^2+(y-5)^2=40$
Notes		4 The evidence for ●[7] and ●[8] may appear in (a). 5 Centre (-5, 5) **without working** in method 1 may still gain ●[12] but not ●[10] or ●[11], in method 2 may still gain ●[12] but not ●[9], ●[10] or ●[11]. Any other centre **without working** in method 1 does not gain ●[10], ●[11] or ●[12], in method 2 does not gain ●[9], ●[10], ●[11] or ●[12]. 6 The centre must have been clearly indicated before it is used at the ●[12] stage. 7 Do not accept, eg $\sqrt{40}^2$ or $39\cdot69$, or any other approximations for ●[12]. 8 The evidence for ●[8] may not appear until the candidate states the radius or equation of the second circle.		
3		$-7<p<5$	4	
		●[1] substitute into discriminant		●[1] $(p+1)^2-4\times1\times9$
		●[2] know condition for no real roots		●[2] $b^2-4ac<0$
		●[3] factorise		●[3] $(p-5)(p+7)<0$
		●[4] solve for p		●[4] $-7<p<5$

4		$\frac{27}{4}$	5	
		•¹ know to integrate and interpret limits •² use "upper − lower" •³ integrate •⁴ substitute limits •⁵ evaluate area		•¹ $\int_{-3}^{0} \ldots \ldots \ldots$ •² $\int_{-3}^{0}\left(x^3+3x^2+2x+3\right)-(2x+3)\,dx$ •³ $\frac{1}{4}x^4+x^3$ •⁴ $0-\left(\frac{1}{4}(-3)^4+(-3)^3\right)$ •⁵ $\frac{27}{4}$ units^2
Notes		1 Where a candidate differentiates one or more terms at •³ then •⁴ and •⁵ are not available. 2 Candidates who substitute without integrating at •² do not gain •³, •⁴ and •⁵. 3 Candidates must show evidence that they have considered the upper limit 0 at •⁴. 4 Where candidates show no evidence for both •³ and •⁴, but arrive at the correct area, then •³, •⁴ and •⁵ are not available. 5 The omission of dx at •² should not be penalised.		
5	(a)	$\overrightarrow{OB} = 4\mathbf{i} + 4\mathbf{j}$	1	
		•¹ state $\overrightarrow{OB}$ in unit vector form		•¹ $4\mathbf{i}+4\mathbf{j}$
5	(b)	$\overrightarrow{DB} = \begin{pmatrix} 2 \\ 2 \\ -6 \end{pmatrix}$ $\overrightarrow{DM} = \begin{pmatrix} 0 \\ -2 \\ -6 \end{pmatrix}$	3	
		•² state components of $\overrightarrow{DB}$ •³ state coordinates of M •⁴ state components of $\overrightarrow{DM}$		•² $\begin{pmatrix} 2 \\ 2 \\ -6 \end{pmatrix}$ •³ $(2,0,0)$ stated, or implied by •⁴ •⁴ $\begin{pmatrix} 0 \\ -2 \\ -6 \end{pmatrix}$

5	**(c)**	$40 \cdot 3°$ or $0 \cdot 703$ rads	5	
		●[5] know to use scalar product ●[6] find scalar product ●[7] find magnitude of a vector ●[8] find magnitude of a vector ●[9] evaluate angle BDM		●[5] $\cos B\hat{D}M = \dfrac{\overrightarrow{DB}.\ \overrightarrow{DM}}{\lvert\overrightarrow{DB}\rvert . \lvert\overrightarrow{DM}\rvert}$ **stated or implied by ●[9]** ●[6] $\overrightarrow{DB}.\overrightarrow{DM} = 32$ ●[7] $\lvert\overrightarrow{DB}\rvert = \sqrt{44}$ ●[8] $\lvert\overrightarrow{DM}\rvert = \sqrt{40}$ ●[9] $40 \cdot 3°$ or $0 \cdot 703$ rads
Notes		1 ●[5] is not available to candidates who evaluate the wrong angle. 2 If candidates do not attempt ●[9], then ●[5] is only available if the formula quoted relates to the labelling in the question. 3 ●[9] should be awarded to any answer which rounds to $40°$ or $0{\cdot}7$ radians. 4 In the event that both magnitudes are equal **or** there is only one non-zero component, ●[8] is not available.		
6		$\dfrac{27}{2}$	4	
		●[1] use distributive law ●[2] calculate scalar product ●[3] calculate scalar product ●[4] process scalar product $=0$ and complete		●[1] $\mathbf{p.p} + \mathbf{p.q} + \mathbf{p.r}$ ●[2] $\mathbf{p.p} = 9$ ●[3] $\mathbf{p.q} = \dfrac{9}{2}$ ●[4] $\mathbf{p.r} = 0$ and $\dfrac{27}{2}$
7	**(a)**	$k \approx 0 \cdot 028$	4	
		●[1] interpret half-life ●[2] process equation ●[3] write in logarithmic form ●[4] process for k		●[1] $\dfrac{1}{2}P_0 = P_0 e^{-25k}$ **stated or implied by ●[2]** ●[2] $e^{-25k} = \dfrac{1}{2}$ ●[3] $\log_e \dfrac{1}{2} = -25k$ ●[4] $k \approx 0 \cdot 028$
Notes		1 Do not penalise candidates who substitute a numerical value for P_0 in part (a).		

7	**(b)**	No, with reason	4	
		•[5] interpret equation •[6] process •[7] state percentage decrease •[8] justify answer		•[5] $P_t = P_0 e^{-80\times 0\cdot 028}$ •[6] $P_t \approx 0\cdot 1065 P_0$ •[7] 89% •[8] No, the concentration will not have decreased by over 90%. 89% decrease.
Notes		2 For candidates who use a value of k which does not round to $0\cdot 028$, •[5] is not available unless already penalised in part (a). 3 For a value of k ex-nihilo then •[5], •[6] and •[7] are not available. 4 •[6] is only available for candidates who express P_t as a multiple of P_0. 5 Beware of candidates using proportion. This is not a valid strategy.		
8		$\frac{3\pi}{8}$	6	
		•[1] start to integrate •[2] complete integration •[3] process limits •[4] simplify numeric term and equate to $\frac{10}{4}$ •[5] start to solve equation •[6] solve for a		•[1] $-\frac{5}{4}\cos\ldots$ •[2] $-\frac{5}{4}\cos\left(4x-\frac{\pi}{2}\right)$ •[3] $-\frac{5}{4}\cos\left(4a-\frac{\pi}{2}\right)+\frac{5}{4}\cos\left(\frac{4\pi}{8}-\frac{\pi}{2}\right)$ •[4] $-\frac{5}{4}\cos\left(4a-\frac{\pi}{2}\right)+\frac{5}{4}=\frac{10}{4}$ •[5] $\cos\left(4a-\frac{\pi}{2}\right)=-1$ •[6] $a=\frac{3\pi}{8}$
Notes		1 Candidates who include solutions outwith the range cannot gain •[6]. 2 The inclusion of $+c$ at •[1] or •[2] should be treated as bad form. 3 •[6] is only available for a valid numerical answer. 4 Where the candidate differentiates, •[1], •[2] and •[3] are not available. 5 Where the candidate integrates incorrectly, •[3], •[4], •[5] and •[6] are still available. 6 The value of a **must** be given in radians.		

9	**(a)**	4 cm	5	
		•1 prepare to differentiate •2 differentiate •3 equate derivative to 0 •4 process for x •5 verify nature		•1 $\ldots 48x^{-1}$ •2 $3-48x^{-2}$ •3 $3-48x^{-2}=0$ •4 $x=4$ •5 nature table or 2nd derivative
Notes		1 Do not penalise the non-appearance of -4 at •4.		
9	**(b)**	No, (£198 > £195)	2	
		•6 evaluate L •7 calculate cost and justify answer		•6 $L=24$ •7 $24\times £8\cdot 25$ = £198. No and reason (£198 > £195)
Notes		2 Candidates who process $x=-4$ to obtain $L=-24$ do not gain •6. 3 $y=24$ is not awarded •6.		
10	**(a)**	$a(t)=-16\sin\left(2t-\frac{\pi}{2}\right)$	3	
		•1 know to differentiate •2 differentiate trig function •3 applies chain rule		•1 $a=v'(t)$ •2 $-8\sin\left(2t-\frac{\pi}{2}\right)\ldots\ldots$ •3 $\ldots\ldots\times 2$ and complete $a(t)=-16\sin\left(2t-\frac{\pi}{2}\right)$
Notes		1 Alternatively, $8\cos\left(2t-\frac{\pi}{2}\right)=8\sin 2t$ •1 $v'(t)\ldots\ldots$ •2 $=8\cos 2t\ldots$ •3 $=\ldots\times 2$		

10	**(b)**	$a(10) > 0$ therefore increasing	2	
		●4 know to and evaluate $a(10)$ ●5 interpret result		●4 $a(10) = 6 \cdot 53$ ●5 $a(10) > 0$ therefore increasing
Notes		1 ●5 is available only as a consequence of substituting into a derivative. 2 ●4 and ●5 are not available to candidates who work in degrees. 3 ●2 and ●3 may be awarded if they appear in the working for 10(b). However, ●1 requires a clear link between acceleration and $v'(t)$.		
10	**(c)**	$s(t) = 4\sin\left(2t - \frac{\pi}{2}\right) + 8$	3	
		●6 know to integrate ●7 integrate correctly ●8 determine constant and complete		●6 $s(t) = \int v(t)dt$ ●7 $s(t) = 4\sin\left(2t - \frac{\pi}{2}\right) + c$ ●8 $c = 8$ so $s(t) = 4\sin\left(2t - \frac{\pi}{2}\right) + 8$
Notes		4 ●7 and ●8 are not available to candidates who work in degrees. However, accept $\int 8\cos(2t - 90)dt$ for ●6.		

[END OF EXEMPLAR MARKING INSTRUCTIONS]