H

National
Qualifications
SPECIMEN ONLY

SQ30/H/01

Mathematics
Paper 1
(Non-Calculator)

Date — Not applicable

Duration — 1 hour and 10 minutes

Total marks — 60

Attempt ALL questions.

You may NOT use a calculator.

Full credit will be given only to solutions which contain appropriate working.

State the units for your answer where appropriate.

Write your answers clearly in the answer booklet provided. In the answer booklet you must clearly identify the question number you are attempting.

Use **blue** or **black** ink.

Before leaving the examination room you must give your answer booklet to the Invigilator; if you do not you may lose all the marks for this paper.

SQ30H01

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\mathbf{a.b} = |\mathbf{a}||\mathbf{b}| \cos \theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Trigonometric formulae:

$$
\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2\sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}
$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
$\cos ax$	$\frac{1}{a} \sin ax + C$

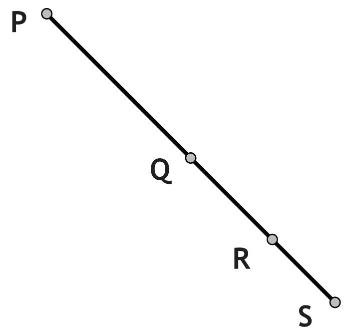
Attempt ALL questions **MARKS**

Total marks — 60

1. Find $\int \frac{3x^3+1}{2x^2}\,dx,\ x \neq 0$. **4**

2. Find the coordinates of the points of intersection of the curve $y = x^3 - 2x^2 + x + 4$ and the line $y = 4x + 4$. **5**

3. In the diagram, P has coordinates $(-6, 3, 9)$,



$\overrightarrow{PQ} = 6\mathbf{i} + 12\mathbf{j} - 6\mathbf{k}$ and $\overrightarrow{PQ} = 2\overrightarrow{QR} = 3\overrightarrow{RS}$.

Find the coordinates of S. **5**

4. Given that $2x^2 + px + p + 6 = 0$ has no real roots, find the range of values for p, where $p \in \mathbb{R}$. **4**

5. Line l_1 has equation $\sqrt{3}y - x = 0$.

(a) Line l_2 is perpendicular to l_1. Find the gradient of l_2. **2**

(b) Calculate the angle l_2 makes with the positive direction of the x-axis. **2**

6. (a) Find an equivalent expression for $\sin(x + 60)^\circ$. **1**

(b) Hence, or otherwise, determine the exact value of $\sin 105^\circ$. **3**

7. (a) Show that $(x + 1)$ is a factor of $x^3 - 13x - 12$. **3**

(b) Factorise $x^3 - 13x - 12$ fully. **2**

MARKS

8. $f(x)$ and $g(x)$ are functions, defined on the set of real numbers, such that

$f(x) = 1 - \frac{1}{2}x$ and $g(x) = 8x^2 - 3$.

(a) Given that $h(x) = g(f(x))$, show that $h(x) = 2x^2 - 8x + 5$. **3**

(b) Express $h(x)$ in the form $a(x+p)^2 + q$. **3**

(c) Hence, or otherwise, state the coordinates of the turning point on the graph of $y = h(x)$. **1**

(d) Sketch the graph of $y = h(x) + 3$, showing clearly the coordinates of the turning point and the y-axis intercept. **2**

9. (a) AB is a line parallel to the line with equation $y + 3x = 25$.

A has coordinates $(-1, 10)$.

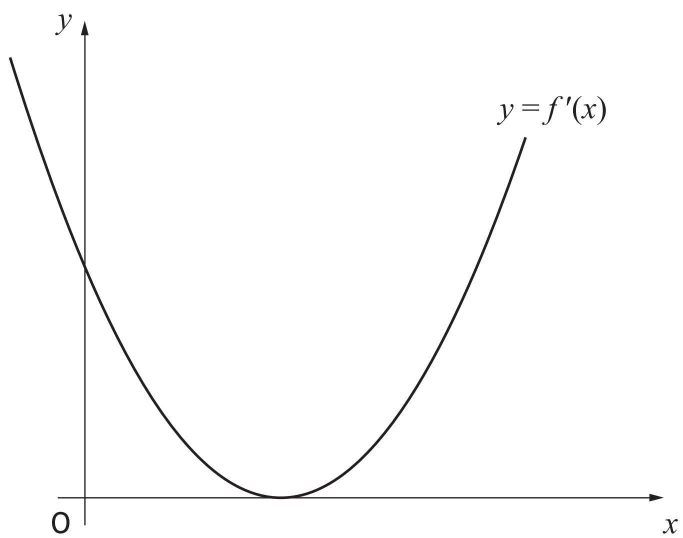
Find the equation of AB. **1**

(b) $3y = x + 11$ is the perpendicular bisector of AB.

Determine the coordinates of B. **5**

10. Find the rate of change of the function $f(x) = 4\sin^3 x$ when $x = \frac{5\pi}{6}$. **3**

11. The diagram shows the graph of $y = f'(x)$. The x-axis is a tangent to this graph.



(a) Explain why the function $f(x)$ is never decreasing. **1**

(b) On a graph of $y = f(x)$, the y-coordinate of the stationary point is negative. Sketch a possible graph for $y = f(x)$. **2**

MARKS

12. The voltage, $V(t)$, produced by a generator is described by the function $V(t) = 120\sin 100\pi t$, $t > 0$, where t is the time in seconds.

(a) Determine the period of $V(t)$. **2**

(b) Find the first three times for which $V(t) = -60$. **6**

[END OF SPECIMEN QUESTION PAPER]

H

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Mathematics
Paper 1
(Non-Calculator)

Marking Instructions

These Marking Instructions have been provided to show how SQA would mark this Specimen Question Paper.

General Marking Principles for Higher Mathematics

This information is provided to help you understand the general principles you must apply when marking candidate responses to questions in this Paper. These principles must be read in conjunction with the detailed marking instructions, which identify the key features required in candidate responses.

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- Working subsequent to a correct answer
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- Repeated error within a question

Definitions of Mathematics-specific command words used in this Specimen Question Paper.

Determine: find a numerical value or values from the information given.

Expand: multiply out an algebraic expression by making use of the distributive law or a compound trigonometric expression by making use of one of the addition formulae for $\sin(A \pm B)$ or $\cos(A \pm B)$.

Show that: use mathematics to prove something, eg that a statement or given value is correct — all steps, including the required conclusion, must be shown.

Express: use given information to rewrite an expression in a specified form.

Hence: use the previous answer to proceed.

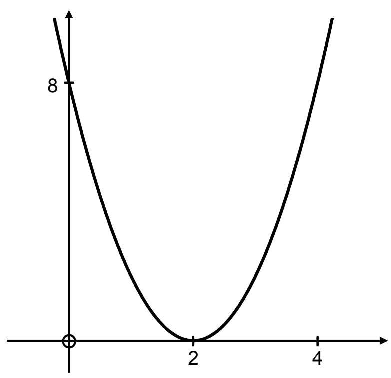
Hence, or otherwise: use the previous answer to proceed; however, another method may alternatively be used.

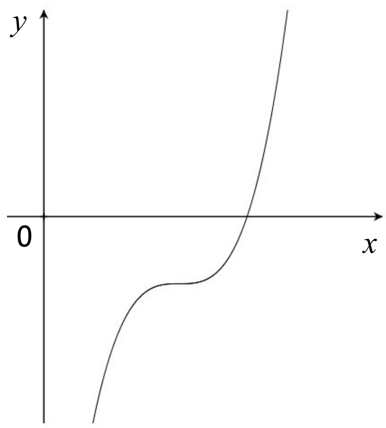
Justify: show good reason(s) for the conclusion(s) reached.

Specific Marking Instructions for each question

Question			Marking scheme. Give one mark for each •	Max mark	Illustration of evidence for awarding a mark at each •
1			Ans: $\frac{3}{4}x^2 - \frac{1}{2}x^{-1} + C$	4	
			•¹ preparation for integration		•¹ $\frac{3}{2}x + \frac{1}{2}x^{-2}$
			•² correct integration of first term		•² $\frac{3}{2} \bullet \frac{x^2}{2} + \ldots$
			•³ correct integration of second term		•³ $\ldots + \frac{1}{2} \bullet \frac{x^{-1}}{-1}$
			•⁴ includes constant of integration		•⁴ $\frac{3}{4}x^2 - \frac{1}{2}x^{-1} + C$
2			Ans: $(-1,0),(0,4),(3,16)$	5	
			•¹ sets equation of curve equal to equation of line		•¹ $x^3 - 2x^2 + x + 4 = 4x + 4$
			•² equates to zero		•² $x^3 - 2x^2 - 3x = 0$
			•³ factorises fully		•³ $x(x+1)(x-3) = 0$
			•⁴ calculates x-coordinates		•⁴ $x = 0, x = -1, x = 3$
			•⁵ calculates y-coordinates		•⁵ $(0,4),(-1,0),(3,16)$
3			Ans: $\mathrm{S}(5,25,-2)$	5	
			•¹ find coordinate of Q or component vector $\mathbf{q}$		•¹ $\mathbf{q} = \mathbf{p} + \overrightarrow{PQ} = \begin{pmatrix} 0 \\ 15 \\ 3 \end{pmatrix}$ or $\mathrm{Q}(0,15,3)$
			•² sets up vector equation for $\mathbf{r}$		•² $\mathbf{r} = \mathbf{q} + \overrightarrow{QR} = \begin{pmatrix} 0 \\ 15 \\ 3 \end{pmatrix} + \begin{pmatrix} 3 \\ 6 \\ -3 \end{pmatrix}$
			•³ find coordinate of R or component vector $\mathbf{r}$		•³ $\mathbf{r} = \begin{pmatrix} 3 \\ 21 \\ 0 \end{pmatrix}$ or $\mathrm{R}(3,21,0)$
			•⁴ sets up vector equation for $\mathbf{s}$		•⁴ $\mathbf{s} = \mathbf{r} + \overrightarrow{RS} = \begin{pmatrix} 3 \\ 21 \\ 0 \end{pmatrix} + \begin{pmatrix} 2 \\ 4 \\ -2 \end{pmatrix}$
			•⁵ find coordinate of S		•⁵ $\mathrm{S}(5,25,-2)$

Question			Marking scheme. Give one mark for each •	Max mark	Illustration of evidence for awarding a mark at each •
4			Ans: $-4 < p < 12$ •¹ know discriminant < 0 •² simplify •³ factorise LHS •⁴ correct range	4	•¹ $b^2 - 4ac < 0$ and $a = 2,\ b = p,\ c = p + 6$ stated or implied by •² •² $p^2 - 8p - 48 < 0$ •³ $(p-12)(p+4) < 0$ •⁴ $-4 < p < 12$
5	(a)		Ans: $m_{l_2} = -\sqrt{3}$ •¹ rearranging equation to calculate gradient of line l_1 •² calculating gradient of l_2	2	•¹ $y = \frac{1}{\sqrt{3}}x \quad m = \frac{1}{\sqrt{3}}$ •² $m_{l_2} = -\sqrt{3}$
	(b)		Ans: $\theta = \frac{2\pi}{3}$ or 120° •³ using $m = \tan\theta$ •⁴ calculating angle	2	•³ $\tan\theta = -\sqrt{3}$ •⁴ $\theta = \frac{2\pi}{3}$ or 120°
6	 (a) (b)		Ans: $\frac{1+\sqrt{3}}{2\sqrt{2}}$ or $\frac{\sqrt{2}+\sqrt{6}}{4}$ •¹ **correct** expansion •² **any** expression equivalent to $\sin 105^\circ$ •³ correct exact value equivalents •⁴ correct answer	4	•¹ $\sin x^\circ \cos 60^\circ + \cos x^\circ \sin 60^\circ$ •² $\sin(45+60)^\circ$ or equivalent •³ $\frac{1}{\sqrt{2}} \times \frac{1}{2} + \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2}$ •⁴ $\frac{1+\sqrt{3}}{2\sqrt{2}}$ or $\frac{\sqrt{2}+\sqrt{6}}{4}$
7	(a)		•¹ know to use $x = -1$ •² complete synthetic division •³ recognition of zero remainder	3	•¹ $\begin{array}{r\|rrrr} -1 & 1 & 0 & -13 & -12 \\ & & -1 & 1 & 12 \\ \hline & 1 & -1 & -12 & 0 \end{array}$ •² $\begin{array}{r\|rrrr} -1 & 1 & 0 & -13 & -12 \\ & & -1 & 1 & 12 \\ \hline & \mathbf{1} & \mathbf{-1} & \mathbf{-12} & \mathbf{0} \end{array}$ •³ $(x+1)$ is a factor as remainder is zero
	(b)		Ans: $(x+1)(x+3)(x-4)$ •⁴ identify quotient •⁵ factorised fully	2	•⁴ $x^2 - x - 12$ •⁵ $(x+1)(x+3)(x-4)$
Notes			Alternative methods of showing $(x+1)$ is a factor, such as long division, inspection and evaluating are perfectly acceptable.		

Question			Marking scheme. Give one mark for each •	Max mark	Illustration of evidence for awarding a mark at each •
8	(a)		Ans: $h(x)=2x^2-8x+5$ •[1] correct substitution •[2] squaring •[3] expanding and simplifying	3	•[1] $h(x)=8\left(1-\frac{1}{2}x\right)^2-3$ •[2] $1-x+\frac{1}{4}x^2$ •[3] $h(x)=2x^2-8x+5$
	(b)		Ans: $2(x-2)^2-3$ •[4] identify common factor •[5] complete the square •[6] process for q	3	•[4] $2(x^2-4x\ldots$ **stated or implied by •[3]** •[5] $2(x-2)^2\ldots$ •[6] $2(x-2)^2-3$
Notes			Values for p and q must be consistent with the value for a.		
	(c)		Ans: $(2,-3)$ •[7] state turning point	1	•[7] $(2,-3)$
	(d)		Ans:  •[8] correct shape •[9] annotation, including y-axis intercept	2	•[8] parabola with minimum turning point labelled (positioned consistently with answer to (b)) •[9] (0,8)

Question			Marking scheme. Give one mark for each •	Max mark	Illustration of evidence for awarding a mark at each •
9	(a)		Ans: $y-10=-3(x+1)$ •[1] finding equation of line	1	•[1] $y-10=-3(x+1)$ or equivalent
	(b)		Ans: $B(3,-2)$ •[2] use of simultaneous equations •[3] solving to find one coordinate of midpoint •[4] finding remaining coordinate of midpoint •[5] using midpoint formula or 'stepping out' •[6] finding coordinates of B	5	•[2] $y=-3x+7$ and $3y=x+11$ •[3] either $x=1$ or $y=4$ •[4] M (1, 4) •[5] either $x=3$ or $y=-2$ •[6] $B(3,-2)$
10			Ans: $\frac{3\sqrt{3}}{2}$ •[1] start to differentiate •[2] complete differentiation •[3] evaluate $f'\left(\frac{5\pi}{6}\right)$	3	•[1] $3\times 4\sin^2 x$ •[2] $\times\cos x$ •[3] $12\left(\frac{1}{2}\right)^2\times\frac{-\sqrt{3}}{2}=12\times\frac{1}{4}\times\frac{-\sqrt{3}}{2}=\frac{-3\sqrt{3}}{2}$
11	(a)		•[1] knows derived function represents gradient and that the minimum value of $f'(x)$ is zero	1	•[1] $m=f'(x)\geq 0$ stated explicitly
	(b)		•[2] interprets information correctly •[3] completes sketch	2	•[2] stationary point plotted in fourth quadrant •[3] point of inflexion on an increasing graph 

Question			Marking scheme. Give one mark for each •	Max mark	Illustration of evidence for awarding a mark at each •
12	(a)		Ans: $\frac{1}{50}$ sec or 0·02 sec	**2**	
			•[1] knows how to find period		•[1] $T = \frac{2\pi}{100\pi}$
			•[2] correct answer		•[2] $\frac{1}{50}$ or 0·02
	(b)		Ans: $\frac{7}{600}$, $\frac{11}{600}$, and $\frac{19}{600}$ sec	**6**	
			•[1] equating function with –60		•[1] $120\sin 100\pi t = -60$
			•[2] rearranging		•[2] $\sin 100\pi t = -\frac{1}{2}$
			•[3] solve equation for $100\pi t$		•[3] $100\pi t = \frac{7\pi}{6}$ and $\frac{11\pi}{6}$
			•[4] process solutions for t		•[4] $t = \frac{7}{600}$ and $\frac{11}{600}$
			•[5] knowing to use period or demonstrating another solution from the third quadrant		•[5] $T = \frac{1}{50}$ or $100\pi t = 3\pi + \frac{\pi}{6}$
			•[6] third value for t		•[6] $\frac{19}{600}$

[END OF SPECIMEN MARKING INSTRUCTIONS]

H

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SQ30/H/02

Mathematics
Paper 2

Date — Not applicable

Duration — 1 hour and 30 minutes

Total marks — 70

Attempt ALL questions.

You may use a calculator.

Full credit will be given only to solutions which contain appropriate working.

State the units for your answer where appropriate.

Write your answers clearly in the answer booklet provided. In the answer booklet you must clearly identify the question number you are attempting.

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SQ30H02

FORMULAE LIST

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Scalar Product: $\mathbf{a}.\mathbf{b} = |\mathbf{a}||\mathbf{b}| \cos \theta$, where θ is the angle between $\mathbf{a}$ and $\mathbf{b}$

or $\mathbf{a}.\mathbf{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$

Trigonometric formulae:

$$\begin{aligned} \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \sin 2A &= 2\sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$ $\cos ax$	$a \cos ax$ $-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a} \cos ax + C$
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Attempt ALL questions

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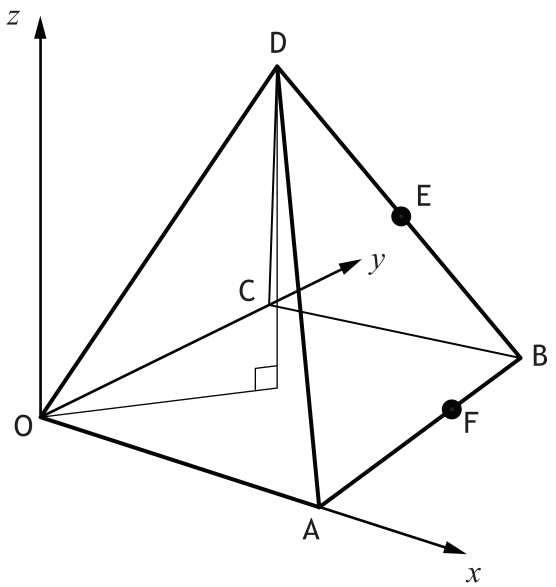
MARKS

1.



A square based right pyramid is shown in the diagram.

Square OABC has a side length of 60 units with edges OA and OC lying on the x-axis and y-axis respectively.

The coordinates of D are (30, 30, 80).

E is the midpoint of BD and F divides AB in the ratio 2:1.

(a) Find the coordinates of E and F. **2**

(b) Calculate $\overrightarrow{ED}.\overrightarrow{EF}$. **2**

(c) Hence, or otherwise, calculate the size of angle DEF. **4**

2. A wildlife reserve has introduced conservation measures to build up the population of an endangered mammal. Initially the reserve population of the mammal was 2000. By the end of the first year there were 2500 and by the end of the second year there were 2980.

It is believed that the population can be modelled by the recurrence relation:

$$u_{n+1} = au_n + b\,,$$

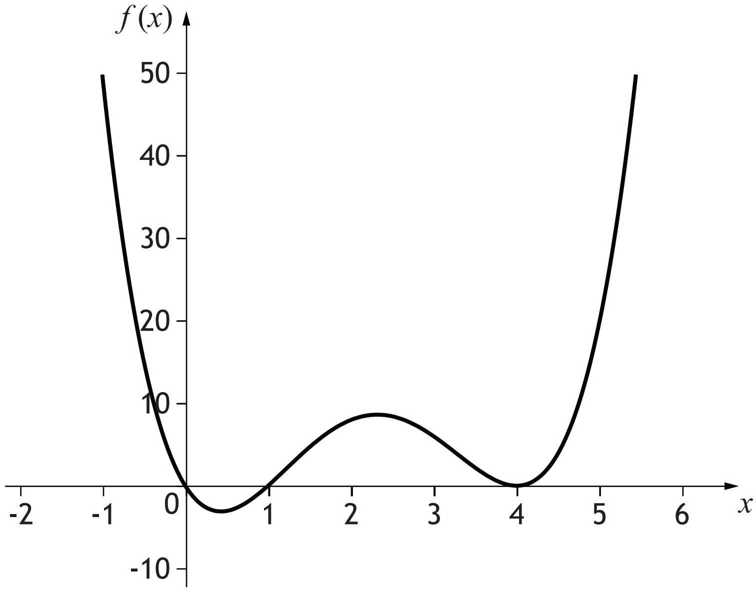
where a and b are constants and n is the number of years since the reserve was set up.

(a) Use the information above to find the values of a and b. **4**

(b) Conservation measures will end if the population stabilises at over 13 000. Will this happen? Justify your answer. **3**

MARKS

3. The diagram shows the graph of $f(x) = x(x-p)(x-q)^2$.



(a) Determine the values of p and q. **1**

(b) Find the equation of the tangent to the curve when $x = 1$. **4**

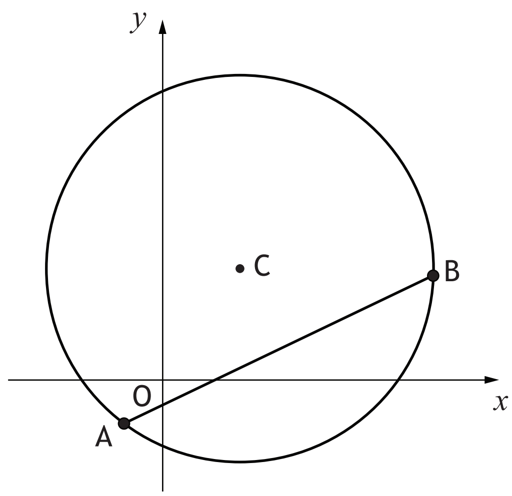
4. (a) Express $y = \log_4 2x$ in the form $y = \log_4 x + k$, clearly stating the value of k. **2**

(b) Hence, or otherwise, describe the relationship between the graphs of $y = \log_4 2x$ and $y = \log_4 x$. **1**

(c) Determine the coordinates of the point where the graph of $y = \log_4 2x$ intersects the x-axis. **2**

(d) Sketch and annotate the graph of $y = f^{-1}(x)$, where $f(x) = \log_4 2x$. **3**

5.



Points A(–1, –1) and B(7, 3) lie on the circumference of a circle with centre C, as shown in the diagram.

(a) Find the equation of the perpendicular bisector of AB. **4**

CB is parallel to the x-axis.

(b) Find the equation of the circle, passing through A and B, with centre C. **4**

6. The points A(0, 9, 7), B(5, –1, 2), C(4, 1, 3) and D(x, –2, 2) are such that AB is perpendicular to CD.

Determine the value of x. **5**

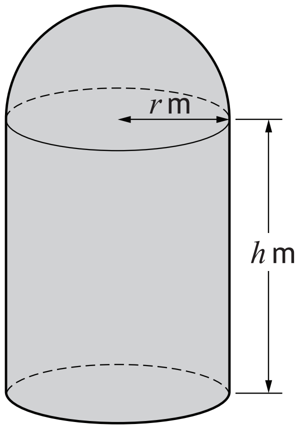
7. Given that $P(t) = 30e^{t-2}$ decide whether each of the statements below is true or false. Justify your answers.

Statement A $\quad P(0) = 30$.

Statement B $\quad$ When $P(t) = 15$, the only possible value of t is 1·3 to one decimal place. **6**

8. A design for a new grain container is in the shape of a cylinder with a hemispherical roof and a flat circular base. The radius of the cylinder is r metres, and the height is h metres.

The volume of the **cylindrical** part of the container needs to be 100 cubic metres.



(a) Given that the curved surface area of a hemisphere of radius r is $2\pi r^2$ show that the surface area of metal needed to build the grain container is given by:

$$A = \frac{200}{r} + 3\pi r^2 \text{ square metres}$$ 3

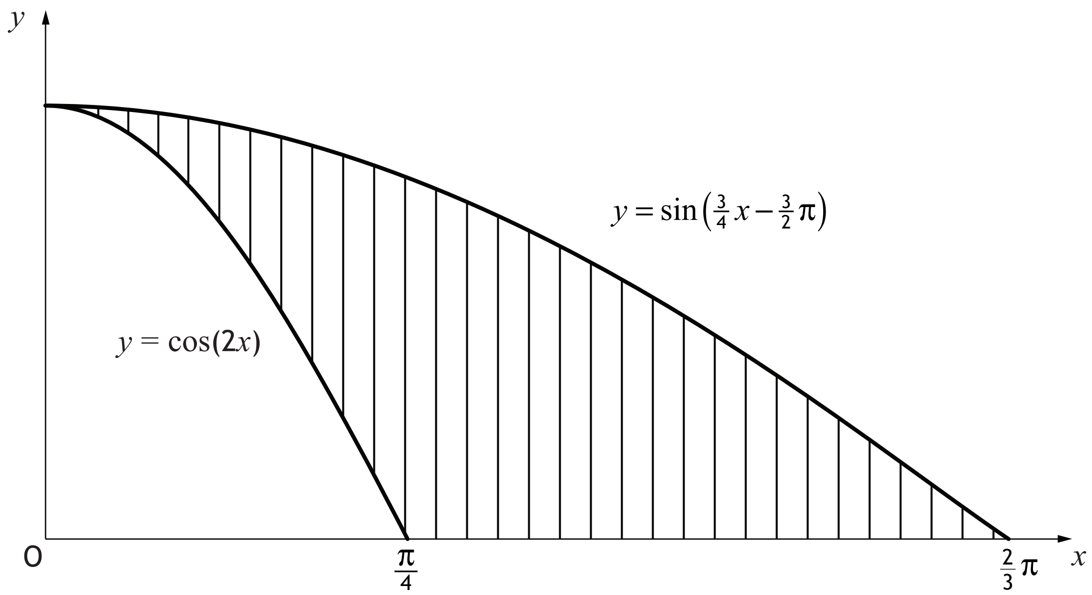
(b) Determine the value of r which minimises the amount of metal needed to build the container. 6

9. A sea-life visitor attraction has a new logo in the shape of a shark fin.

The outline of the logo can be represented by parts of

- the x axis
- the curve with equation $y = \cos(2x)$
- the curve with equation $y = \sin\left(\frac{3}{4}x - \frac{3}{2}\pi\right)$

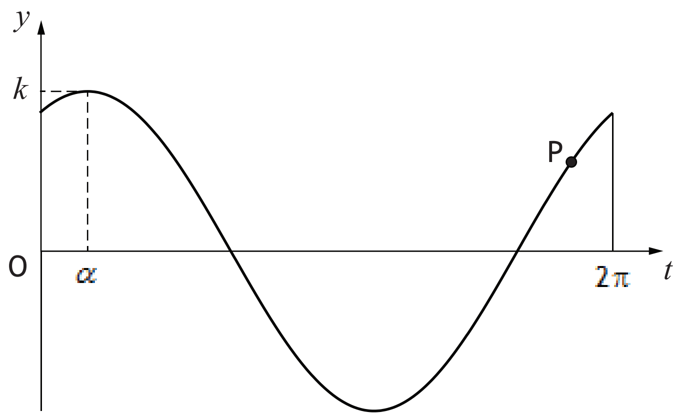
as shown in the diagram.



Calculate the shaded area. **6**

10. Two sound sources produce the waves $y = \sin t$ and $y = \sqrt{3}\cos t$.

An investigation into the addition of these two waves produces the graph shown, with equation $y = k\cos(t - \alpha)$ for $0 \leq t \leq 2\pi$.



(a) Calculate the values of k and α. **4**

The point P has a y-coordinate of 1·2.

(c) Hence calculate the value of the t-coordinate of point P. **4**

[END OF SPECIMEN QUESTION PAPER]

H

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Mathematics
Paper 2

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- Correct working in the wrong part of a question
- Legitimate variations in solutions
- Repeated error within a question

Definitions of Mathematics-specific command words used in this Specimen Question Paper.

Determine: find a numerical value or values from the information given.

Expand: multiply out an algebraic expression by making use of the distributive law or a compound trigonometric expression by making use of one of the addition formulae for $\sin(A \pm B)$ or $\cos(A \pm B)$.

Show that: use mathematics to prove something, eg that a statement or given value is correct — all steps, including the required conclusion, must be shown.

Express: use given information to rewrite an expression in a specified form.

Hence: use the previous answer to proceed.

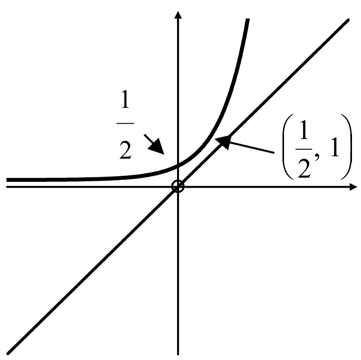
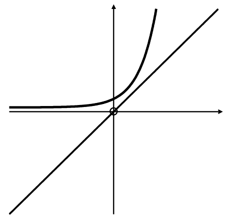
Hence, or otherwise: use the previous answer to proceed; however, another method may alternatively be used.

Justify: show good reason(s) for the conclusion(s) reached.

Specific Marking Instructions for each question

Question			Marking scheme. Give one mark for each •	Max mark	Illustration of evidence for awarding a mark at each •
1	(a)		•¹ find coordinates of E •² find coordinates of F	2	•¹ E(45, 45, 40) •² F(60, 40, 0)
	(b)		Ans: -1750 •³ find $\overrightarrow{ED}$ and $\overrightarrow{EF}$ •⁴ correct calculation of scalar product	2	•³ $\overrightarrow{ED}=\begin{pmatrix}-15\\-15\\40\end{pmatrix}$, $\overrightarrow{EF}=\begin{pmatrix}15\\-5\\-40\end{pmatrix}$ •⁴ $\overrightarrow{ED}.\overrightarrow{EF}=-225+75-1600=-1750$
	(c)		Ans: 154° •⁵ know how to find angle DEF using formula •⁶ find $\lvert\overrightarrow{ED}\rvert$ •⁷ find $\lvert\overrightarrow{EF}\rvert$ •⁸ calculates angle DEF	4	•⁵ $\cos DEF=\dfrac{\overrightarrow{ED}.\overrightarrow{EF}}{\lvert\overrightarrow{ED}\rvert\lvert\overrightarrow{EF}\rvert}$ or equivalent •⁶ $\lvert\overrightarrow{ED}\rvert=\sqrt{2050}$ •⁷ $\lvert\overrightarrow{EF}\rvert=\sqrt{1850}$ •⁸ $\cos DEF=\dfrac{-1750}{\sqrt{2050}\sqrt{1850}}$ $DEF=153\cdot977...=154°$
2	(a)		Ans: $a=0\cdot96$, $b=580$ •¹ set up one equation •² set up second equation •³ solve for one variable •⁴ solve for second variable	4	•¹ $2500=2000a+b$ •² $2980=2500a+b$ •³ $480=500a$ or $12500=10000a+5b$ $a=\dfrac{480}{500}$ $\quad 11920=10000a+4b$ $a=0\cdot96$ $\quad 580=b$ •⁴ $b=2500-2000(0\cdot96)$ $b=2500-1920$ $b=580$ or $2000a=2500-580$ $a=\dfrac{1920}{2000}$ $a=0\cdot96$

Question			Marking scheme. Give one mark for each •	Max mark	Illustration of evidence for awarding a mark at each •
	(b)		Ans: Yes. Stabilises at 14 500 •5 knows how to find the limit •6 calculate limit •7 conclusion	3	•5 $u_{n+1} = 0\cdot96u_n + 580, \quad -1 < a < 1$ $L = \dfrac{b}{1-a}$ $L = \dfrac{580}{1-0\cdot96}$ •6 $L = 14500$ •7 yes, conservation measures will end, since the predicted population stabilises at 14 500 and 14 500 > 13 000
3	(a)		Ans: $p = 1, q = 4$ •1 state values of p and q	1	•1 $p = 1, q = 4$
	(b)		Ans: $y = 9(x-1)$ •2 expand brackets •3 differentiate •4 calculate gradient of tangent •5 substitutes gradient and (1,0) into equation of line	4	•2 $f(x) = x^4 - 9x^3 + 24x^2 - 16x$ •3 $f'(x) = 4x^3 - 27x^2 + 48x - 16$ •4 $f'(1) = 4 - 27 + 48 - 16 = 9$ •5 $y = 9(x-1)$

Question			Marking scheme. Give one mark for each •	Max mark	Illustration of evidence for awarding a mark at each •
4	(a)		Ans: $y = \log_4 x + \frac{1}{2}$ •[1] using law of logarithms •[2] evaluating $\log_4 2$	2	•[1] $\log_4 2x = \log_4 2 + \log_4 x$ •[2] $\log_4 2 = \frac{1}{2}$
	(b)		Ans: Graph of $y = \log_4 x$ moved up by $\frac{1}{2}$ or graph of $y = \log_4 x$ compressed horizontally by a factor of 2. •[3] valid description of relationship	1	•[3] valid description — see answer
	(c)		Ans: $x = \frac{1}{2}$ •[4] setting $y = 0$ •[5] solving for x	2	•[4] $\log_4 2x = 0$ •[5] $x = \frac{1}{2}$
	(d)		Ans:  •[6] reflecting $y = \log_4 2x$ in the line $y = x$ •[7] correct shape •[8] annotating (2 points) **(or other valid method)**	3	•[6] reflect in $y = x$ •[7]  •[8] $\left(0, \frac{1}{2}\right)$ and $\left(\frac{1}{2}, 1\right)$

Question			Marking scheme. Give one mark for each •	Max mark	Illustration of evidence for awarding a mark at each •
5	(a)		Ans: $y-1=-2(x-3)$ •¹ calculate midpoint of AB •² calculate gradient of AB •³ state gradient of perpendicular bisector •⁴ substitute into equation of line	4	•¹ $(3,\ 1)$ •² $\frac{1}{2}$ •³ -2 •⁴ $y-1=-2(x-3)$
	(b)		Ans: $(x-2)^2+(y-3)^2=25$ •⁵ knowing and using $y=3$ •⁶ solving for x •⁷ identifying the radius •⁸ obtain circle equation	4	•⁵ $3=-2x+7$ •⁶ $x=2$ •⁷ $r=5$ •⁸ $(x-2)^2+(y-3)^2=25$
6			Ans: $x=-3$ •¹ use perpendicular property •² find $\overrightarrow{CD}$ •³ find $\overrightarrow{AB}$ •⁴ correct substitution into scalar product formula •⁵ calculates value of x	5	•¹ If $\overrightarrow{CD}$ is perpendicular to $\overrightarrow{AB}$ then $\overrightarrow{CD}.\overrightarrow{AB}=0$ •² $\begin{pmatrix} x-4 \\ -3 \\ -1 \end{pmatrix}$ •³ $\begin{pmatrix} 5 \\ -10 \\ -5 \end{pmatrix}$ •⁴ $5(x-4)+(-10)(-3)+(-5)(-1)=0$ •⁵ $x=-3$

Question			Marking scheme. Give one mark for each •	Max mark	Illustration of evidence for awarding a mark at each •
7			Ans: A False and B True •1 valid reason for statement A •2 selecting true or false for statement A with valid reason •3 setting $P(t)=15$ •4 taking log to base e •5 completing valid reason •6 selecting true or false for statement B with valid reason	6	•1 $P(0)=30e^{-2}=4\cdot06$ •2 false, since $P(0)\neq30$ (do not award without valid reason) •3 $15=30e^{t-2}$ •4 $\ln e^{t-2}=\ln0\cdot5$ •5 $t-2=\ln0\cdot5$ $t=\ln0\cdot5+2$ (1·3) •6 true, since $t=1\cdot3$ to one decimal place and there is only one solution (do not award without valid reason)
Notes			Substituting $t=1\cdot3$ into $P(t)=30e^{t-2}$ is not sufficient to show that statement B is true, since it does not prove that $t=1\cdot3$ is the <u>only</u> solution.		
8	(a)		•1 know to equate volume to 100 •2 obtain an expression for h •3 complete area evaluation	3	•1 $V=\pi r^2h=100$ •2 $h=\dfrac{100}{\pi r^2}$ •3 $A=\pi r^2+2\pi r^2+2\pi r\times\dfrac{100}{\pi r^2}$
	(b)		Ans: $r=2\cdot20$ m •4 know to and start to differentiate •5 complete differentiation •6 set derivative to zero •7 obtain r •8 justify nature of stationary point •9 interpret result	6	•4 $A'(r)=6\pi r\ldots$ •5 $A'(r)=6\pi r-\dfrac{200}{r^2}$ •6 $6\pi r-\dfrac{200}{r^2}=0$ •7 $r=2\cdot20$ metres •8 $A''(r)=6\pi+\dfrac{400}{r^3}\Rightarrow A''(2\cdot1974\ldots)=56\cdot5\ldots$ •9 minimum (when $r=2\cdot20$ m)
Notes			Candidates may use a nature table at •8 to justify a minimum turning point when $r=2\cdot1974\ldots$		

Question			Marking scheme. Give one mark for each •	Max mark	Illustration of evidence for awarding a mark at each •
9			Ans: $\frac{5}{6}$ •1 knowing to use integration •2 using correct limits •3 integrating correctly •4 integrating correctly •5 substituting limits correctly •6 evaluating correctly	6	•1 $\int \sin(\frac{3}{4}x-\frac{3}{2}\pi)dx-\int \cos(2x)dx$ •2 $\int_0^{\frac{2}{3}\pi} \sin(\frac{3}{4}x-\frac{3}{2}\pi)dx-\int_0^{\frac{\pi}{4}} \cos(2x)dx$ •3 $\left[-\frac{4}{3}\cos(\frac{3}{4}x-\frac{3}{2}\pi)\right]\ldots\ldots$ •4 $-[\frac{1}{2}\sin(2x)]$ •5 See * below •6 $(\frac{4}{3}-0)-(\frac{1}{2}-0)=\frac{5}{6}$
* $([-\frac{4}{3}\cos(\frac{3}{4}\times\frac{2}{3}\pi-\frac{3}{2}\pi)]-[-\frac{4}{3}\cos(0-\frac{3}{2}\pi)])-([\frac{1}{2}\sin(2\times\frac{1}{4}\pi)]-[\frac{1}{2}\sin(2\times 0)])$					
10	(a)		Ans: $k=2, \alpha=\frac{\pi}{6}$ or equivalent •1 knows to set wave function equal to addition of individual waves •2 knows to expand •3 knows to compare coefficients •4 interpret comparison	4	•1 $\sin t+\sqrt{3}\cos t=k\cos(t-\alpha)$ or equivalent •2 $k\cos\alpha\cos t+k\sin\alpha\sin t$ or equivalent •3 $k\sin\alpha=1, \quad k\cos\alpha=\sqrt{3}$ or equivalent •4 $k=2, \quad \alpha=\frac{\pi}{6}$ or equivalent
			Ans: 5·9 •5 equates wave function with y-coordinate of P •6 rearranges correctly •7 solve equation for $t-\frac{\pi}{6}$ •8 find t-coordinate of P by interpreting diagram	4	•5 $2\cos\left(t-\frac{\pi}{6}\right)=1\cdot 2$ or equivalent •6 $\cos\left(t-\frac{\pi}{6}\right)=0\cdot 6$ or equivalent •7 •7 $t-\frac{\pi}{6}=0\cdot 927\ldots$ & $5\cdot 355\ldots$ $1\cdot 45\ldots$ & •8 $5\cdot 879\ldots$

[END OF SPECIMEN MARKING INSTRUCTIONS]