X056/301

NATIONAL
QUALIFICATIONS
2000

THURSDAY, 25 MAY
9.00 AM – 10.10 AM

MATHEMATICS
HIGHER
Paper 1
(Non-calculator)

Read Carefully

1 **Calculators may <u>NOT</u> be used in this paper.**

2 There are three Sections in this paper.

Section A assesses the compulsory units Mathematics 1 and 2.
Section B assesses the optional unit Mathematics 3.
Section C assesses the optional unit Statistics.

Candidates must attempt **all** questions in Section A (Mathematics 1 and 2) **and**

either Section B (Mathematics 3)

or Section C (Statistics).

3 Full credit will be given only where the solution contains appropriate working.

4 Answers obtained by readings from scale drawings will not receive any credit.

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FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos \theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\sin (A \pm B) = \sin A \cos B \pm \cos A \sin B$$

$$\cos (A \pm B) = \cos A \cos B \mp \sin A \sin B$$

$$\sin 2A = 2\sin A \cos A$$

$$\cos 2A = \cos^2 A - \sin^2 A = 2\cos^2 A - 1 = 1 - 2\sin^2 A$$

Table of standard derivatives and integrals:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

Statistics:

Sample standard deviation: $s = \sqrt{\frac{1}{n-1}\Sigma(x_i - \bar{x})^2} = \sqrt{\frac{1}{n-1}\left(\Sigma x_i^2 - \frac{1}{n}\left(\Sigma x_i\right)^2\right)}$ where n is the sample size.

Sums of squares and products: $S_{xx} = \Sigma\left(x_i - \bar{x}\right)^2 = \Sigma x_i^2 - \frac{1}{n}\left(\Sigma x_i\right)^2$

$$S_{yy} = \Sigma\left(y_i - \bar{y}\right)^2 = \Sigma y_i^2 - \frac{1}{n}\left(\Sigma y_i\right)^2$$

$$S_{xy} = \Sigma\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right) = \Sigma x_i y_i - \frac{1}{n}\Sigma x_i \Sigma y_i$$

Linear regression: The equation of the least squares regression line of y on x is given by $y = \alpha + \beta x$, where estimates for α and β, a and b, are given by: $a = \bar{y} - b\bar{x}$

$$b = \frac{\Sigma\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)}{\Sigma\left(x_i - \bar{x}\right)^2} = \frac{S_{xy}}{S_{xx}}$$

Product moment correlation coefficient r: $r = \frac{\Sigma\left(x_i - \bar{x}\right)\left(y_i - \bar{y}\right)}{\sqrt{\Sigma\left(x_i - \bar{x}\right)^2 \Sigma\left(y_i - \bar{y}\right)^2}} = \frac{S_{xy}}{\sqrt{S_{xx}S_{yy}}}$

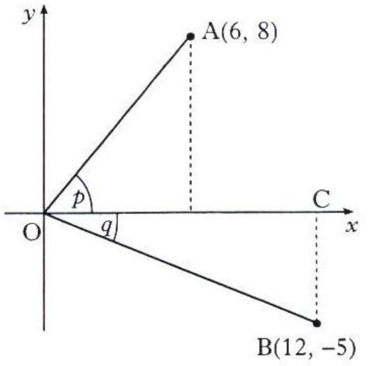
SECTION A (Mathematics 1 and 2)

Marks

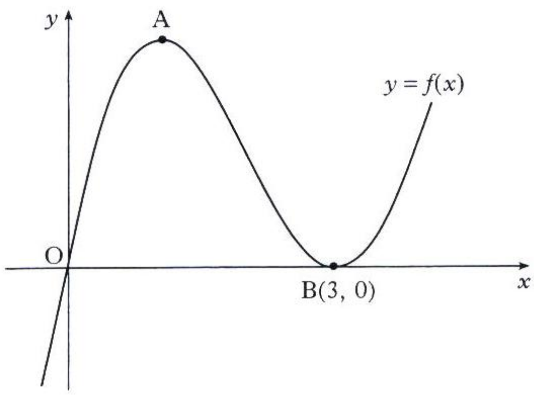
ALL candidates should attempt this Section.

A1. On the coordinate diagram shown, A is the point (6, 8) and B is the point (12, –5). Angle AOC = p and angle COB = q.

Find the exact value of $\sin(p + q)$. **4**



A2. A sketch of the graph of $y = f(x)$ where $f(x) = x^3 - 6x^2 + 9x$ is shown below. The graph has a maximum at A and a minimum at B(3, 0).



(a) Find the coordinates of the turning point at A. **4**

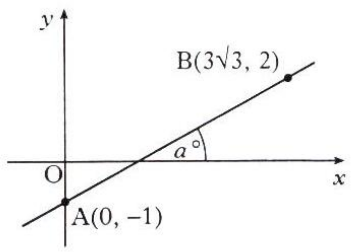
(b) Hence sketch the graph of $y = g(x)$ where $g(x) = f(x + 2) + 4$.

Indicate the coordinates of the turning points. There is no need to calculate the coordinates of the points of intersection with the axes. **2**

(c) Write down the range of values of k for which $g(x) = k$ has 3 real roots. **1**

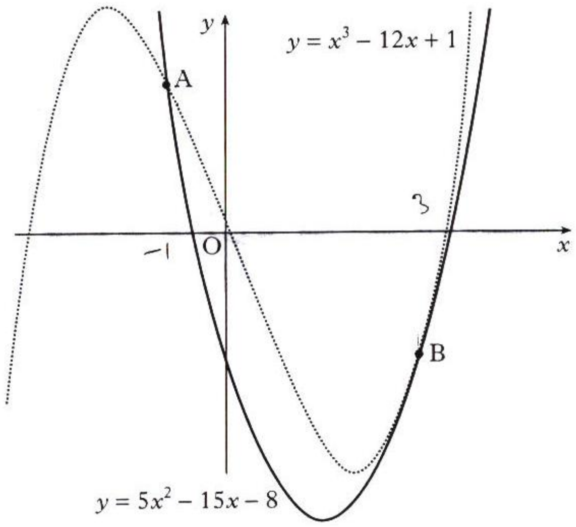
[Turn over

A3. Find the size of the angle $a°$ that the line joining the points A(0, −1) and B($3\sqrt{3}$, 2) makes with the positive direction of the x-axis. **3**



A4. The diagram shows a sketch of the graphs of $y = 5x^2 - 15x - 8$ and $y = x^3 - 12x + 1$.

The two curves intersect at A and touch at B, ie at B the curves have a common tangent.



(*a*) (i) Find the x-coordinates of the points on the curves where the gradients are equal. **4**

(ii) By considering the corresponding y-coordinates, or otherwise, distinguish geometrically between the two cases found in part (i). **1**

(*b*) The point A is (−1, 12) and B is (3, −8).

Find the area enclosed between the two curves. **5**

Marks

A5. Two sequences are generated by the recurrence relations $u_{n+1} = au_n + 10$ and $v_{n+1} = a^2v_n + 16$.

The two sequences approach the same limit as $n \to \infty$.

Determine the value of a and evaluate the limit. **5**

A6. For what range of values of k does the equation $x^2 + y^2 + 4kx - 2ky - k - 2 = 0$ represent a circle? **5**

[*END OF SECTION A*]

Candidates should now attempt

EITHER Section B (Mathematics 3) on *Page six*

OR Section C (Statistics) on *Pages seven and eight*

[Turn over

SECTION B (Mathematics 3)

Marks

ONLY candidates doing the course Mathematics 1, 2 and 3 should attempt this Section.

B7. VABCD is a pyramid with a rectangular base ABCD.

Relative to some appropriate axes,

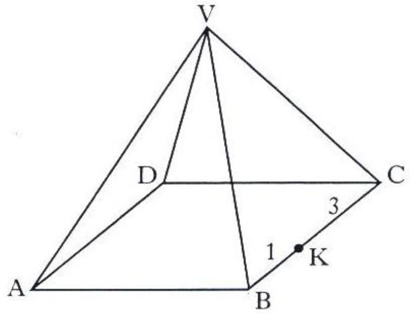
$\overrightarrow{VA}$ represents $-7\boldsymbol{i} - 13\boldsymbol{j} - 11\boldsymbol{k}$

$\overrightarrow{AB}$ represents $6\boldsymbol{i} + 6\boldsymbol{j} - 6\boldsymbol{k}$

$\overrightarrow{AD}$ represents $8\boldsymbol{i} - 4\boldsymbol{j} + 4\boldsymbol{k}$.

K divides BC in the ratio 1:3.

Find $\overrightarrow{VK}$ in component form. **3**



B8. The graph of $y = f(x)$ passes through the point $\left(\frac{\pi}{9}, 1\right)$.

If $f'(x) = \sin(3x)$, express y in terms of x. **4**

B9. Evaluate $\log_5 2 + \log_5 50 - \log_5 4$. **3**

B10. Find the maximum value of $\cos x - \sin x$ and the value of x for which it occurs in the interval $0 \leq x \leq 2\pi$. **6**

[END OF SECTION B]

X056/302

NATIONAL
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2000

THURSDAY, 25 MAY
10.30 AM – 12.00 NOON

MATHEMATICS
HIGHER
Paper 2

Read Carefully

1 **Calculators may be used in this paper.**

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Section A assesses the compulsory units Mathematics 1 and 2.
Section B assesses the optional unit Mathematics 3.
Section C assesses the optional unit Statistics.

Candidates must attempt **all** questions in Section A (Mathematics 1 and 2) **and**

either Section B (Mathematics 3)

or Section C (Statistics).

3 Full credit will be given only where the solution contains appropriate working.

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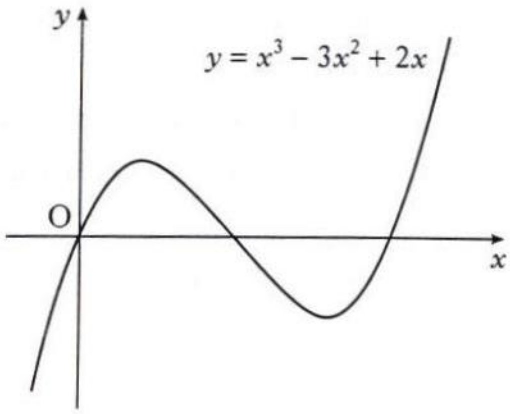
SECTION A (Mathematics 1 and 2)

ALL candidates should attempt this Section.

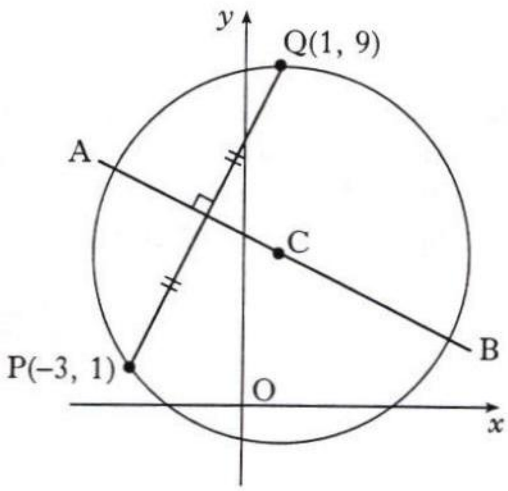
A1. The diagram shows a sketch of the graph of $y = x^3 - 3x^2 + 2x$.



(*a*) Find the equation of the tangent to this curve at the point where $x = 1$.

(*b*) The tangent at the point (2, 0) has equation $y = 2x - 4$. Find the coordinates of the point where this tangent meets the curve again.

A2. (*a*) Find the equation of AB, the perpendicular bisector of the line joining the points P(−3, 1) and Q(1, 9).



(*b*) C is the centre of a circle passing through P and Q. Given that QC is parallel to the *y*-axis, determine the equation of the circle.

(*c*) The tangents at P and Q intersect at T.

Write down

(i) the equation of the tangent at Q

(ii) the coordinates of T.

A3. $f(x) = 3 - x$ and $g(x) = \frac{3}{x}$, $x \neq 0$.

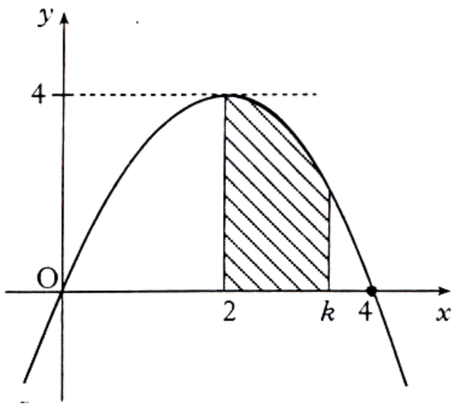
(*a*) Find $p(x)$ where $p(x) = f(g(x))$.

(*b*) If $q(x) = \frac{3}{3 - x}$, $x \neq 3$, find $p(q(x))$ in its simplest form.

Marks

A4. The parabola shown crosses the x-axis at (0, 0) and (4, 0), and has a maximum at (2, 4).

The shaded area is bounded by the parabola, the x-axis and the lines $x = 2$ and $x = k$.



(*a*) Find the equation of the parabola. **2**

(*b*) Hence show that the shaded area, A, is given by

$$A = -\frac{1}{3}k^3 + 2k^2 - \frac{16}{3}.$$ **3**

A5. Solve the equation $3\cos 2x° + \cos x° = -1$ in the interval $0 \le x \le 360$. **5**

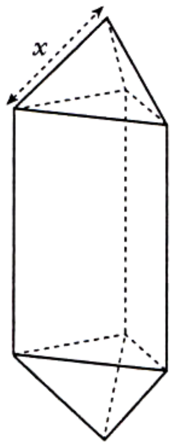
A6. A goldsmith has built up a solid which consists of a triangular prism of fixed volume with a regular tetrahedron at each end.

The surface area, A, of the solid is given by

$$A(x) = \frac{3\sqrt{3}}{2}\left(x^2 + \frac{16}{x}\right)$$

where x is the length of each edge of the tetrahedron.

Find the value of x which the goldsmith should use to minimise the amount of gold plating required to cover the solid. **6**



[*END OF SECTION A*]

Candidates should now attempt

EITHER Section B (Mathematics 3) on *Pages five and six*

OR Section C (Statistics) on *Pages seven and eight*

SECTION B (Mathematics 3)

Marks

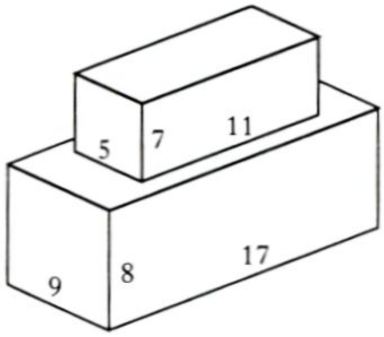
ONLY candidates doing the course Mathematics 1, 2 and 3 should attempt this Section.

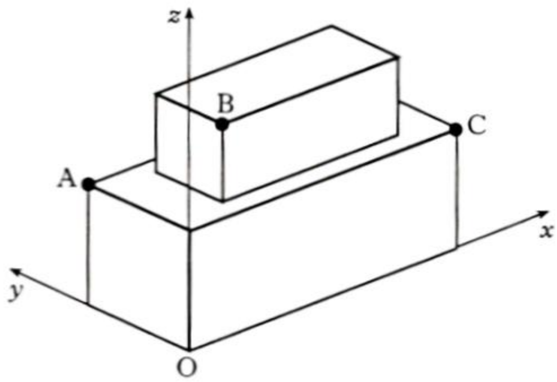
B7. For what value of t are the vectors $u = \begin{pmatrix} t \\ -2 \\ 3 \end{pmatrix}$ and $v = \begin{pmatrix} 2 \\ 10 \\ t \end{pmatrix}$ perpendicular? **2**

B8. Given that $f(x) = (5x-4)^{\frac{1}{2}}$, evaluate $f'(4)$. **3**

B9. A cuboid measuring 11 cm by 5 cm by 7 cm is placed centrally on top of another cuboid measuring 17 cm by 9 cm by 8 cm.

Coordinate axes are taken as shown.





(*a*) The point A has coordinates (0, 9, 8) and C has coordinates (17, 0, 8). Write down the coordinates of B. **1**

(*b*) Calculate the size of angle ABC. **6**

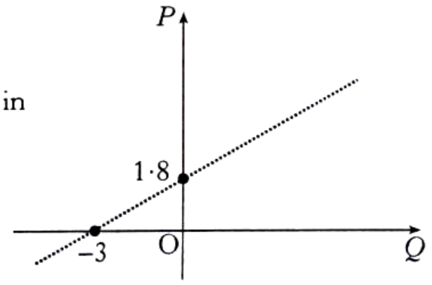
[Turn over

Marks

B10. Find $\int \frac{1}{(7-3x)^2}\,dx$. 2

B11. The results of an experiment give rise to the graph shown.

(*a*) Write down the equation of the line in terms of P and Q. 2



It is given that $P = \log_e p$ and $Q = \log_e q$.

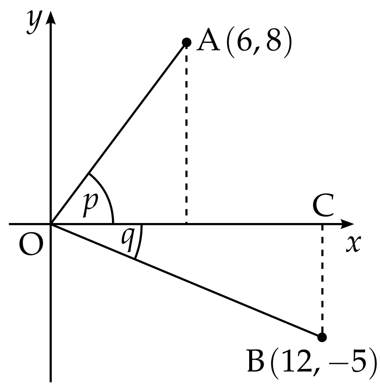
(*b*) Show that p and q satisfy a relationship of the form $p = aq^b$, stating the values of a and b. 4

[*END OF SECTION B*]

Higher 2000 - Paper 1 Solutions

[SQA] 1. On the coordinate diagram shown, A is the point $(6,8)$ and B is the point $(12,-5)$. Angle AOC $= p$ and angle COB $= q$.

Find the exact value of $\sin(p+q)$. **4**

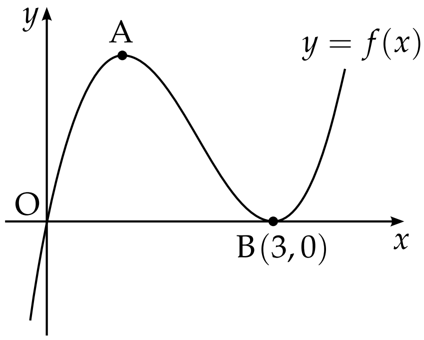


Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	4	C	NC	T9	$\frac{63}{65}$	2000 P1 Q1

•1 ss: know to use trig expansion •2 pd: process missing sides •3 ic: interpret data •4 pd: process	•1 $\sin p \cos q + \cos p \sin q$ •2 10 **and** 13 •3 $\frac{8}{10} \cdot \frac{12}{13} + \frac{6}{10} \cdot \frac{5}{13}$ •4 $\frac{126}{130}$

[SQA] 2. A sketch of the graph of $y = f(x)$ where $f(x) = x^3 - 6x^2 + 9x$ is shown below.

The graph has a maximum at A and a minimum at B$(3, 0)$.



(*a*) Find the coordinates of the turning point at A. **4**

(*b*) Hence sketch the graph of $y = g(x)$ where $g(x) = f(x + 2) + 4$.

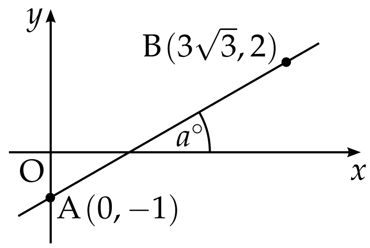
Indicate the coordinates of the turning points. There is no need to calculate the coordinates of the points of intersection with the axes. **2**

(*c*) Write down the range of values of k for which $g(x) = k$ has 3 real roots. **1**

Part	Marks	Level	Calc.	Content	Answer	U1 OC3
(*a*)	4	C	NC	C8	A$(1,4)$	2000 P1 Q2
(*b*)	2	C	NC	A3	sketch (translate 4 up, 2 left)	
(*c*)	1	A/B	NC	A2	$4 < k < 8$	

•[1] ss: know to differentiate	•[1] $\frac{dy}{dx} = \ldots$
•[2] pd: differentiate correctly	•[2] $\frac{dy}{dx} = 3x^2 - 12x + 9$
•[3] ss: know gradient $= 0$	•[3] $3x^2 - 12x + 9 = 0$
•[4] pd: process	•[4] $A = (1, 4)$
	translate $f(x)$ 4 units up, 2 units left
•[5] ic: interpret transformation	•[5] sketch with coord. of $A'(-1, 8)$
•[6] ic: interpret transformation	•[6] sketch with coord. of $B'(1, 4)$
•[7] ic: interpret sketch	•[7] $4 < k < 8$ (accept $4 \leq k \leq 8$)

[SQA] 3. Find the size of the angle $a°$ that the line joining the points $A(0, -1)$ and $B(3\sqrt{3}, 2)$ makes with the positive direction of the x-axis. **3**

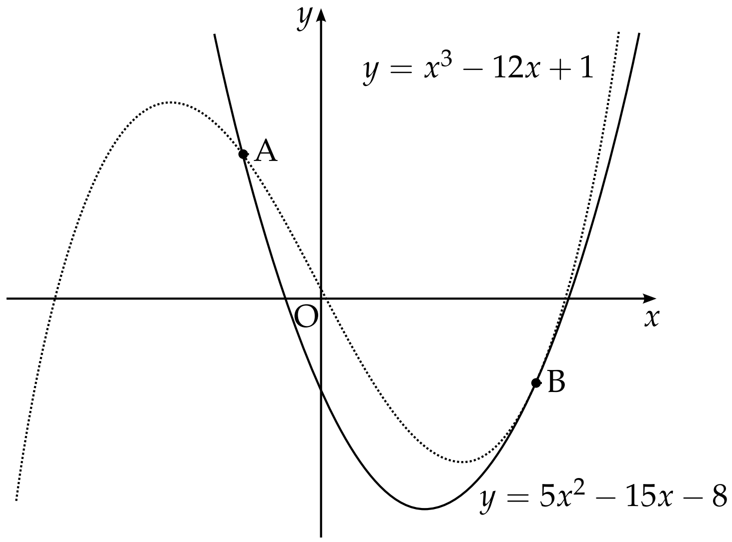


Part	Marks	Level	Calc.	Content	Answer	U1 OC1
	3	C	NC	G2	30	2000 P1 Q3

•[1] ss: know how to find gradient or equ. •[2] pd: process •[3] ic: interpret exact value	•[1] $\frac{2-(-1)}{3\sqrt{3}-0}$ •[2] $\tan a$ = gradient *stated or implied by* •[3] •[3] $a = 30$

[SQA] 4. The diagram shows a sketch of the graphs of $y = 5x^2 - 15x - 8$ and $y = x^3 - 12x + 1$.

The two curves intersect at A and touch at B, i.e. at B the curves have a common tangent.



(*a*) (i) Find the x-coordinates of the point of the curves where the gradients are equal. **4**

(ii) By considering the corresponding y-coordinates, or otherwise, distinguish geometrically between the two cases found in part (i). **1**

(*b*) The point A is $(-1, 12)$ and B is $(3, -8)$.

Find the area enclosed between the two curves. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC2
(*ai*)	4	C	NC	C4	$x = \frac{1}{3}$ and $x = 3$	2000 P1 Q4
(*aii*)	1	C	NC	CGD	parallel and coincident	
(*b*)	5	C	NC	C17	$21\frac{1}{3}$	

•1 ss: know to diff. and equate	•1 find derivatives and equate
•2 pd: differentiate	•2 $3x^2 - 12$ **and** $10x - 15$
•3 pd: form equation	•3 $3x^2 - 10x + 3 = 0$
•4 ic: interpret solution	•4 $x = 3$, $x = \frac{1}{3}$
•5 ic: interpret diagram	•5 tangents at $x = \frac{1}{3}$ are parallel, at $x = 3$ coincident
•6 ss: know how to find area between curves	•6 $\int(\text{cubic} - \text{parabola})$ **or** $\int(\text{cubic}) - \int(\text{parabola})$
•7 ic: interpret limits	•7 $\int_{-1}^{3} \cdots dx$
•8 pd: form integral	•8 $\int(x^3 - 5x^2 + 3x + 9)dx$ or equiv.
•9 pd: process integration	•9 $\left[\frac{1}{4}x^4 - \frac{5}{3}x^3 + \frac{3}{2}x^2 + 9x\right]_{-1}^{3}$ or equiv.
•10 pd: process limits	•10 $21\frac{1}{3}$

[SQA] 5. Two sequences are generated by the recurrence relations $u_{n+1} = au_n + 10$ and $v_{n+1} = a^2 v_n + 16$.

The two sequences approach the same limit as $n \to \infty$.

Determine the value of a and evaluate the limit. **5**

Part	Marks	Level	Calc.	Content	Answer	U1 OC4
	4	C	NC	A13	$a = \frac{3}{5}, L = 25$	2000 P1 Q5
	1	A/B	NC	A12		

•1 ss: know how to find limit •2 pd: process •3 pd: process •4 ic: interpret coeff. of u_n •5 pd: process	•1 $L = aL + 10$ **or** $L = a^2L + 16$ **or** $L = \frac{b}{1-a}$ •2 $L = \frac{10}{1-a}$ **or** $L = \frac{16}{1-a^2}$ •3 $\frac{10}{1-a}$ **or** $\frac{16}{1-a^2}$ •4 $10a^2 - 16a + 6 = 0$ •5 $a = \frac{3}{5}$ **and** $L = 25$

[SQA] 6. For what range of values of k does the equation $x^2 + y^2 + 4kx - 2ky - k - 2 = 0$ represent a circle? **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC4
	5	A	NC	G9, A17	for all k	2000 P1 Q6

•1 ss: know to examine radius •2 pd: process •3 pd: process •4 ic: interpret quadratic inequation •5 ic: interpret quadratic inequation	•1 $g = 2k, f = -k, c = -k - 2$ *stated or implied by* •2 •2 $r^2 = 5k^2 + k + 2$ •3 (real $r \Rightarrow$) $5k^2 + k + 2 > 0$ (*accept* $\geq$) •4 use discr. **or** complete sq. **or** diff. •5 true for all k

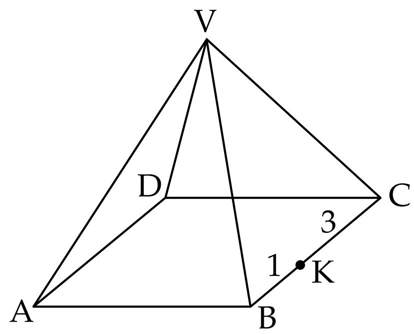
[SQA] 7. VABCD is a pyramid with a rectangular base ABCD.

Relative to some appropriate axes,

$\overrightarrow{VA}$ represents $-7\boldsymbol{i} - 13\boldsymbol{j} - 11\boldsymbol{k}$

$\overrightarrow{AB}$ represents $6\boldsymbol{i} + 6\boldsymbol{j} - 6\boldsymbol{k}$

$\overrightarrow{AD}$ represents $8\boldsymbol{i} - 4\boldsymbol{j} + 4\boldsymbol{k}$.



K divides BC in the ratio 1 : 3.

Find $\overrightarrow{VK}$ in component form. **3**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	3	C	CN	G25, G21, G20	$\begin{pmatrix} 1 \\ -8 \\ -16 \end{pmatrix}$	2000 P1 Q7

•1 ss: recognise crucial aspect •2 ic: interpret ratio •3 pd: process components	•1 $\overrightarrow{VK} = \overrightarrow{VA} + \overrightarrow{AB} + \overrightarrow{BK}$ **or** $\overrightarrow{VK} = \overrightarrow{VB} + \overrightarrow{BK}$ •2 $\overrightarrow{BK} = \frac{1}{4}\overrightarrow{BC}$ **or** $\frac{1}{4}\overrightarrow{AD}$ **or** $\begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$ **or** $\begin{pmatrix} -1 \\ -7 \\ -17 \end{pmatrix}$ •3 $\overrightarrow{VK} = \begin{pmatrix} 1 \\ -8 \\ -16 \end{pmatrix}$

[SQA] 8. The graph of $y = f(x)$ passes through the point $\left(\frac{\pi}{9}, 1\right)$.

If $f'(x) = \sin(3x)$ express y in terms of x. **4**

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	4	A/B	NC	C18, C23	$y = -\frac{1}{3}\cos(3x) + \frac{7}{6}$	2000 P1 Q8

•1 ss: know to integrate •2 pd: integrate •3 ic: interpret $(\frac{\pi}{9}, 1)$ •4 pd: process	•1 $y = \int \sin(3x)\,dx$ *stated or implied by* •2 •2 $-\frac{1}{3}\cos(3x)$ •3 $1 = -\frac{1}{3}\cos(\frac{3\pi}{9}) + c$ or equiv. •4 $c = \frac{7}{6}$

[SQA] 9. Evaluate $\log_5 2 + \log_5 50 - \log_5 4$. **3**

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
	2	C	NC	A28	2	2000 P1 Q9
	1	A/B	NC	A28		

•1 pd: use $\log_a x + \log_a y = \log_a xy$ •2 pd: use $\log_a x - \log_a y = \log_a \frac{x}{y}$ •3 pd: use $\log_a a = 1$	•1 $\log_5 100 - \log_5 4$ •2 $\log_5 25$ •3 2

[SQA] 10. Find the maximum value of $\cos x - \sin x$ and the value of x for which it occurs in the interval $0 \leq x \leq 2\pi$. **6**

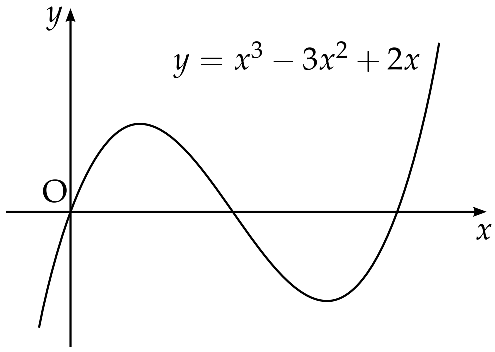
Part	Marks	Level	Calc.	Content	Answer	U3 OC4
	6	A/B	CN	T14	max value $\sqrt{2}$ when $x = \frac{7\pi}{4}$	2000 P1 Q10

•1 ss: use e.g. $k\cos(x + a)$ •2 ic: expand chosen rule •3 pd: compare coefficients •4 pd: process •5 pd: process •6 ic: interpret trig expression	•1 e.g. use $k\cos(x + a)$ •2 $k\cos x \cos a - k\sin x \sin a$ •3 $k\cos a = 1$ and $k\sin a = 1$ •4 $k = \sqrt{2}$ •5 $\tan a = 1$, $a = \frac{\pi}{4}$ ($45°$ *is bad form*) •6 max. value $= \sqrt{2}$ when $x = \frac{7\pi}{4}$ (*do not accept* $45°$)

[END OF QUESTIONS]

Higher 2000 - Paper 2 Solutions

[SQA] 1. The diagram shows a sketch of the graph of $y = x^3 - 3x^2 + 2x$.



(*a*) Find the equation of the tangent to this curve at the point where $x = 1$. **5**

(*b*) The tangent at the point $(2, 0)$ has equation $y = 2x - 4$. Find the coordinates of the point where this tangent meets the curve again. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC1
(*a*)	5	C	CN	C5	$x + y = 1$	2000 P2 Q1
(*b*)	5	C	CN	A23, A22, A21	$(-1, -6)$	

•[1] ss: know to differentiate	•[1] $y' = \dots$
•[2] pd: differentiate correctly	•[2] $3x^2 - 6x + 2$
•[3] ss: know that gradient $= f'(1)$	•[3] $y'(1) = -1$
•[4] ss: know that y-coord $= f(1)$	•[4] $y(1) = 0$
•[5] ic: state equ. of line	•[5] $y - 0 = -1(x - 1)$
•[6] ss: equate equations	•[6] $2x - 4 = x^3 - 3x^2 + 2x$
•[7] pd: arrange in standard form	•[7] $x^3 - 3x^2 + 4 = 0$
•[8] ss: know how to solve cubic	•[8] $\begin{array}{c\|cccc} \cdots & 1 & -3 & 0 & 4 \\ & & \cdots & \cdots & \cdots \\ \hline & \cdots & \cdots & \cdots & \cdots \end{array}$
•[9] pd: process	•[9] identify $x = -1$ from working
•[10] ic: interpret	•[10] $(-1, -6)$

[SQA] 2. (*a*) Find the equation of AB, the perpendicular bisector of the line joing the points P$(-3,1)$ and Q$(1,9)$. **4**

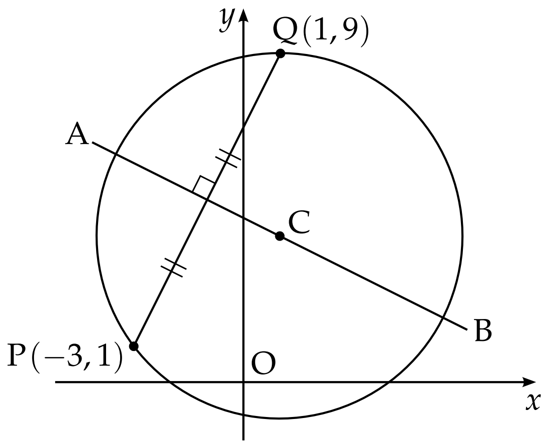
(*b*) C is the centre of a circle passing through P and Q. Given that QC is parallel to the y-axis, determine the equation of the circle. **3**

(*c*) The tangents at P and Q intersect at T.

Write down

(i) the equation of the tangent at Q

(ii) the coordinates of T. **2**



Part	Marks	Level	Calc.	Content	Answer	U2 OC4
(*a*)	4	C	CN	G7	$x+2y=9$	2000 P2 Q2
(*b*)	3	C	CN	G10	$(x-1)^2+(y-4)^2=25$	
(*c*)	2	C	CN	G11, G8	(i) $y=9$, (ii) T$(-9,9)$	

•1 ss: know to use midpoint	•1 midpoint $=(-1,5)$
•2 pd: process gradient of PQ	•2 $m_{PQ}=\frac{9-1}{1-(-1)}$
•3 ss: know how to find perp. gradient	•3 $m_{\perp}=-\frac{1}{2}$
•4 ic: state equ. of line	•4 $y-5=-\frac{1}{2}(x-(-1))$
•5 ic: interpret "parallel to y-axis"	•5 $y_C=4$ *stated or implied by* •7
•6 pd: process radius	•6 radius $=5$ or equiv. *stated or implied by* •7
•7 ic: state equ. of circle	•7 $(x-1)^2+(y-4)^2=25$
•8 ic: interpret diagram	•8 $y=9$
•9 ss: know to use equ. of AB	•9 T$=(-9,9)$

[SQA] 3. $f(x) = 3 - x$ and $g(x) = \dfrac{3}{x}, x \neq 0$.

(*a*) Find $p(x)$ where $p(x) = f(g(x))$. **2**

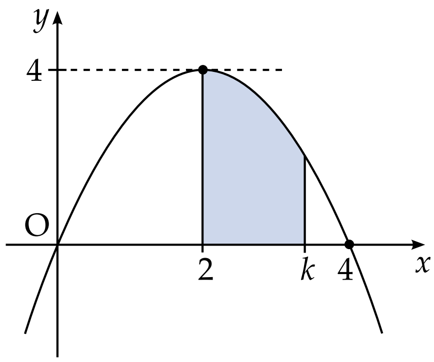
(*b*) If $q(x) = \dfrac{3}{3-x}, x \neq 3$, find $p(q(x))$ in its simplest form. **3**

Part	Marks	Level	Calc.	Content	Answer	U1 OC2
(*a*)	2	C	CN	A4	$3 - \frac{3}{x}$	2000 P2 Q3
(*b*)	2	C	CN	A4	x	
(*b*)	1	A/B	CN	A4		

•[1] ic: interpret composite func. •[2] pd: process •[3] ic: interpret composite func. •[4] pd: process •[5] pd: process	•[1] $f\left(\frac{3}{x}\right)$ *stated or implied by* •[2] •[2] $3 - \frac{3}{x}$ •[3] $p\left(\frac{3}{3-x}\right)$ *stated or implied by* •[4] •[4] $3 - \frac{3}{\frac{3}{3-x}}$ •[5] x

[SQA] 4. The parabola shown crosses the x-axis at $(0,0)$ and $(4,0)$, and has a maximum at $(2,4)$.

The shaded area is bounded by the parabola, the x-axis and the lines $x = 2$ and $x = k$.



(*a*) Find the equation of the parabola. **2**

(*b*) Hence show that the shaded area, A, is given by

$A = -\frac{1}{3}k^3 + 2k^2 - \frac{16}{3}$. **3**

Part	Marks	Level	Calc.	Content	Answer	U2 OC2
(*a*)	2	C	CN	A19	$y = 4x - x^2$	2000 P2 Q4
(*b*)	3	C	CN	C16	proof	

•[1] ic: state standard form •[2] pd: process for x^2 coeff. •[3] ss: know to integrate •[4] pd: integrate correctly •[5] pd: process limits and complete proof	•[1] $ax(x-4)$ •[2] $a = -1$ •[3] $\int_2^k$(function from (a)) •[4] $-\frac{1}{3}x^3 + 2x^2$ •[5] $-\frac{1}{3}k^3 + 2k^2 - \left(-\frac{8}{3} + 8\right)$

[SQA] 5. Solve the equation $3\cos 2x^\circ + \cos x^\circ = -1$ in the interval $0 \leq x \leq 360$. **5**

Part	Marks	Level	Calc.	Content	Answer	U2 OC3
	5	A/B	CR	T10	$60, 131{\cdot}8, 228{\cdot}2, 300$	2000 P2 Q5

•1 ss: know to use $\cos 2x = 2\cos^2 x - 1$	•1 $3(2\cos^2 x^\circ - 1)$
•2 pd: process	•2 $6\cos^2 x^\circ + \cos x^\circ - 2 = 0$
•3 ss: know to/and factorise quadratic	•3 $(2\cos x^\circ - 1)(3\cos x^\circ + 2)$
•4 pd: process	•4 $\cos x^\circ = \frac{1}{2}$, $x = 60, 30$
•5 pd: process	•5 $\cos x^\circ = -\frac{2}{3}$, $x = 132, 228$

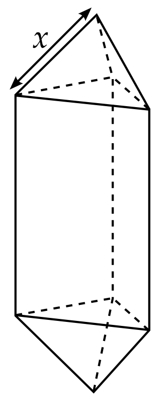
[SQA] 6. A goldsmith has built up a solid which consists of a triangular prism of fixed volume with a regular tetrahedron at each end.

The surface area, A, of the solid is given by

$$A(x) = \frac{3\sqrt{3}}{2}\left(x^2 + \frac{16}{x}\right)$$

where x is the length of each edge of the tetrahedron.

Find the value of x which the goldsmith should use to minimise the amount of gold plating required to cover the solid. **6**



Part	Marks	Level	Calc.	Content	Answer	U1 OC3
	6	A/B	CN	C11	$x = 2$	2000 P2 Q6

•1 ss: know to differentiate	•1 $A'(x) = \ldots$
•2 pd: process	•2 $\frac{3\sqrt{3}}{2}(2x - 16x^{-2})$ **or** $3\sqrt{3}x - 24\sqrt{3}x^{-2}$
•3 ss: know to set $f'(x) = 0$	•3 $A'(x) = 0$
•4 pd: deal with x^{-2}	•4 $-\frac{16}{x^2}$ **or** $-\frac{24\sqrt{3}}{x^2}$
•5 pd: process	•5 $x = 2$
•6 ic: check for minimum	•6 see table below; so $x = 2$ is min.

x	2^-	2	2^+
$A'(x)$	$-ve$	0	$+ve$

so $x = 2$ is min.

[SQA] 7. For what value of t are the vectors $\boldsymbol{u} = \begin{pmatrix} t \\ -2 \\ 3 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 2 \\ 10 \\ t \end{pmatrix}$ perpendicular? **2**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
	2	C	CN	G27	$t = 4$	2000 P2 Q7

•¹ ss: know to use scalar product •² ic: interpret scalar product	•¹ $\boldsymbol{u}.\boldsymbol{v} = 2t - 20 + 3t$ •² $\boldsymbol{u}.\boldsymbol{v} = 0 \Rightarrow t = 4$

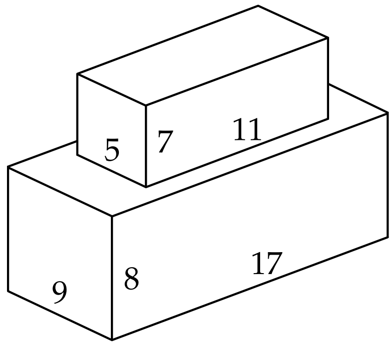
[SQA] 8. Given that $f(x) = (5x-4)^{\frac{1}{2}}$, evaluate $f'(4)$. **3**

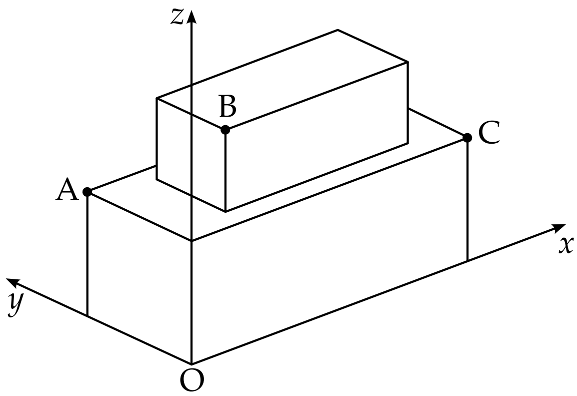
Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	1	C	CN	C21	$\frac{5}{8}$	2000 P2 Q8
	2	A/B	CN	C21		

•¹ pd: differentiate power •² pd: differentiate 2nd function •³ pd: evaluate $f'(x)$	•¹ $\frac{1}{2}(5x-4)^{-\frac{1}{2}}$ •² $\times 5$ •³ $f'(4) = \frac{5}{8}$

[SQA] 9. A cuboid measuring 11 cm by 5 cm by 7 cm is placed centrally on top of another cuboid measuring 17 cm by 9 cm by 8 cm.

Coordinates axes are taken as shown.





(*a*) The point A has coordinates $(0, 9, 8)$ and C has coordinates $(17, 0, 8)$.
Write down the coordinates of B. **1**

(*b*) Calculate the size of angle ABC. **6**

Part	Marks	Level	Calc.	Content	Answer	U3 OC1
(*a*)	1	C	CN	G22	$B(3, 2, 15)$	2000 P2 Q9
(*b*)	6	C	CR	G28	$92{\cdot}5°$	

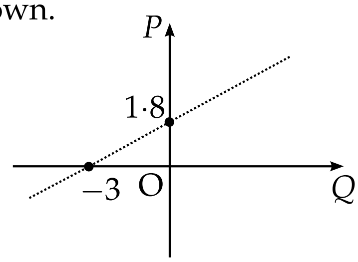
•[1] ic: interpret 3-d representation	•[1] $B = (3, 2, 15)$ *treat* $\begin{pmatrix} 3 \\ 2 \\ 15 \end{pmatrix}$ *as bad form*
•[2] ss: know to use scalar product	•[2] $\cos A\hat{B}C = \frac{\overrightarrow{BA}.\overrightarrow{BC}}{\lvert\overrightarrow{BA}\rvert\lvert\overrightarrow{BC}\rvert}$
•[3] pd: process vectors	•[3] $\overrightarrow{BA} = \begin{pmatrix} -3 \\ 7 \\ -7 \end{pmatrix}$
•[4] pd: process vectors	•[4] $\overrightarrow{BC} = \begin{pmatrix} 14 \\ -2 \\ -7 \end{pmatrix}$
•[5] pd: process lengths	•[5] $\lvert\overrightarrow{BA}\rvert = \sqrt{107}$, $\lvert\overrightarrow{BC}\rvert = \sqrt{249}$
•[6] pd: process scalar product	•[6] $\overrightarrow{BA}.\overrightarrow{BC} = -7$
•[7] pd: evaluate scalar product	•[7] $A\hat{B}C = 92{\cdot}5°$

[SQA] 10. Find $\int \frac{1}{(7-3x)^2}\,dx$. **2**

Part	Marks	Level	Calc.	Content	Answer	U3 OC2
	2	A/B	CN	C22, C14	$\frac{1}{3(7-3x)}+c$	2000 P2 Q10

•1 pd: integrate function •2 pd: deal with function of function	•1 $\frac{1}{-1}(7-3x)^{-1}$ •2 $\times\frac{1}{-3}$

[SQA] 11. The results of an experiment give rise to the graph shown.



(*a*) Write down the equation of the line in terms of P and Q. **2**

It is given that $P = \log_e p$ and $Q = \log_e q$.

(*b*) Show that p and q satisfy a relationship of the form $p = aq^b$, stating the values of a and b. **4**

Part	Marks	Level	Calc.	Content	Answer	U3 OC3
(*a*)	2	A/B	CR	G3	$P = 0{\cdot}6Q + 1{\cdot}8$	2000 P2 Q11
(*b*)	4	A/B	CR	A33	$a = 6{\cdot}05$, $b = 0{\cdot}6$	

•1 ic: interpret gradient •2 ic: state equ. of line •3 ic: interpret straight line •4 ss: know how to deal with x of $x \log y$ •5 ss: know how to express number as log •6 ic: interpret sum of two logs	•1 $m = \frac{1{\cdot}8}{3} = 0{\cdot}6$ •2 $P = 0{\cdot}6Q + 1{\cdot}8$ Method 1 •3 $\log_e p = 0{\cdot}6 \log_e q + 1{\cdot}8$ •4 $\log_e q^{0{\cdot}6}$ •5 $\log_e 6{\cdot}05$ •6 $p = 6{\cdot}05 q^{0{\cdot}6}$ Method 2 $\ln p = \ln aq^b$ •3 $\ln p = \ln a + b \ln q$ •4 $\ln p = 0{\cdot}6 \ln q + 1{\cdot}8$ *stated or implied by •5 or •6* •5 $\ln a = 1{\cdot}8$ •6 $a = 6{\cdot}05$, $b = 0{\cdot}6$

[END OF QUESTIONS]