X100/301

NATIONAL
QUALIFICATIONS
2002

MONDAY, 27 MAY
9.00 AM – 10.10 AM

MATHEMATICS
HIGHER
Units 1, 2 and 3
Paper 1
(Non-calculator)

Read Carefully

1 **Calculators may NOT be used in this paper.**

2 Full credit will be given only where the solution contains appropriate working.

3 Answers obtained by readings from scale drawings will not receive any credit.

LIB X100/301 6/27070

FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\boldsymbol{a}.\boldsymbol{b} = |\boldsymbol{a}|\,|\boldsymbol{b}| \cos\theta$, where θ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$

or $\boldsymbol{a}.\boldsymbol{b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\boldsymbol{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\begin{aligned}
\sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\
\cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\
\sin 2A &= 2\sin A \cos A \\
\cos 2A &= \cos^2 A - \sin^2 A \\
&= 2\cos^2 A - 1 \\
&= 1 - 2\sin^2 A
\end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a\cos ax$
$\cos ax$	$-a\sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)\,dx$
$\sin ax$	$-\frac{1}{a}\cos ax + C$
$\cos ax$	$\frac{1}{a}\sin ax + C$

ALL questions should be attempted.

Marks

1. The point P(2, 3) lies on the circle $(x + 1)^2 + (y - 1)^2 = 13$. Find the equation of the tangent at P. **4**

2. The point Q divides the line joining P(−1, −1, 0) to R(5, 2, −3) in the ratio 2 : 1. Find the coordinates of Q. **3**

3. Functions f and g are defined on suitable domains by $f(x) = \sin(x°)$ and $g(x) = 2x$.

   (*a*) Find expressions for:

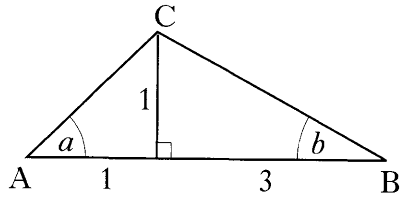
   (i) $f(g(x))$;

   (ii) $g(f(x))$. **2**

   (*b*) Solve $2f(g(x)) = g(f(x))$ for $0 \leq x \leq 360$. **5**

4. Find the coordinates of the point on the curve $y = 2x^2 - 7x + 10$ where the tangent to the curve makes an angle of 45° with the positive direction of the x-axis. **4**

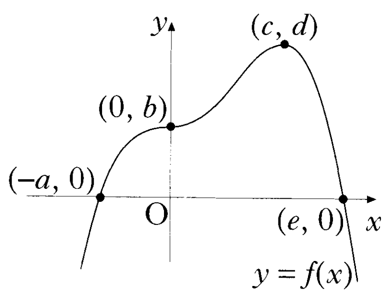
5. In triangle ABC, show that the exact value of $\sin(a + b)$ is $\frac{2}{\sqrt{5}}$. **4**



6. The graph of a function f intersects the x-axis at $(-a, 0)$ and $(e, 0)$ as shown.

   There is a point of inflexion at $(0, b)$ and a maximum turning point at (c, d).

   Sketch the graph of the derived function f'. **3**



[Turn over for Questions 7 to 11 on *Page four*

Marks

7. (*a*) Express $f(x) = x^2 - 4x + 5$ in the form $f(x) = (x - a)^2 + b$. **2**

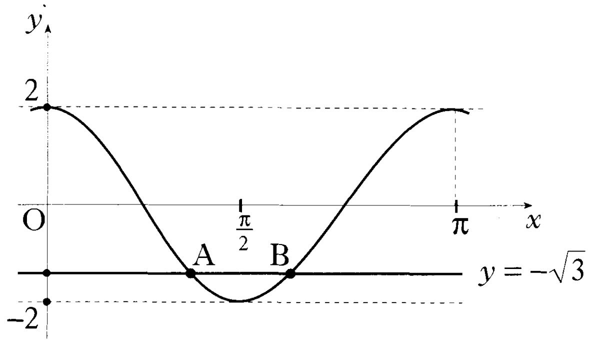
(*b*) On the same diagram sketch:

(i) the graph of $y = f(x)$;

(ii) the graph of $y = 10 - f(x)$. **4**

(*c*) Find the range of values of x for which $10 - f(x)$ is positive. **1**

8. The diagram shows the graph of a cosine function from 0 to π.



(*a*) State the equation of the graph. **1**

(*b*) The line with equation $y = -\sqrt{3}$ intersects this graph at points A and B.

Find the coordinates of B. **3**

9. (*a*) Write $\sin(x) - \cos(x)$ in the form $k\sin(x - a)$ stating the values of k and a where $k > 0$ and $0 \leq a \leq 2\pi$. **4**

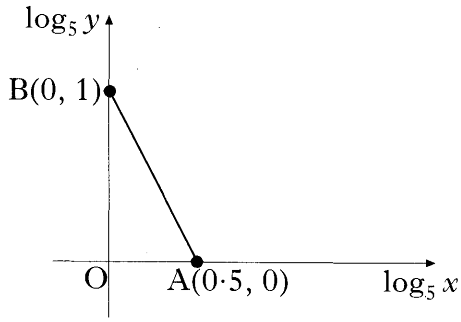
(*b*) Sketch the graph of $y = \sin(x) - \cos(x)$ for $0 \leq x \leq 2\pi$, showing clearly the graph's maximum and minimum values and where it cuts the x-axis and the y-axis. **3**

10. (*a*) Find the derivative of the function $f(x) = (8 - x^3)^{\frac{1}{2}}$, $x < 2$. **2**

(*b*) Hence write down $\int \frac{x^2}{(8 - x^3)^{\frac{1}{2}}}\,dx$. **1**

11. The graph illustrates the law $y = kx^n$.

If the straight line passes through A(0·5, 0) and B(0, 1), find the values of k and n. **4**



[*END OF QUESTION PAPER*]

X100/303

NATIONAL QUALIFICATIONS 2002

MONDAY, 27 MAY
10.30 AM – 12.00 NOON

MATHEMATICS
HIGHER
Units 1, 2 and 3
Paper 2

Read Carefully

1 **Calculators may be used in this paper.**

2 Full credit will be given only where the solution contains appropriate working.

3 Answers obtained by readings from scale drawings will not receive any credit.

LIB X100/303 6/27070

ALL questions should be attempted.

Marks

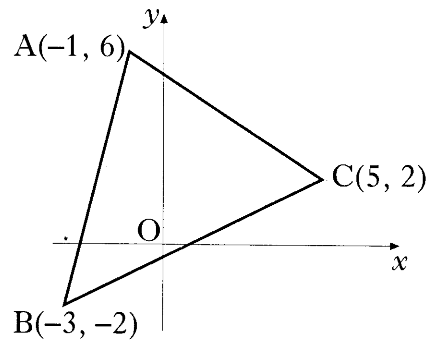
1. Triangle ABC has vertices A(–1, 6), B(–3, –2) and C(5, 2).

Find

(*a*) the equation of the line p, the median from C of triangle ABC. **3**

(*b*) the equation of the line q, the perpendicular bisector of BC. **4**

(*c*) the coordinates of the point of intersection of the lines p and q. **1**



2. The diagram shows a square-based pyramid of height 8 units.

Square OABC has a side length of 6 units.

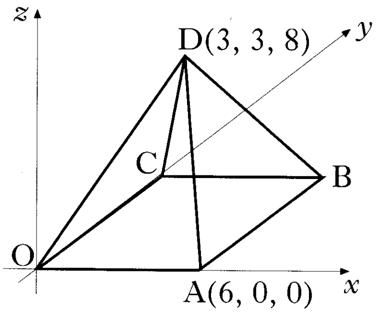
The coordinates of A and D are (6, 0, 0) and (3, 3, 8).

C lies on the y-axis.

(*a*) Write down the coordinates of B. **1**

(*b*) Determine the components of $\overrightarrow{DA}$ and $\overrightarrow{DB}$. **2**

(*c*) Calculate the size of angle ADB. **4**

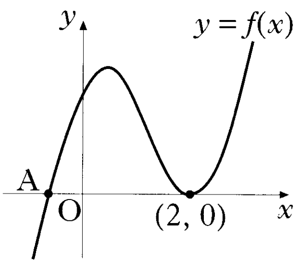


3. The diagram shows part of the graph of the curve with equation $y = 2x^3 - 7x^2 + 4x + 4$.

(*a*) Find the x-coordinate of the maximum turning point. **5**

(*b*) Factorise $2x^3 - 7x^2 + 4x + 4$. **3**

(*c*) State the coordinates of the point A and hence find the values of x for which $2x^3 - 7x^2 + 4x + 4 < 0$. **2**



[Turn over

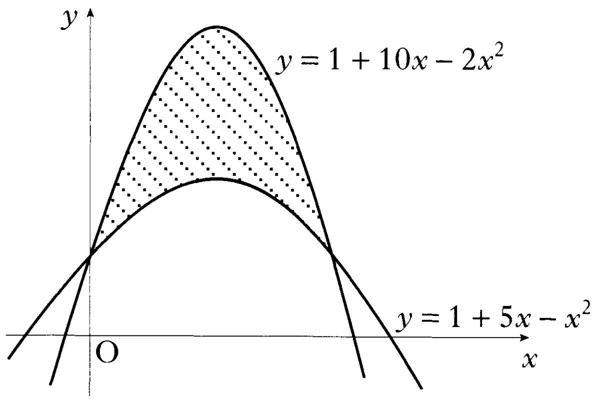
Marks

4. A man decides to plant a number of fast-growing trees as a boundary between his property and the property of his next door neighbour. He has been warned, however, by the local garden centre that, during any year, the trees are expected to increase in height by 0·5 metres. In response to this warning he decides to trim 20% off the height of the trees at the start of any year.

(*a*) If he adopts the "20% pruning policy", to what height will he expect the trees to grow in the long run? **3**

(*b*) His neighbour is concerned that the trees are growing at an alarming rate and wants assurances that the trees will grow no taller than 2 metres. What is the minimum percentage that the trees will need to be trimmed each year so as to meet this condition? **3**

5. Calculate the shaded area enclosed between the parabolas with equations $y = 1 + 10x - 2x^2$ and $y = 1 + 5x - x^2$. **6**



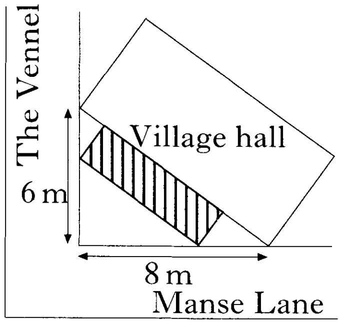
6. Find the equation of the tangent to the curve $y = 2\sin\left(x - \frac{\pi}{6}\right)$ at the point where $x = \frac{\pi}{3}$. **4**

7. Find the x-coordinate of the point where the graph of the curve with equation $y = \log_3 (x - 2) + 1$ intersects the x-axis. **3**

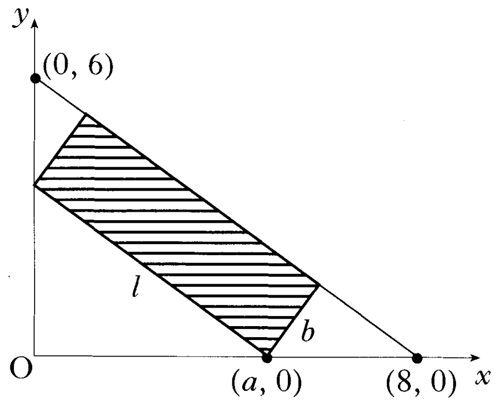
8. A point moves in a straight line such that its acceleration a is given by $a = 2(4 - t)^{\frac{1}{2}}$, $0 \leq t \leq 4$. If it starts at rest, find an expression for the velocity v where $a = \frac{dv}{dt}$. **4**

9. Show that the equation $(1 - 2k)x^2 - 5kx - 2k = 0$ has real roots for all integer values of k. **5**

10. The shaded rectangle on this map represents the planned extension to the village hall. It is hoped to provide the largest possible area for the extension.



The coordinate diagram represents the right angled triangle of ground behind the hall. The extension has length l metres and breadth b metres, as shown. One corner of the extension is at the point $(a, 0)$.



(*a*) (i) Show that $l = \frac{5}{4}a$.

(ii) Express b in terms of a and hence deduce that the area, $A\,\text{m}^2$, of the extension is given by $A = \frac{3}{4}a(8 - a)$. **3**

(*b*) Find the value of a which produces the largest area of the extension. **4**

[*END OF QUESTION PAPER*]

SCOTTISH QUALIFICATIONS AUTHORITY

2002 Mathematics

Higher – Paper 1

Finalised Marking Instructions

1. Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than marks deducted for what is wrong.
2. Award one mark for each 'bullet' point. Each error should be underlined in RED at the point in the working where it first occurs, and not at any subsequent stage of the working.
3. The working subsequent to an error must be followed through by the marker with possible full marks for the subsequent working, provided that the difficulty involved is approximately similar. Where, subsequent to an error, the working is eased, a deduction(s) of mark(s) should be made.
   This may happen where a question is divided into parts. In fact, failure to even answer an earlier section does not preclude a candidate from assuming the result of that section and obtaining full marks for a later section.

4. Correct working should be ticked (✓).This is essential for later stages of the SQA procedures. Where working subsequent to an error(s) is correct and scores marks, it should be marked with a crossed tick (✗). In appropriate cases attention may be directed to work which is not quite correct (e.g. bad form) but which has not been penalised, by underlining with a dotted or wavy line.
   Work which is correct but inadequate to score any marks should be corrected with a double cross tick (✗✗).

5. • The total mark for each section of a question should be entered in red in the **outer** right hand margin, opposite the end of the working concerned.
   • Only the mark should be written, **not** a fraction of the possible marks.
   • These marks should correspond to those on the question paper and these instructions.

6. It is of great importance that the utmost care should be exercised in adding up the marks. Where appropriate, all summations for totals and grand totals must be carefully checked.
   Where a candidate has scored zero marks for any question attempted, "0" should be shown against the answer.

7. As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Accept answers arrived at by inspection or mentally where it is possible for the answer so to have been obtained. Situations where you may accept such working will normally be indicated in the marking instructions.

cont/

8. Do not penalise:
   - working subsequent to a correct answer
   - omission of units
   - bad form
   - legitimate variations in numerical answers
   - correct working in the "wrong" part of a question

9. No marks should awarded for a part of an answer which shows a complete misunderstanding of any fundamental principle or complete ignorance of any process involved in that part.

10. If in doubt between two marks, give an intermediate mark, but without fractions. When in doubt between consecutive numbers, give the higher mark.

11. In cases of difficulty covered neither in detail nor in principle in the Instructions, attention may be directed to the assessment of particular answers by making a PA referral. Write PA at the top left of the front cover of the script and complete the PA referral sheet. This reference must be restricted to genuine cases of difficulty. **Also,** write the letters "**PA**" (in red) on Form Ex6 immediately after the candidate's name.

12. No marks should be deducted at this stage for careless or badly arranged work. In cases where the writing or arrangement is very bad, a note may be made on the upper left-hand corner of the front cover of the script.

13 Do not write any comments on the scripts. A summary of acceptable notation is given on page 4.

Summary

Throughout the examination procedures many scripts are remarked. It is essential that markers follow common procedures:

1 **Tick** correct working.
2 Put a mark in the **right-hand margin to match the marks allocations on the question paper.**
3 Do **not** write marks as fractions.
4 Put each mark **at the end** of the candidate's response to the question.
5 **Follow through** errors to see if candidates can score marks subsequent to the error.
6 Do **not** write any comments on the scripts.

Higher Mathematics : A Guide to Standard Signs and Abbreviations

Remember - No comments on the scripts. Please use the following and nothing else.

Signs

✓ The tick. You are not expected to tick every line but of course you smust check through the whole of a response.

___ ✗ The cross and underline. Underline an error and place a cross at the end of the line.

The tick-cross. Use this to show correct work where you are following through subsequent to an error.

The double cross-tick. Use this to show correct work but which is inadequate to score any marks.

$\wedge$ The roof. Use this to show something is missing such as a crucial step in a proof or a 'condition' etc.

The tilde. Use this to indicate a minor transgression which is not being penalised (such as bad form).

RE Repeated error (which would generally not be penalised within the same question).

BoD Benefit of Doubt. Use this where you have to decide between two consecutive marks and award the higher.

EA Eased. Where working is found correct whilst following through subsequent to an error, the working has been eased sufficiently for a mark not to be awarded.

Marks being allotted e.g. (•) would not normally be shown on scripts

Working		margins
$\frac{dy}{dx}=4x-7$	✓ •	
$4x-7=\underline{0}$	✗	
$x=\frac{7}{4}$		
$y=3\frac{7}{8}$	•	2
$C=\underline{(1,-1)}$	✗	
$m=\frac{3-(-1)}{4-1}$		
$m_{rad}=\frac{4}{3}$	•	
$m_{tgt}=\frac{-1}{\frac{4}{3}}$		
$m_{tgt}=-\frac{3}{4}$	•	
$y-3=-\frac{3}{4}(x-2)$	•	3
$x^2-3x=28$	✓ •	
$x=7$	$\wedge$	1
$\sin 2A=2\sin A\cos A$	✓ •	
$=2\times\frac{1}{3}\times\underline{\frac{3}{4}}$	✗	
$=\frac{1}{2}$	•	
$\cos 2A=\cos^2 A-\sin^2 A$	✓ •	
$=\frac{9}{16}-\frac{1}{9}$	RE •	
$=\frac{65}{144}$	•	5
$\log_3(x-2)=\underline{1}$	✗	
$(x-2)=3^1$	•	
$x-2=3$		
$x=5$	EA	1

All of these are to help us be more consistent and **accurate.**

It goes without saying that however accurate you are in marking, it is to no avail unless you have added the marks up correctly. Please double check totals!!

	Give 1 mark for each •	Illustrations for awarding each •
1	The point P(2, 3) lies on the circle $(x+1)^2+(y-1)^2=13$. Find the equation of the tangent to the circle at P.	4
1	**2.4.1, 1.1.10** **CN. C 02/35** **ans:** $2y+3x=12$ **4 marks** •1 ic : interpret centre from equ of circle •2 ss : know to find gradient of radius •3 ss : know find perpendicular gradient •4 ic : state equation of tangent	•1 $C=(-1,1)$ *ic* •2 $m_{rad}=\frac{2}{3}$ *ss* •3 $m_{tgt}=-\frac{3}{2}$ *ss* •4 $y-3=-\frac{3}{2}(x-2)$ *ic*

Notes

1. •4 is not available unless an attempt has been made to find a perpendicular gradient.
2. •2, •3 and •4 are not available to candidates who incorrectly attempt to use calculus.
3. Please make a PA referral for any candidate who uses implicit differentiation correctly.
4. •4 is not available unless the gradient e.g. $\left(-\frac{1}{\frac{2}{3}}\right)$ has been simplified.

Example

$C=(1,-1)$	✗
$m_{rad}=4$	√✗ •2
$m_{tgt}=-\frac{1}{4}$	√✗ •3
$y-3=-\frac{1}{4}(x-2)$	√✗ •4
	3 marks given

	Give 1 mark for each •	Illustrations for awarding each •

2 The point Q divides the line joining P(–1, –1, 0) to R(5, 2, –3) in the ratio 2 : 1.
Find the coordinates of Q. **3**

2	**3.1.6** NC C 02/64	•[1] $\vec{PR} = \begin{pmatrix} 6 \\ 3 \\ -3 \end{pmatrix}$ *pd*
	ans: (3, 1, –2) **3 marks**	•[2] $\vec{PQ} = \frac{2}{3}\vec{PR}$ *ss*
	•[1] pd : find vector components	•[3] $Q = (3,1,-2)$ *pd*
	•[2] ss : use parallel vectors	
	•[3] pd : process vectors	

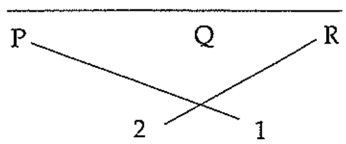
Note

1 An incorrect ratio loses •[2]

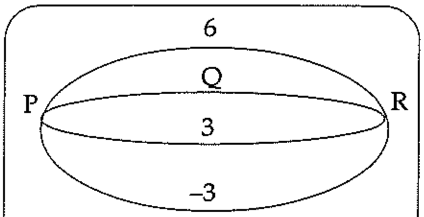
2 Treat $Q = \begin{pmatrix} 3 \\ 1 \\ -2 \end{pmatrix}$ as bad form.

3 The coordinates of Q are only worth •[3]. So an answer with no working is only worth 1 mark (ie •[3]).

4 The justification (ie •[1] and •[2]) may take the form of a diagram eg below are two examples of a diagram and the answer for Q . Each may be awarded 3 marks.



Q = (3, 1, –2)



Q = (3, 1, –2)

Further examples

1

•[1] $\vec{PR} = \begin{pmatrix} 6 \\ 3 \\ -3 \end{pmatrix}$ ✓ •1

•[2] $\vec{PQ} = \underline{\frac{1}{3}\vec{PR}}$ ✗

•[3] $Q = (1,0,-1)$ ✓✗ •3

2 marks

2

•[1] $q = \dfrac{1p + 2r}{3}$ ✓ •1

•[2] *substitute col v.* ✓ •2

•[3] $Q = (3,1,-2)$ ✓ •3

3 marks

3

•[1] $q - p = 2(r - q)$ ✓ •1

$\left[\textit{maybe preceded by } \vec{PQ} = 2\vec{QR} \right]$

•[2] *substitute col v.* ✓ •2

•[3] $Q = (3,1,-2)$ ✓ •3

3 marks

	Give 1 mark for each •	Illustrations for awarding each •

3 Functions f and g are defined on suitable domains by $f(x)=\sin(x^\circ)$ and $g(x)=2x$.

(*a*) Find expressions for (i) $f(g(x))$ **2**

(ii) $g(f(x))$.

(*b* Solve $2f(g(x))=g(f(x))$ for $0 \le x \le 360$ **5**

3	1.2.6, 2.3.3+ CN C 02/5	•[1] $\sin(2x^\circ)$ *ic*
a	ans: $\sin(2x^\circ)$ & $2\sin(x^\circ)$ **2 marks**	•[2] $2\sin(x^\circ)$ *ic*
b	$0^\circ, 60^\circ, 180^\circ, 300^\circ, 360^\circ$ **5 marks**	•[3] $2\sin(2x^\circ)=2\sin(x^\circ)$ *ss*
		•[4] appearance of $2\sin(x^\circ)\cos(x^\circ)$ *ss*
	•[1] ic : interpret f(g(x))	•[5] $2\sin(x^\circ)(2\cos(x^\circ)-1)$ *pd*
	•[2] ic : interpret g(f(x))	•[6] $\sin(x^\circ)=0$ **and** 0, 180, 360 *pd*
	•[3] ss : equate for intersection	•[7] $\cos(x^\circ)=\frac{1}{2}$ **and** 60,300 *pd*
	•[4] ss : substitute for sin $2x$	**or**
	•[5] pd : extract a common factor	•[6] $\sin(x^\circ)=0$ **and** $\cos(x^\circ)=\frac{1}{2}$ *pd*
	•[6] pd : solve a 'common factor' equation	•[7] 0,60,180,300,360 *pd*
	•[7] pd : solve a 'linear' equation	

Notes

1 1 mark may be given for $f(2x)$ **and** $g(\sin x^\circ)$ where final f and g are both wrong.

2 Example 2 illustrates an ***easing*** where a mark is not awarded (although the working is correct). Similarly, solving $\cos(x^\circ)=1 \ldots x=0$ or 360 would be considered as easing the working.

Examples

1

$2\sin(2x^\circ)=2\sin(x^\circ)$ ✓ •3
$2\times 2\sin x^\circ \cos x^\circ = 2\sin x^\circ$ ✓ •4
$2\cos(x^\circ)=1$ ✗
$\cos x^\circ = \frac{1}{2}$ ✓✗
$x=60,300$ ✓✗ •7
3 marks

2

$g(f(x))=\sin(2x^\circ)$ ✗
$f(g(x))=2\sin(x^\circ)$ ✓✗ •2
$2\times 2\sin(x^\circ)=\sin(2x^\circ)$ ✓✗ •3
appearance of $2\sin(x^\circ)\cos(x^\circ)$ ✓✗ •4
$2\sin(x^\circ)(2-\cos(x^\circ))=0$ ✓✗ •5
$\sin(x^\circ)=0$ *and* 0, 180, 360 ✓✗ •6
$\cos(x^\circ)=2$ *and no solution* ✓✗ eased
5 marks

Examples : some endings

3

....
$2\sin x^\circ \cos x^\circ = 0$
$\sin 2x^\circ = 0$
$2x = 0,180,360,\ldots 720$
$x=0,90,180,270,360$ ✓✗ •7

4

....
$2\sin x^\circ \cos x^\circ = 0$
$\sin x^\circ = 0$ **and** $\cos x^\circ = 0$
$x=0,180,360$ **and** $90,270$ ✓✗ •7

	Give 1 mark for each •	Illustrations for awarding each •

4 Find the coordinates of the point on the curve $y = 2x^2 - 7x + 10$ where the tangent to the curve makes an angle of 45° with the positive direction of the x-axis. **4**

	Give 1 mark for each •	Illustrations for awarding each •
4	**1.3.10, 1.1.3** NC C 02/15 **ans: (2, 4)** **4 marks** •[1] sp : know to diff and differentiate •[2] pd : process gradient from angle •[3] ss : equate equivalent expressions •[4] pd : solve and complete	•[1] $\frac{dy}{dx} = 4x - 7$ *sp* •[2] $m_{\text{tang}} = \tan 45° = 1$ *pd* •[3] $4x - 7 = 1$ *ss* •[4] $(2,4)$ *pd*

Notes

1 Simply stating that tan 45° = 1 earns no marks.

Examples

1

$\frac{dy}{dx} = 4x - 7$ ✓ •1

$4x - 7 = \underline{0}$ ✗

$x = \frac{7}{4}$

$y = 3\frac{7}{8}$ ✓✗ •4

2 marks

2

$y = x + c$ ✓ •1

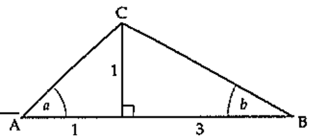
$2x^2 - 8x + 10 - c = 0$ ✓ •2

$\Delta = 64 - 8(10 - c) = 0$ ✓ •3

$c = 2$ and point $= (2,4)$ ✓ •4

4 marks

5 In triangle ABC, show that the exact value of $\sin(a+b)$ is $\frac{2}{\sqrt{5}}$. **4**



	Give 1 mark for each •	Illustrations for awarding each •
5	**2.3.2** NC C 02/32 **ans: proof** **4 marks** •[1] pd : process the missing sides •[2] ss : expand •[3] pd : substitute •[4] pc : process & complete proof	•[1] $AC = \sqrt{2}$ **and** $BC = \sqrt{10}$ *pd* *stated or implied by* •[3] •[2] $\sin(a+b) = \sin a \cos b + \cos a \sin b$ *ss* •[3] $\frac{1}{\sqrt{2}} \cdot \frac{3}{\sqrt{10}} + \frac{1}{\sqrt{2}} \cdot \frac{1}{\sqrt{10}}$ *pd* •[4] $\frac{4}{\sqrt{20}} = \ldots\ldots = \frac{2}{\sqrt{5}}$ *pc*

Notes

1 Partial credit: 2 marks may be given for either of $\frac{1}{\sqrt{2}}\cos b + \frac{1}{\sqrt{2}}\sin b$ *or* $\frac{3}{\sqrt{10}}\sin a + \frac{1}{\sqrt{10}}\cos a$.

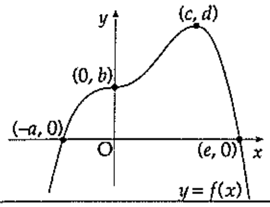
2 At •[4] the completion of the proof might be as $\frac{4}{\sqrt{20}} = \frac{4}{\sqrt{4 \times 5}} = \frac{2}{\sqrt{5}}$ *OR* $\frac{4}{\sqrt{20}} = \frac{4}{2\sqrt{5}} = \frac{2}{\sqrt{5}}$.

	Give 1 mark for each •	Illustrations for awarding each •

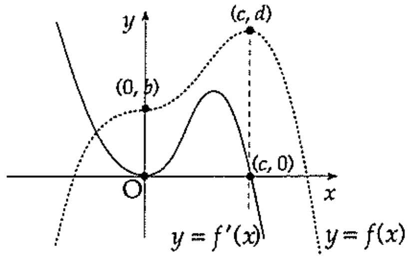
6 The graph of a function f intersects the x-axis at $(-a, 0)$ and $(e, 0)$ as shown.

There is a point of inflexion at $(0, b)$ and a maximum turning point at (c, d).

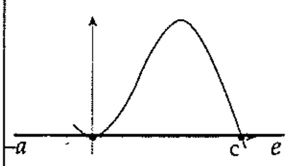
Sketch the graph of the derived function f'. 3



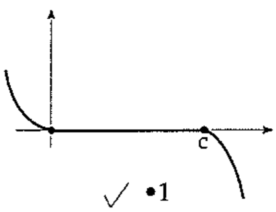
6	**1.2.4** **CN C 02/8** **ans: sketch** **3 marks** •1 ic : interpret stationary points •2 ic : interpret main body of f •3 ic : interpret tails of f	•1 roots at 0 and c (accept a statement to this effect) *ic* •2 min at LH root, max between roots *ic* •3 both 'tails' correct *ic*



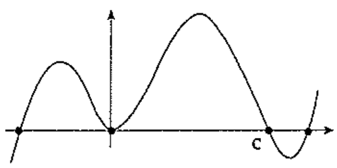
Examples



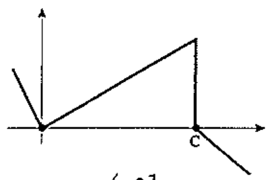
✓ •1
✓ •2
✗ •3



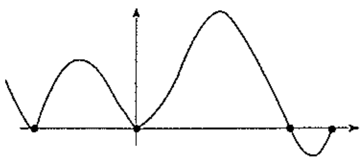
✓ •1
✗ •2
✓ •3



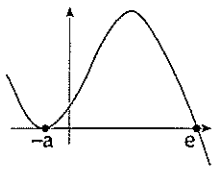
✓ •1
✓ •2
✗ •3



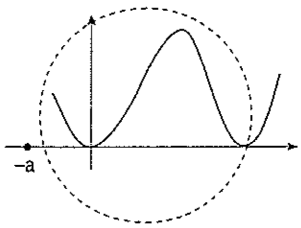
✓ •1
✗ •2
✓ •3



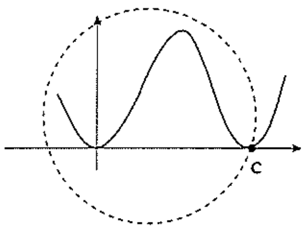
award 1 mark



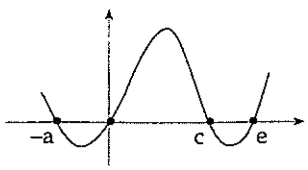
award 1 mark



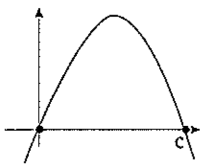
award 1 mark



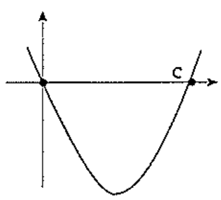
award 2 marks



award 1 mark



✓ •1
✗ •2
✗ •3



✓ •1
✗ •2
✗ •3

	Give 1 mark for each •	Illustrations for awarding each •

7 (*a*) Express $f(x)=x^2-4x+5$ in the form $f(x)=(x-a)^2+b$. 2

(*b*) On the same diagram sketch (i) the graph of $y=f(x)$ (ii) the graph of $y=10-f(x)$. 4

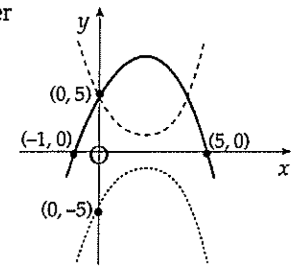
(*c*) Find the range of values of x for which $10-f(x)$ is positive. 1

7	**1.2.8 +, 1.2.4**	**NC C**	**02/9**	
a	**ans:** $a=2$, $b=1$	**2 marks**		
b	**ans: sketch**	**4 marks**		
c	**ans: sketch**	**1 mark**		

•1 pd : process eg completing the square
•2 pd : process eg completing the square
•3 ic : interpret minimum
•4 ic : interpret y-intercept
•5 ss : reflect in x-axis
•6 ss : translate parallel to y-axis
•7 ic : interpret graph

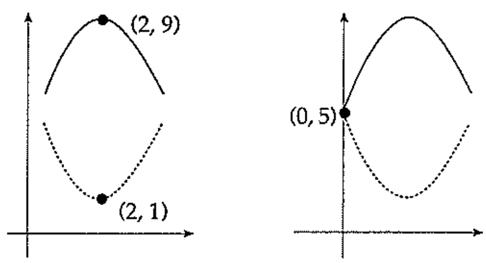
•1	$a=2$	*pd*
•2	$b=1$	*pd*
•3	*any 2 from* parabola; min tp(2,1); (0,5)	*ic*
•4	the remaining one from above list	*ic*
•5	reflecting in x-axis	*ss*
•6	translating +10units $\parallel^{el}$ to y-axis	*ss*
•7	$(-1,5)$ *ie* $-1<x<5$	*ic*

Answer



Note

1. At •7, do not penalise $-1\le x\le 5$
2. If transformations done in wrong order, then •6 is not available.
3. Accept phrase " reflect in x-axis and then translate up 10 units" for 1 mark (in lieu of •5 and •6).
4. Either of the two diagrams below illustrates the minimum requirements for •5 and •6.



5. •5 and •6 are still available for candidates who do not produce a parabola at •3/•4 stage. However a straight line would be a case of easing the working and so for a straight line a maximum of 1 mark (from •5 & •6) could be awarded for a correct reflection and translation.

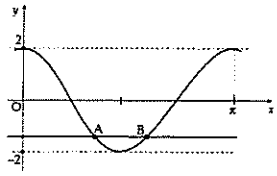
Give 1 mark for each •	Illustrations for awarding each •

8 The diagram shows the graph of a cosine function from 0 to π.

(*a*) State the equation of the graph. 1

(*b*) The line with equation $y=-\sqrt{3}$ intersects this graph at points A and B. 3

Find the coordinates of B.



8	1.2.3, 2.3.1	NC C	02/14
a	ans: y = 2cos 2x	1 mark	
b	ans: $B\left(\frac{7\pi}{12}, -\sqrt{3}\right)$	3 marks	

•1 ic : interpret graph	•1	$2\cos 2x$	*ic*
•2 ss : equate equal parts	•2	$2\cos 2x = -\sqrt{3}$	*ss*
•3 pd : solve linear trig equation in radians	•3	$2x = \frac{5\pi}{6}, \frac{7\pi}{6}$	*pd*
•4 ic : interpret result	•4	$x = \frac{7\pi}{12}$	*ic*

Notes

1 •2 is available for whatever function is obtained in (a).

2 As a consequence of an incorrect function at •1, for •4 to be available, the two roots from •3 must lie as follows:
1 root lies in the interval $0\ldots\pi/2$ and
1 root lies in the interval $\pi/2\ldots\pi$.

3 $x = 105°$ does not earn •4

Examples

1

$y = \underline{2\cos x}$	✗
$2\cos x = -\sqrt{3}$	✓✗ •2
$\cos x = -\frac{\sqrt{3}}{2}$	
$x = \frac{5\pi}{6}, \frac{7\pi}{6}$	✓✗ •3
$x = \ldots\ldots$	✓✗
	2 marks max. (The answers bear no relation to diagram)

2

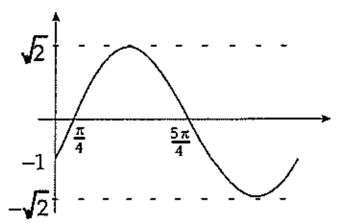
$y = \underline{2\cos x}$	✗
$2\cos x = -\sqrt{3}$	✓✗ •2
$\cos x = -\frac{\sqrt{3}}{2}$	
$x = 150°, 210°$	✓✗ •3
$x = \ldots\ldots$	✓✗
	2 marks max. (The answers bear no relation to diagram)

	Give 1 mark for each •	Illustrations for awarding each •

9 (a) Write $\sin(x)-\cos(x)$ in the form $k\sin(x-a)$ stating the values of k and a where $k>0$ and $0 \le a \le 2\pi$. 4

(b) Sketch the graph of $y=\sin(x)-\cos(x)$ for $0 \le x \le 2\pi$, showing clearly the graph's maximum and minimum values and where it cuts the x-axis and the y-axis. 3

9	**3.4.1** NC C 02/81	•[1] $k\sin(x)\cos(a)-k\cos(x)\sin(a)$ **explicitly stated**
a	**ans:** $\sqrt{2}\sin(x-\frac{\pi}{4})$ **4 marks**	•[2] $k\cos(a)=1$ **and** $k\sin(a)=1$ **explicitly stated**
b	**ans: sketch 3 marks**	•[3] $k=\sqrt{2}$ *pd*
	•[1] ss : know to expand and expand	•[4] $a=\frac{\pi}{4}$ *pd*
	•[2] ic : compare coefficients	•[5] correct shape of graph (*ie* sin) but not passing through the origin *ic*
	•[3] pd : write down the value of k	•[6] graph lies between $\sqrt{2}$ and $-\sqrt{2}$ *ic*
	•[4] pd : process a	•[7] $\left(\frac{\pi}{4},0\right)\left(\frac{5\pi}{4},0\right)$, $(0,-1)$ *pd*
	•[5] ic : sketch a sine curve	
	•[6] ic : int/com max. and min. values	
	•[7] pd : process intercepts	



Notes

1 No justification is required for k ($=\sqrt{2}$)

2 Do not penalise degrees at the •[4] stage

3 •[7] is only available for answers in radians.

4 If k is worked out to be 1, •[6] is not available (eased)

5 Do not penalise graphs which go beyond the interval 0 … 2π.

Example

$$t^2=1^2+1^2 \Rightarrow t=\sqrt{2} \quad ✓ •1$$
$$\sin x-\cos x=\sqrt{2}\left(\sin x\times\tfrac{1}{\sqrt{2}}-\cos x\times\tfrac{1}{\sqrt{2}}\right) \quad ✓ •2$$
$$=\sqrt{2}\left(\sin x\cos\tfrac{\pi}{4}-\cos x\sin\tfrac{\pi}{4}\right) \quad ✓ •3$$
$$=\sqrt{2}\sin\left(x-\tfrac{\pi}{4}\right) \quad ✓ •4$$

	Give 1 mark for each •	Illustrations for awarding each •

10 (*a*) Find the derivative of the function $f(x)=\left(8-x^3\right)^{\frac{1}{2}},\ x<2$. **2**

(*b*) Hence write down $\displaystyle\int \frac{x^2}{\left(8-x^3\right)^{\frac{1}{2}}}\,dx$. **1**

10 **3.2.2** **CN AB 02/62**

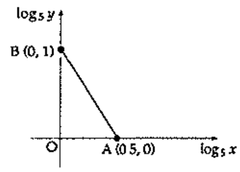
a **ans:** $-\frac{3}{2}x^2\left(8-x^3\right)^{-\frac{1}{2}}$ **2 marks**

b **ans :** $-\frac{2}{3}\left(8-x^3\right)^{\frac{1}{2}}+c$ **1 mark**

- •[1] pd : process differentiation
- •[2] pd : use the chain rule
- •[3] ic : interpret answer from (a)

•[1]	$\frac{1}{2}\left(8-x^3\right)^{-\frac{1}{2}}$	*pd*
•[2]	$\ldots\times -3x^2$	*pd*
•[3]	$-\frac{2}{3}f(x)$ or $-\frac{2}{3}\left(8-x^3\right)^{\frac{1}{2}}$	*ic*

11 The graph illustrates the law $y=kx^n$.
If the straight line passes through A(0·5, 0) and B(0, 1), find the values of k and n. **4**



11 **3.3.7** **NC AB 02/77**

ans : $y=5x^{-2}$ **4 marks**

- •[1] ic : interpret graph
- •[2] ss : use log laws
- •[3] ss : use log laws
- •[3] pd : solve log equation

•[1]	$\log_5 y=-2(\log_5 x)+1$	*ic*
•[2]	$\log_5 y=\log_5 x^{-2}+\ldots$	*ss*
•[3]	$\ldots+\log_5 5$	*pd*
•[4]	$y=5x^{-2}$	*pd*

Note Do not accept $\dfrac{1}{-\frac{1}{2}}$ for the value of n.

Examples

1

No 'theory', just
gradient $=-2$ *so* $n=-2$ ✓ •3
intercept $=1$
$\log_5 k=1$ *so* $k=5$ ✓ •4
2 marks max.

2

$\log_5 y=-2(\log_5 x)+1$ ✓ •1
$\log_5 y+\log_5 x^2=1$ ✓ •2
$\log_5\left(yx^2\right)=1$ ✓ •3
$yx^2=5$ *so* $k=5, n=-2$ ✓ •4
4 marks

3

$\log\ y=\left(\log\ kx^n\right)$ ✓ •1
$\log\ y=n\log\ x+\log\ k$ ✓ •2
gradient $=-2$ *so* $n=-2$ ✓ •3
intercept $=1$
$\log_5 k=1$ *so* $k=5$ ✓ •4
4 marks

4

$\log\ y=\left(\log\ kx^n\right)$ ✓ •1
$\log\ y=n\log\ x+\log\ k$ ✓ •2
followed by sim. equations
$1=n\times 0+\log k$
$0=n\times\left(\frac{1}{2}\right)+\log k$
leads to
$n=-2$ ✓ •3
$\log_5 k=1$ *so* $k=5$ ✓ •4
4 marks

Give 1 mark for each •	Illustrations for awarding each •

for the Mathematics with Statistics paper
Replacing Maths qu 2, 9 & 10,11 .

2 In a survey for a supermarket, 1000 customers were asked to answer these two questions:

A : Would you use the supermarket more often if there were facilities for selling petrol?

B : Would you use the supermarket more often if there were facilities for processing films?

Some of the results are recorded in the table below.

question B

		yes	no	total
question A	yes	102		510
	no	84		
	total			1000

For this set of customers calculate

(i) P(customer responded 'yes' to question A).

(ii) P(customer responded 'no' to both questions).

(iii) P(customer responded 'no' to at least one question). 4

9 A Probus Club went in three buses to a Garden Festival. The frequency distribution of the ages of those on the trip was as follows:

age	60	61	62	63	64	65	66	67	68	69	70	75	77
frequency	10	12	11	8	16	14	9	13	18	15	6	1	1

Would either of the two eldest people be considered as an outlier for this distribution? 5

11 Andrew throws a dart at a circular target of radius a cm. The random variable X represents the distance a dart lands from the centre of the circle. All points in the circle are equally likely to be hit by the dart.

(*a*) Find $P(X < x)$, the probability that a dart lands less than x cm from the centre of the circle. 2

(*b*) Show that the probability density function for the distribution of the distance of a dart from the centre is

$$f(x) = \begin{cases} \dfrac{2x}{a^2} & 0 \le x \le a \\ 0 & \textit{otherwise}. \end{cases}$$ 1

5

(*c*) Find the mean and variance of this distribution.

	Give 1 mark for each •	Illustrations for awarding each •
2	**4.2.3** **CN C 02/S7** **ans : 0·51, 0·406, 0·898 4 marks** •[1] ic : complete table •[2] ic : interpret table •[3] ic : interpret the table •[4] ss : know to add relevant numbers	•[1] complete relevant bits of table *i.e.* (490, 406) *s/i by* •[3] *ic* •[2] 0.51 *ic* •[3] 0.406 *pd* •[4] 0.898 *ss*
9	**4.1.4** **NC AB 02/S2** **ans : only eldest 5 marks** •[1] ss : know how to find Qs eg c.f.s •[2] pd : process median •[3] pd : process quartiles •[4] pd : process fences •[5] ic : interpret value of fence	•[1] cum.frequency column *s/i by* •[2] *or* •[3] *ss* •[2] $Q_1 = 63$ *pd* •[3] $Q_3 = 68$ *pd* •[4] fence $= 75\frac{1}{2}$ *pd* •[5] comment:only the eldest is outlier *ic*
11 a b c	**4.2.3, 4.3.3,4** **CN AB 02/S14** **ans :** $\frac{x^2}{a^2}$ **2 marks** **ans :** $\frac{2x}{a^2}$ **1 mark** **ans :** $\frac{2}{3}a,\ \frac{1}{18}a^2$ **5 marks** •[1] ss : know how to find probability •[2] pd : process probability •[3] ss : know that $f(x) = F'(x)$ •[4] ss : know how to find mean •[5] pd : process $E(X$ •[6] ss : know how to find variance •[7] pd : process $E(X^2)$ •[8] pd : process variance	•[1] probability $= \frac{\text{favourable area}}{\text{possible area}}$ *s/i* *ss* •[2] probability $= \frac{x^2}{a^2}$ *pd* •[3] $f(x) = F'(x) = \frac{2x}{a^2}$ *ss* •[4] mean $= E(X) = \int_0^a \frac{2x^2}{a^2}\,dx$ *ss* •[5] $\frac{2}{3}a$ *pd* •[6] $VarX = E(X^2) - (E(X))^2$ *ss* •[7] $E(X^2) = \int_0^a \frac{2x^3}{a^2}\,dx = \frac{1}{2}a^2$ *pd* •[8] $\frac{1}{18}a^2$ *pd*

SCOTTISH
QUALIFICATIONS
AUTHORITY

2002 Mathematics

Higher – Paper 2

Finalised Marking Instructions

1. Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than marks deducted for what is wrong.
2. Award one mark for each 'bullet' point. Each error should be underlined in RED at the point in the working where it first occurs, and not at any subsequent stage of the working.
3. The working subsequent to an error must be followed through by the marker with possible full marks for the subsequent working, provided that the difficulty involved is approximately similar. Where, subsequent to an error, the working is eased, a deduction(s) of mark(s) should be made.
   This may happen where a question is divided into parts. In fact, failure to even answer an earlier section does not preclude a candidate from assuming the result of that section and obtaining full marks for a later section.

4. Correct working should be ticked (✓).This is essential for later stages of the SQA procedures. Where working subsequent to an error(s) is correct and scores marks, it should be marked with a crossed tick (✗). In appropriate cases attention may be directed to work which is not quite correct (e.g. bad form) but which has not been penalised, by underlining with a dotted or wavy line.
   Work which is correct but inadequate to score any marks should be corrected with a double cross tick (✗).

5. • The total mark for each section of a question should be entered in red in the **outer** right hand margin, opposite the end of the working concerned.
   • Only the mark should be written, **not** a fraction of the possible marks.
   • These marks should correspond to those on the question paper and these instructions.

6. It is of great importance that the utmost care should be exercised in adding up the marks. Where appropriate, all summations for totals and grand totals must be carefully checked.
   Where a candidate has scored zero marks for any question attempted, "0" should be shown against the answer.

7. As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Accept answers arrived at by inspection or mentally where it is possible for the answer so to have been obtained. Situations where you may accept such working will normally be indicated in the marking instructions.

cont/

8. Do not penalise:
   - working subsequent to a correct answer
   - omission of units
   - bad form
   - legitimate variations in numerical answers
   - correct working in the "wrong" part of a question

9. No marks should awarded for a part of an answer which shows a complete misunderstanding of any fundamental principle or complete ignorance of any process involved in that part.

10. If in doubt between two marks, give an intermediate mark, but without fractions. When in doubt between consecutive numbers, give the higher mark.

11. In cases of difficulty covered neither in detail nor in principle in the Instructions, attention may be directed to the assessment of particular answers by making a PA referral. Write PA at the top left of the front cover of the script and complete the PA referral sheet. This reference must be restricted to genuine cases of difficulty. **Also,** write the letters "**PA**" (in red) on Form Ex6 immediately after the candidate's name.

12. No marks should be deducted at this stage for careless or badly arranged work. In cases where writing or arrangement is very bad, a note may be made on the upper left-hand corner of the front cover of the script.

13 Do not write any comments on the scripts. A summary of acceptable notation is given on page 4.

Summary

Throughout the examination procedures many scripts are remarked. It is essential that markers follow common procedures:

1 **Tick** correct working.
2 Put a mark in the **right-hand margin to match the marks allocations on the question paper.**
3 Do **not** write marks as fractions.
4 Put each mark **at the end** of the candidate's response to the question.
5 **Follow through** errors to see if candidates can score marks subsequent to the error.
6 Do **not** write any comments on the scripts.

Higher Mathematics : A Guide to Standard Signs and Abbreviations

Remember - No comments on the scripts. Please use the following and nothing else.

Signs

✓ The tick. You are not expected to tick every line but of course you smust check through the whole of a response.

___ ✗ The cross and underline. Underline an error and place a cross at the end of the line.

The tick-cross. Use this to show correct work where you are following through subsequent to an error.

The double cross-tick. Use this to show correct work but which is inadequate to score any marks.

∧ The roof. Use this to show something is missing such as a crucial step in a proof or a 'condition' etc.

~ The tilde. Use this to indicate a minor transgression which is not being penalised (such as bad form).

RE Repeated error (which would generally not be penalised within the same question).

BoD Benefit of Doubt. Use this where you have to decide between two consecutive marks and award the higher.

EA Eased. Where working is found correct whilst following through subsequent to an error, the working has been eased sufficiently for a mark not to be awarded.

Marks being allotted e.g. (•) would not normally be shown on scripts

		margins
$\frac{dy}{dx}=4x-7$	✓ •	
$4x-7=\underline{0}$	✗	
$x=\frac{7}{4}$		
$y=3\frac{7}{8}$	✗ •	2
$C=\underline{(1,-1)}$	✗	
$m=\frac{3-(-1)}{4-1}$		
$m_{rad}=\frac{4}{3}$	✗ •	
$m_{tgt}=\frac{-1}{\frac{4}{3}}$		
$m_{tgt}=-\frac{3}{4}$	✗ •	
$y-3=-\frac{3}{4}(x-2)$	✗ •	3
$x^2-3x=28$	✓ •	
$x=7$ ∧		1
$\sin 2A=2\sin A\cos A$	✓ •	
$=2\times\frac{1}{3}\times\underline{\frac{3}{4}}$	✗	
$=\frac{1}{2}$	✗ •	
$\cos 2A=\cos^2 A-\sin^2 A$	✓ •	
$=\frac{9}{16}-\frac{1}{9}$ RE	✗ •	
$=\frac{65}{144}$	✗ •	5
$\log_3(\underline{x-2})=1$	✗	
$(x-2)=3^1$	✗ •	
$x-2=3$		
$x=5$ EA		1

All of these are to help us be more consistent and **accurate**.

It goes without saying that however accurate you are in marking, it is to no avail unless you have added the marks up correctly. Please double check totals!!

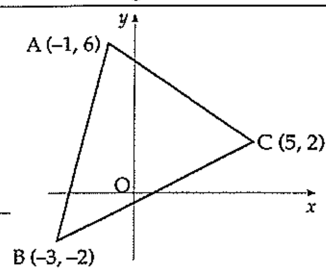
	Give 1 mark for each •	Illustrations of evidence for awarding each •

1 Triangle ABC has vertices A(–1, 6), B(–3, –2) and C(5, 2). Find

(*a*) the equation of the line p, the median from C of triangle ABC. 3

(*b*) the equation of the line q, the perpendicular bisector of BC. 4

(*c*) the coordinates of the point of intersection of the lines p and q. 1



1	**1.1.1 ++**	**CN**	**C**	**02/2**
a	**ans : $y = 2$**	**3 marks**		
b	**ans : $y = -2x + 2$**	**4 marks**		
c	**(0, 2)**	**1 mark**		

- •¹ ss : determine midpoint coordinates
- •² pd : determine gradient thr' 2 pts
- •³ ic : state equation of st line
- •⁴ ss : determine midpoint coordinates
- •⁵ pd : determine gradient thr' 2 pts
- •⁶ ss : determine gradient at right angles to •⁵
- •⁷ ic : state equation of st line
- •⁸ pd : process intersection

•¹	$F = \text{mid}_{AB} = (-2,2)$	*ss*
•²	$m_{FC} = 0$ stated or implied by •³	*pd*
•³	equ FC is $y = 2$	*ic*
•⁴	$M = \text{mid}_{BC} = (1,0)$	*ss*
•⁵	$m_{BC} = \frac{1}{2}$	*pd*
•⁶	$m_{\perp} = -2$	*ss*
•⁷	$y - 0 = -2(x-1)$	*ic*
•⁸	(0,2)	*pd*

Notes

1 For •³, accept $y-2=0(x+2)$ **or** $y-2=(\textit{value of } m\textit{, simplified or not})(x+2)$

2 •⁷ is only available as a consequence of finding a perpendicular gradient

3 The diagrams below illustrate some of the 'lines' which can be accorded any marks. For example, in (b) a line perp. to BC passing through C earns marks •⁵ and •⁶ only.

4 For introducing decimals, work to 2 d.p.

For (a) Perp. bis of AB

$midpt = (-2,2)$	✓ •1
$m_{AB} = 4$	✗
$m_{\perp} = -\frac{1}{4}$	✗
$y-2 = -\frac{1}{4}(x+2)$	✗ •3
	2 marks given

For (a) Altitude thr' C

$m_{AB} = 4$	✓ •2
$m_{\perp} = -\frac{1}{4}$	✗
$y-2 = -\frac{1}{4}(x-5)$	✓ •3
	2 marks given

(b) Median thr' A

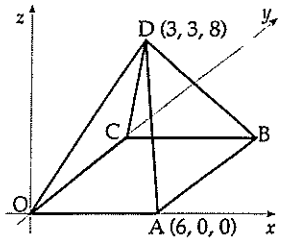
$midpt = (1,0)$	✓ •4
$m_{med} = -3$	✓ •5
$y-0 = \underline{-3}(x-1)$	✗
	2 marks given

(b) Altitude thr' A

$m_{BC} = \frac{1}{2}$	✓ •5
$m_{perp} = -2$	✓ •6
$\underline{y-6} = -2\underline{(x+1)}$	✗
	2 marks given

	Give 1 mark for each •	Illustrations of evidence for awarding each •

2 The diagram shows a square-based pyramid of height 8 units. Square OABC has a side length of 6 units. The coordinates of A and D are (6, 0, 0) and (3, 3, 8). C lies on the y-axis.



(*a*) Write down the coordinates of B. **1**

(*b*) Determine the components of $\overrightarrow{DA}$ and $\overrightarrow{DB}$. **2**

(*c*) Calculate the size of angle ADB. **4**

	Give 1 mark for each •	Illustrations of evidence for awarding each •
2	**3.1.11** **C** **C** **02/66**	
a	**ans : (6, 6, 0)** **1 mark**	•[1] $B=(6,6,0)$ *ic*
b	**ans :** $\overrightarrow{DA}=\begin{pmatrix}3\\-3\\-8\end{pmatrix}$ $\overrightarrow{DB}=\begin{pmatrix}3\\3\\-8\end{pmatrix}$ **2 marks**	•[2] $\overrightarrow{DA}=\begin{pmatrix}3\\-3\\-8\end{pmatrix}$ *ic* •[3] $\overrightarrow{DB}=\begin{pmatrix}3\\3\\-8\end{pmatrix}$ *ic*
c	**ans : 38·7°** **4 marks** •[1] ic : interpret diagram •[2] ic : write down components of a vector •[3] ic : write down components of a vector •[4] ss : use e.g. scalar product formula •[5] pd : process lengths •[6] pd : process scalar product •[7] pd : process angle	•[4] $\cos A\hat{D}B=\dfrac{\overrightarrow{DA}.\overrightarrow{DB}}{\lvert\overrightarrow{DA}\rvert\lvert\overrightarrow{DB}\rvert}$ stated/implied by •[7] *ss* •[5] $\lvert\overrightarrow{DA}\rvert=\sqrt{82},\ \lvert\overrightarrow{DB}\rvert=\sqrt{82}$ *pd* •[6] $\overrightarrow{DA}.\overrightarrow{DB}=64$ *pd* •[7] $A\hat{D}B=38.7°$ *pd*

Alternative method 1 for •[4] to •[7].

- •[4] $\cos A\hat{D}B=\dfrac{a^2+b^2-d^2}{2ab}$ *ss*
- •[5] $a=b=\sqrt{82}$ *pd*
- •[6] $\overrightarrow{BA}=\begin{pmatrix}0\\6\\0\end{pmatrix}\Rightarrow d=6$ *pd*
- •[7] $A\hat{D}B=38.7°$ *pd*

Alternative method 2 for •[4] to •[7].

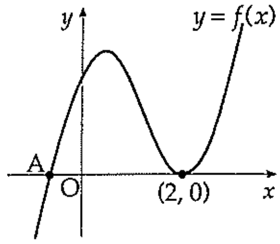
- •[4] ΔADB *isosceles, half base* $=3$ *ss*
- •[5] $a=b=\sqrt{82}$ *pd*
- •[6] $\sin\frac{1}{2}ADB=\dfrac{3}{\sqrt{82}}$ *ss*
- •[7] $A\hat{D}B=38.7°$ *pd*

Note

1. For •[7] accept 38·6° or 0·67 radians or 0·68 radians and answers which round to these values.
2. Do not penalise premature rounding before •[7]. •[7] is the only mark available for calculations.
3. Any formula quoted at •[4] must relate to the data or labelling in this question.
4. Treat $\overrightarrow{DA}=d-a$ and $\overrightarrow{DB}=d-b$ as a repeated error (RE) (•[2] not available).
5. Treat $\overrightarrow{DA}=a+d$ and $\overrightarrow{DB}=b+d$ as a repeated error (RE) (•[2] not available).
6. Calculations of the angles AOB (45) or AOD (70.7) may earn 3 of the last 4 marks provided the correct use of the scalar product has been demonstrated.

	Give 1 mark for each •	Illustrations of evidence for awarding each •

3 The diagram shows part of the graph of the curve with equation $y = 2x^3 - 7x^2 + 4x + 4$.



(*a*) Find the x-coordinate of the maximum turning point. 5

(*b*) Factorise $2x^3 - 7x^2 + 4x + 4$. 3

(*c*) State the coordinates of the point A and hence find the values of x for which $2x^3 - 7x^2 + 4x + 4 < 0$. 2

3	**1.3.12, 2.1.3** **NC. C** **02/23**	•¹ $f'(x) = \ldots\ldots$	*ss*
a	**ans :** $x = \frac{1}{3}$ **5 marks**	•² $6x^2 - 14x + 4$	*pd*
b	**ans :** $(x-2)(2x+1)(x-2)$ **3 marks**	•³ $6x^2 - 14x + 4 = 0$	*ss*
c	**ans :** $A(-\frac{1}{2}, 0)$, $x < -\frac{1}{2}$ **2 marks**	•⁴ $(3x-1)(x-2)$	*pd*
		•⁵ $x = \frac{1}{3}$	*pd*
	•¹ ss : know to differentiate		
	•² pd : differentiate		
	•³ ss : know to set derivative to zero	•⁶ $\begin{array}{c\|cccc} \ldots & 2 & -7 & 4 & 4 \\ & \ldots & \ldots & \ldots & \ldots \\ \hline & \ldots & \ldots & \ldots & 0 \end{array}$	*ss*
	•⁴ pd : start solving process of equation		
	•⁵ pd : complete solving process		
	•⁶ ss : strategy for cubic eg synth division	•⁷ $2x^2 - 3x - 2$	*ic*
	•⁷ ic : extract quadratic factor	•⁸ $(x-2)(2x+1)(x-2)$	*pd*
	•⁸ pd : complete the cubic factorisation	•⁹ $A(-\frac{1}{2}, 0)$	*ic*
	•⁹ ic : interpret the factors	•¹⁰ $x < -\frac{1}{2}$	*ic*
	•¹⁰ ic : interpret the diagram		

Notes

1. •⁷ may be awarded for repeated synthetic divisions to arrive at **another two with** zero remainders.
2. The "= 0" shown at •3 must appear at least once somewhere in the working between •¹ and •⁵ (but not necessarily at •³)
3. At •⁴, the common factor of 2 may be included at the front or inside one of the binomials.
4. In (b) if $(x-2)^2(x+\frac{1}{2})$ appears *ex nihilo* award 1 mark out of the 3 available.
5. Candidates who attempt to find a solution via a graphics calculator earn no marks. The only acceptable method is via calculus.
6. For •⁹ accept $x = -\frac{1}{2}$
7. For •¹⁰ accept $x \le -\frac{1}{2}$.

	Give 1 mark for each •	Illustrations of evidence for awarding each •

4 A man decides to plant a number of fast-growing trees as a boundary between his property and the property of his next door neighbour. He has been warned, however, by the local garden centre that, during any year, the trees are expected to increase in height by 0·5 metres. In response to this warning he decides to trim 20% off the height of the trees at the start of any year.

(*a*) If he adopts the "20% pruning policy", to what height will he expect the trees to grow in the long run? 3

(*b*) His neighbour is concerned that the trees are growing at an alarming rate and wants assurances that the trees will grow no taller than 2 metres. What is the minimum percentage that the trees will need to be trimmed each year so as to meet this condition? 3

4	**1.4.3 ++**	**CN C**	**02/21**
a	**ans : 2·5 metres**	**3 marks**	
b	**ans : trim 25%**	**3 marks**	

•1 ic : interpret the decay factor	•1 0.8 stated or implied	*ic*
•2 ss : strategy for limit	•2 *eg* $l = 0.8l + 0.5$ *or* $l = \frac{0.5}{1-0.8}$	*ss*
•3 pd : process limit	•3 $-1 < 0.8 < 1$ so $l = 2.5$ *metres*	*pd*
•4 ss : reverse strategy for limit	•4 $2 = 2m + 0.5$	*ss*
•5 pd : process	•5 $m = 0.75$	*pd*
•6 ic : interpret scale factor	•6 trim 25%	*ic*

marked example

0.2	X		
$l = 0.2l + 0.5$	✓X	•2	
$l = 0.625$ *metres*	X	^	1
$2 = 2m + 0.5$	✓X	•4	
$m = 0.75$	✓X	•5	
trim 75%	X		2

Note

1 Correct answers with no working gain no marks.

in (a)

1 For •3 accept $0 < 0.8 < 1$ so $l = 2.5$ metres

2 For $a = 1.2$
$L = 1.2L - 0.5$
$L = 2.5$ award NO marks.

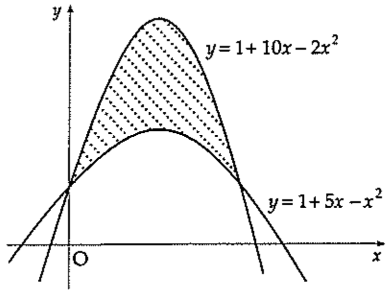
in (b)

3 Trial & improvement is barely acceptable but the three marks are available as follows
for 1st trial with a guess > 20% give •1
for 2nd trial + improvement give •2
for more trial(s) leading to 25% give •3

Candidates may strike lucky by :
Try pruning at 25%
Then $L = 0.75L + 0.5 = 2$ giving $L = 2$ (metres)
This needs to be accorded 3 marks.

	Give 1 mark for each •	Illustrations of evidence for awarding each •

5 Calculate the shaded area enclosed between the parabolas with equations $y=1+10x-2x^2$ and $y=1+5x-x^2$. 6



5 2.2.7 CN C 02/84

ans : $20\frac{5}{6}$ **6 marks**

•1	ss : find intersections	$1+10x-2x^2=1+5x-x^2$	ss
•2	ss : know to find limits	$x=0,5$ and $\int_0^5\left(\ \right)$	ss
•3	ss : know to integrate (upper – lower)	$\int\left(\left(1+10x-2x^2\right)-\left(1+5x-x^2\right)\right)dx$	ss
•4	pd : simplify	$\int\left(5x-x^2\right)\,dx$	pd
•5	pd : integrate	$\frac{5}{2}x^2-\frac{1}{3}x^3$	pd
•6	pd : process limits	$20\frac{5}{6}$	pd

Notes

1 Candidates who do not simplify (•4) before integrating may be awarded •4 for integrating the upper function correctly and may be awarded •5 for integrating the lower function. The last mark is for evaluating and stating the area.

2 For candidates who find two separate areas and subtract - use the illustration below as a guide.

2 Candidates who attempt to find a solution via a graphics calculator earn no marks. The only acceptable method is via calculus.

illustration for Note 2

$1+10x-2x^2=1+5x-x^2$	✓ •1
$x^2-5x=0$	
$x(x-5)=0$	
$x=0,5$	✓
$\int_0^5$ *or* $]_0^5$ (*somewhere below*)	✓ •2
$\int\left(1+10x-2x^2\right)dx$	✓ •3
$x+5x^2-\frac{2}{3}x^3$	✓ •4
$\int\left(1+5x-x^2\right)dx$	
$x+\frac{5}{2}x^2-\frac{1}{3}x^3$	✓ •5
$46\frac{2}{3}$, $25\frac{5}{6}$, and area $=20\frac{5}{6}$	✓ •6
	6 marks

variation on a theme

$1+5x-x^2=1+10x-2x^2$	✓ •1
leading to $x=0,5$	✓
$\int_0^5\left(\left(1+5x-x^2\right)-\left(1+10x-2x^2\right)\right)dx$	✓ •2
	✗ •3
$\int_0^5\left(-5x+x^2\right)\,dx$	✗ •4
$\left[-\frac{5}{2}x^2+\frac{1}{3}x^3\right]_0^5$	✗ •5
$-20\frac{5}{6}$ so area $=20\frac{5}{6}$	✗ •6
	5 marks

	Give 1 mark for each •	Illustrations of evidence for awarding each •
6	Find the equation of the tangent to the curve $y = 2\sin\left(x - \frac{\pi}{6}\right)$ at the point where $x = \frac{\pi}{3}$.	4
6	**1.1.6, 1.3.7, 3.2.2, 1.2.11 CN C 02/63** **ans :** $y = \sqrt{3}x + 1 - \frac{\pi}{\sqrt{3}}$ **4 marks**	
	•[1] pd : find derivative	•[1] $\frac{dy}{dx} = 2\cos\left(x - \frac{\pi}{6}\right)$ *pd*
	•[2] ss : know derivative at x=... represents grad.	•[2] $m = \sqrt{3}$ *ss*
	•[3] pd : find corresponding y-coordinate	•[3] $y_{x=\frac{\pi}{3}} = 1$ *pd*
	•[4] ic : state equation of tangent	•[4] $y - 1 = \sqrt{3}\left(x - \frac{\pi}{3}\right)$ *ic*

Notes

1 Accept decimal equivlent for √3

2 •[4] is only available if an attempt to find m is based on calculus.

$y = 2\sin\left(x - \frac{\pi}{6}\right)$
$y = 2\sin(x - 30)$
$\frac{dy}{dx} = \underline{2\cos(x - 30)}$ ✗ •1

$m = \ldots\ldots = \sqrt{3}$ ✓ •2
$y_{x=30} = \ldots\ldots = 1$ ✓ •3
$y - 1 = \sqrt{3}(x - 60)$ ✗ •4
3 marks

$y = 2\sin\left(x - \frac{\pi}{6}\right)$

$\frac{dy}{dx} = 2\cos\left(x - \frac{\pi}{6}\right)$ ✓ •1
$= 2\cos(x - 30)$
$m = \ldots\ldots = \sqrt{3}$ ✓ •2
$y_{x=30} = \ldots\ldots = 1$ ✓ •3
$y - 1 = \sqrt{3}(x - \underline{60})$ ✗ •4
3 marks

	Give 1 mark for each •	Illustrations of evidence for awarding each •

7 Find the x-coordinate of the point where the graph of the curve with equation $y = \log_3(x-2) + 1$ intersects the x-axis. 3

7	3.3.1 CN C/AB 02/58 **ans :** $x = 2\frac{1}{3}$ **3 marks** •1 ss : know to isolate log term •2 pd : express log equation as expo. equ. •3 pd : process	•1 $\log_3(x-2) = -1$ *ss* •2 $x-2 = 3^{-1}$ *pd* •3 $x = 2\frac{1}{3}$ *pd*

Notes

1 Candidates who sketch the (related) function and conclude that the root is $2 < x < 3$ may be awarded 1 mark. (Do not accept " the root = 2").

$\log_3(x-2) = \underline{1}$ ✗

$(x-2) = 3$ ✗ •2

$x = 5$ ✗ eased

1 mark awarded

$-\log_3(x-2) = 1$ ✓

$\log_3(x-2)^{-1} = 1$ ✓ •1

$(x-2)^{-1} = 3$ ✓ •2

$x = 2\frac{1}{3}$ ✓ •3

3 marks awarded

$\log_3(x-2) + 1 = 0$

$(x-2) \underline{+1} = \ldots\ldots$ ✗

$\ldots\ldots = 3^0$ ✓ •2

$x = 2$ ✗ eased

1 mark awarded

$\log_3(x-2) + \log_3 3 = \ldots$ ✓

$\ldots = \log_3 1$ ✓

$3(x-2) = 1$

$x = 2\frac{1}{3}$ ✓

award 3 marks

	Give 1 mark for each •	Illustrations of evidence for awarding each •

8 A point moves in a straight line such that its acceleration a is given by $a = 2(4-t)^{\frac{1}{2}}$, $0 \le t \le 4$.

If it starts at rest, find an expression for the velocity v where $a = \frac{dv}{dt}$. **4**

8	**3.2.3, 2.2.6** **NC** **C** **02/54** **ans :** $V = -\frac{4}{3}(4-t)^{\frac{3}{2}} + \frac{32}{3}$ **4 marks** •1 ss : know to integrate acceleration •2 pd : integrate •3 ic : use initial conditions with const of int •4 pd : process solution	•1 $V = \int \left(2(4-t)^{\frac{1}{2}}\right)dt$ *ss* stated or implied by •2 •2 $2 \times \frac{1}{-\frac{3}{2}}(4-t)^{\frac{3}{2}}$ *pd* •3 $0 = 2 \times \frac{1}{-\frac{3}{2}}(4-0)^{\frac{3}{2}} + c$ *ic* •4 $c = 10\frac{2}{3}$ *pd* Notes 1 •3 and •4 are only available when a constant of integration is included. 2 Differentiation earns no marks. 3 Note that $\int_0^4 \left(2(4-t)^{\frac{1}{2}}\right)dt$ only earns the first two marks. $= \left[2 \times \frac{1}{-\frac{3}{2}}(4-t)^{\frac{3}{2}}\right]_0^4$ $= 10\frac{2}{3}$

	Give 1 mark for each •	Illustrations of evidence for awarding each •	
9	Show that the equation $(1-2k)x^2 - 5kx - 2k = 0$ has real roots for all integer values of k.		5

9 **2.1.5, 7, 9** **CN AB 02/39**

ans : proof **5 marks**

- •1 ss : know to use discriminant
- •2 ic : pick out discriminant
- •3 pd : simplify to quadratic
- •4 ss : choose to draw table or graph
- •5 pd : complete proof using disc ≥ 0

•1	discriminant = ...	*ss*
•2	disc $= (-5k)^2 - 4(1-2k)(-2k)$	*ic*
•3	$9k^2 + 8k$	*pd*
•4	e.g. draw a table, graph complete the square	*ss*
•5	complete proof and conclusion relating to disc. ≥ 0	*pd*

Notes

1. The evidence for •4 will be
   - $0, -\frac{8}{9}$ and a graph
   - $0, -\frac{8}{9}$ and a table
   - $0, -\frac{8}{9}$ and a completing the square
2. Treat $(5k)^2 - 4(1-2k)(-2k)$ as bad form.
3. Treat $-5k^2 - 4(1-2k)(-2k)$ as bad form.
   $25k^2 + 8k(1-2k)$
6. You will have to penalise $a = \ldots, b = 5k,\ c = \ldots$

$25k^2 + 8k(1-2k)$ ✓ •1, •2

$9k^2 + 8k$ ✓ •3

$y = 9k^2 + 8k$

k	−2	−1	0	1
$9k^2+8k$	4	1	0	17

✓ •4

$9k^2 + 8k \geq 0$

$k(9k+8) \geq 0$

$k \geq 0$ *or* $k \leq -\frac{8}{9}$

There are no integers between 0 and $-\frac{8}{9}$

Hence all roots are real for integer k ✓ •5

5 marks

$\underline{(-5k)^2 - 4(-2k)(-2k)}$ ✓ •1 ✗

$= 25k^2 - 16k^2$

$= 9k^2$ ✗ •2

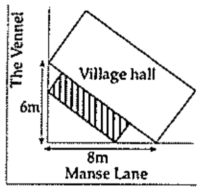
which is not negative anywhere since k^2 is +ve and multiplied by a +ve number. ✗ eased

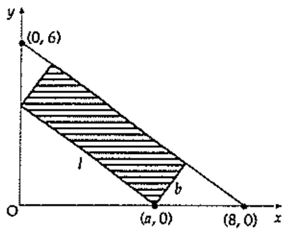
So all the roots are real whatever the value of k. ✗ •3

3 marks

	Give 1 mark for each •	Illustrations of evidence for awarding each •

10 The shaded rectangle on this map represents the planned extension to the village hall.
It is hoped to provide the largest possible area for the extension.
The coordinate diagram represents the right angled triangle of ground behind the hall.
The extension has length l metres and breadth b metres, as shown. One corner of the extension is at the point (a, 0).





(*a*) (i) Show that $l = \frac{5}{4}a$.

(ii) Express b in terms of a and hence deduce that the area, A m^2, of the extension is given by $A = \frac{3}{4}a(8-a)$. **3**

(*b*) Find the value of a which produces the largest area of the extension. **4**

10	**1.3.15**	**CN AB 02/11**	
a	**ans : proof**	**3 marks**	
b	**ans : $a = 4$**	**4 marks**	

•1 ss : select strategy and carry through
•2 ss : select strategy and carry through
•3 ic : complete proof
•4 ss : know to set derivative to zero
•5 pd : differentiate
•6 pd : solve equation
•7 ic : justify maximum eg nature table

•1	proof of $l = \frac{5}{4}a$	*ss*
•2	$b = \frac{3}{5}(8-a)$	*ss*
•3	complete proof leading to $A = \ldots$	*ic*
•4	$\frac{dA}{da} = \ldots 0$	*ss*
•5	$6 - \frac{3}{2}a$	*pd*
•6	$a = 4$	*pd*
•7	*e.g.*nature table, comp the square	*ic*

Notes

1 For •7 the minimum which is acceptable:

a or *x*	3	4	5
'$\frac{dy}{dx}$' or f' or gradient or m	+*ve*	0	−*ve*
		maximum	

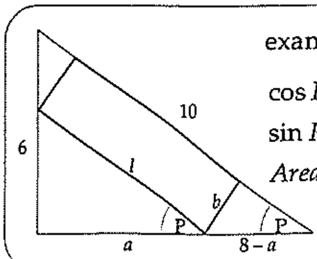
Clearly something like is preferable:

a	4^-	4	4^+
$\frac{dA}{da}$	+*ve*	0	−*ve*
	⋰	…	⋱
		maximum	

Notes

2 Trial & error/improvement earns NO marks

Minimum requirements
(i) from diagram with hyp.=10
$L = \frac{10}{8}a \Rightarrow L = \frac{5}{4}a$
(ii) accept working back from A for •2 but do not accept going forward again for •3.



example of a proof

$\cos P = \frac{8}{10} = \frac{a}{l} \Rightarrow l = \frac{10}{8}a = \frac{5}{4}a$

$\sin P = \frac{6}{10} = \frac{b}{8-a} \Rightarrow b = \frac{6}{10}(8-a)$

$Area = lb = \frac{5}{4}a \times \frac{6}{10}(8-a) = \frac{3}{4}a(8-a)$

Completing the sq 'ending'

$A = -\frac{3}{4}\left(a^2 - 8a\right)$

$= -\frac{3}{4}\left((a-4)^2 - 16\right)$

$= -\frac{3}{4}(a-4)^2 + 12$

max of 12 ✓ •6

when $a = 4$ ✓ •7

Give 1 mark for each • Illustrations of evidence for awarding each •

for the Mathematics with Statistics paper Replacing Maths qu 2, (6 &7), & 8.

2 A TV football commentator claims that "you can't gain points without scoring goals".

A football coach disagrees with this statement as he believes in solid defensive play. He is convinced there is no relationship between the number of goals scored by a team and the number of points they gain.

To test the football coach's claim, a random sample of 8 teams was selected from a football league. The number of points gained (y) was plotted against the number of goals scored (x) and the result is shown on the scattergraph.

Given that $\Sigma x = 343$, $\Sigma y = 260$, $\Sigma x^2 = 17863$, $\Sigma y^2 = 10686$ and $\Sigma xy = 13736$, calculate the product moment correlation coefficient and comment on the football coach's claim that there is no relationship between the number of goals scored and the number of points gained. 5

7 A restaurant caters for both vegetarian and non-vegetarian customers. It is found that the probability of a customer ordering a vegetarian meal is $\frac{2}{5}$.

All meals are classified only as vegetarian or non-vegetarian. Assuming that orders for meals are independent, calculate the probability that, on a particular day,

(*a*) the first three meals ordered are vegetarian. 2

(*b*) that at least one vegetarian meal is ordered in the first five orders. 3

8 (*a*) Show that the expected value of the score when a fair die is rolled is 3·5. 2

(*b*) A computer is programmed to simulate the scores when a six-sided die is rolled.
It produces results such that one of the scores occurs 25% more often than any other score.

(i) Find the probability that the computer selects this score. 3

(ii) The expected value of the score on the simulated die is 3·44 . Find which score occurs 25% more often than any other. 3

	Give 1 mark for each •	Illustrations of evidence for awarding each •
3	**4.4.4** **C C 02/S18** **ans : r = 0.9743, strong +ve correlation** **5 marks** •1 pd : process S_{xy} •2 pd : process S_{xx} •3 pd : process S_{yy} •4 pd : process corr. coefficient •5 ic : comment on value of corr. coeff	•1 $S_{xy} = 2588.5$ *pd* •2 $S_{xx} = 3156.875$ *pd* •3 $S_{yy} = 2236$ *pd* •4 $r = 0.9743$ *pd* •5 strong + ve correlation *or* coach is wrong *or* relationship exists between ...... *ic* Notes 1 Penalise once for premature rounding. 2 Wrong answers like $r = 0.2$ suggest no relationship
6/7 a b	**4.2.10** **C C/A 02/S8** **ans : $\frac{8}{125}$** **2 marks** **ans : $\frac{2882}{3125}$** **3 marks** •1 ss : know how to find P(indep. events) •2 pd : process probability •3 ss : know how to deal with 'at least one' •4 ss : know to deal with 'five orders' •5 pd : process result	•1 $P(VVV) = (P(V))^3$ *ss* •2 $\frac{8}{125}$ (0.064) *pd* •3 $P(N) = \frac{3}{5}$ *ss* •4 $1-(P(N))^5$ *ss* •5 $\frac{2882}{3125}$ (0.92) *pd* Notes 1 $\frac{2}{5}.\left(\frac{3}{5}\right)^4 = \frac{162}{3125}$ may be awarded 1 mark.
8 a b	**4.2.11, 12** **C C/AB 02/S6** **ans : proof** **2 marks** **ans : (i) 0·2** **3 marks** **(ii) 2** **3 marks** •1 ss : table of scores/probabilities •2 ic : complete proof •3 ss : know that $\Sigma p_i = 1$ •4 pd : process •5 pd : process •6 ss : set up equation or start trial & error •7 pd : process •8 pd : process alternative for (bii) fall in E(X) is 0.06 P for all other numbers falls by $\frac{1}{6} - 0.16$ P(a) then rises by $0.2 - 0.16$ $(1+2+..+6)\times\left(\frac{1}{6}-0.16\right) - 0.04\text{a} = 0.06$ $21\times 0.04 - 0.24\text{a} = 0.36$ $\text{a} = 2$	•1 x: 1 2 3 4 5 6; $P(X=x)$: $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ $\frac{1}{6}$ *ss* •2 $E(X) = \frac{1}{6}(1+2+3+4+5+6)$ *ic* •3 $5\times p_{fair} + 1\times\left(p_{fair} + \frac{1}{4}p_{fair}\right) = 1$ *ss* •4 P(fair number) = 0.16 *pd* •5 P(loaded number) = 0.2 *pd* let a be loaded number •6 $a\times 0.2 + (1+..+6 \ \ less \ \ a)\times 0.16 = 3.44$ *ss* •7 $0.2a + (21-a)\times 0.16 = 3.44$ *pd* $0.04a + 3.36 = 3.44$ •8 $a = 2$ *pd* Notes 1 For (a), $\frac{1+2+3+4+5+6}{6} = 3.5$ earns only 1 mark