X100/301

NATIONAL
QUALIFICATIONS
2003

WEDNESDAY, 21 MAY
9.00 AM – 10.10 AM

MATHEMATICS
HIGHER
Units 1, 2 and 3
Paper 1
(Non-calculator)

Read Carefully

1 **Calculators may <u>NOT</u> be used in this paper.**

2 Full credit will be given only where the solution contains appropriate working.

3 Answers obtained by readings from scale drawings will not receive any credit.

SCOTTISH
QUALIFICATIONS
AUTHORITY

ALL questions should be attempted.

Marks

1. Find the equation of the line which passes through the point (–1, 3) and is perpendicular to the line with equation $4x + y - 1 = 0$. **3**

2. (*a*) Write $f(x) = x^2 + 6x + 11$ in the form $(x + a)^2 + b$. **2**

   (*b*) Hence or otherwise sketch the graph of $y = f(x)$. **2**

3. Vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ are defined by $\boldsymbol{u} = 3\boldsymbol{i} + 2\boldsymbol{j}$ and $\boldsymbol{v} = 2\boldsymbol{i} - 3\boldsymbol{j} + 4\boldsymbol{k}$.
   Determine whether or not $\boldsymbol{u}$ and $\boldsymbol{v}$ are perpendicular to each other. **2**

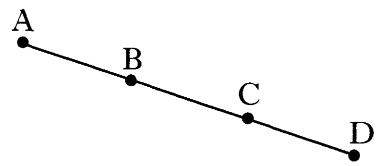
4. A recurrence relation is defined by $u_{n+1} = pu_n + q$, where $-1 < p < 1$ and $u_0 = 12$.

   (*a*) If $u_1 = 15$ and $u_2 = 16$, find the values of p and q. **2**

   (*b*) Find the limit of this recurrence relation as $n \to \infty$. **2**

5. Given that $f(x) = \sqrt{x} + \frac{2}{x^2}$, find $f'(4)$. **5**

6. A and B are the points (–1, –3, 2) and (2, –1, 1) respectively.
   B and C are the points of trisection of AD, that is AB = BC = CD.
   Find the coordinates of D. **3**



7. Show that the line with equation $y = 2x + 1$ does not intersect the parabola with equation $y = x^2 + 3x + 4$. **5**

8. Find $\displaystyle\int_0^1 \frac{dx}{(3x+1)^{\frac{1}{2}}}$. **4**

9. Functions $f(x) = \frac{1}{x-4}$ and $g(x) = 2x + 3$ are defined on suitable domains.

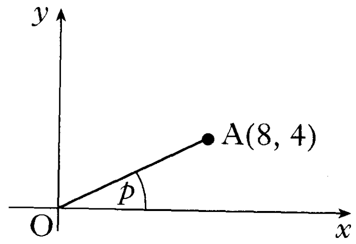
   (*a*) Find an expression for $h(x)$ where $h(x) = f(g(x))$. **2**

   (*b*) Write down any restriction on the domain of h. **1**

[Turn over for Questions 10 to 12 on *Page four*

Marks

10. A is the point (8, 4). The line OA is inclined at an angle p radians to the x-axis.

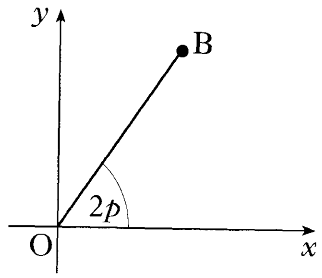


(*a*) Find the exact values of:

(i) $\sin(2p)$;

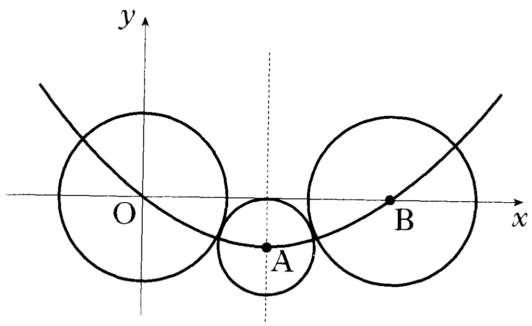
(ii) $\cos(2p)$. **5**

The line OB is inclined at an angle $2p$ radians to the x-axis.



(*b*) Write down the exact value of the gradient of OB. **1**

11.
- O, A and B are the centres of the three circles shown in the diagram below.
- The two outer circles are congruent and each touches the smallest circle.
- Circle centre A has equation $(x - 12)^2 + (y + 5)^2 = 25$.
- The three centres lie on a parabola whose axis of symmetry is shown by the broken line through A.



(*a*) (i) State the coordinates of A and find the length of the line OA. **2**

(ii) Hence find the equation of the circle with centre B. **3**

(*b*) The equation of the parabola can be written in the form $y = px(x + q)$. Find the values of p and q. **2**

12. Simplify $3\log_e(2e) - 2\log_e(3e)$ expressing your answer in the form $A + \log_e B - \log_e C$ where A, B and C are whole numbers. **4**

[*END OF QUESTION PAPER*]

X100/303

NATIONAL QUALIFICATIONS 2003

WEDNESDAY, 21 MAY
10.30 AM – 12.00 NOON

MATHEMATICS
HIGHER
Units 1, 2 and 3
Paper 2

Read Carefully

1 **Calculators may be used in this paper.**
2 Full credit will be given only where the solution contains appropriate working.
3 Answers obtained by readings from scale drawings will not receive any credit.

LIB X100/303 6/3/26570

ALL questions should be attempted.

Marks

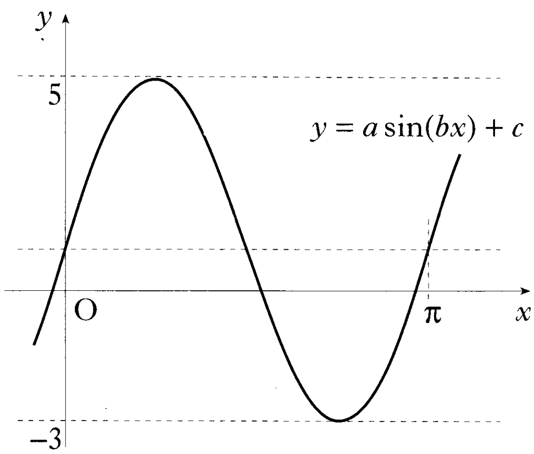
1. $f(x) = 6x^3 - 5x^2 - 17x + 6$.

   (*a*) Show that $(x - 2)$ is a factor of $f(x)$.

   (*b*) Express $f(x)$ in its fully factorised form. **4**

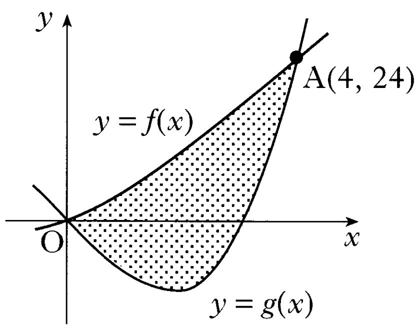
2. The diagram shows a sketch of part of the graph of a trigonometric function whose equation is of the form $y = a\sin(bx) + c$.

   Determine the values of a, b and c. **3**



3. The incomplete graphs of $f(x) = x^2 + 2x$ and $g(x) = x^3 - x^2 - 6x$ are shown in the diagram. The graphs intersect at A(4, 24) and the origin.

   Find the shaded area enclosed between the curves. **5**



4. (*a*) Find the equation of the tangent to the curve with equation $y = x^3 + 2x^2 - 3x + 2$ at the point where $x = 1$. **5**

   (*b*) Show that this line is also a tangent to the circle with equation $x^2 + y^2 - 12x - 10y + 44 = 0$ and state the coordinates of the point of contact. **6**

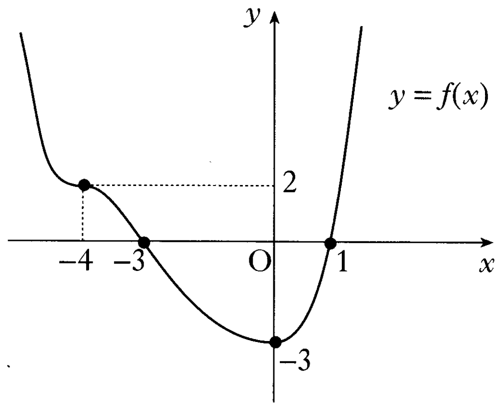
[Turn over

5. The diagram shows the graph of a function f.

f has a minimum turning point at $(0, -3)$ and a point of inflexion at $(-4, 2)$.

(*a*) Sketch the graph of $y = f(-x)$. **2**

(*b*) On the same diagram, sketch the graph of $y = 2f(-x)$. **2**

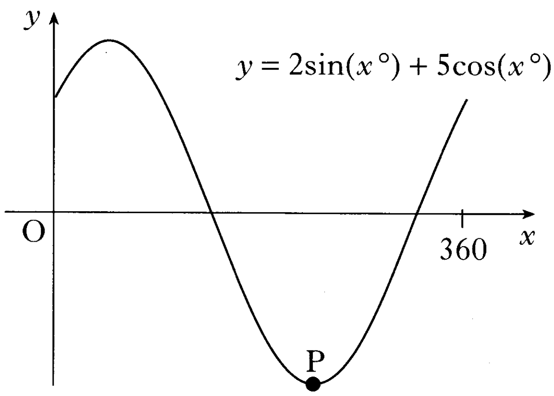


6. If $f(x) = \cos(2x) - 3\sin(4x)$, find the exact value of $f'\left(\frac{\pi}{6}\right)$. **4**

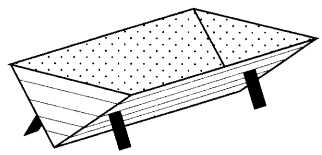
7. Part of the graph of $y = 2\sin(x°) + 5\cos(x°)$ is shown in the diagram.

(*a*) Express $y = 2\sin(x°) + 5\cos(x°)$ in the form $k\sin(x° + a°)$ where $k > 0$ and $0 \le a < 360$. **4**

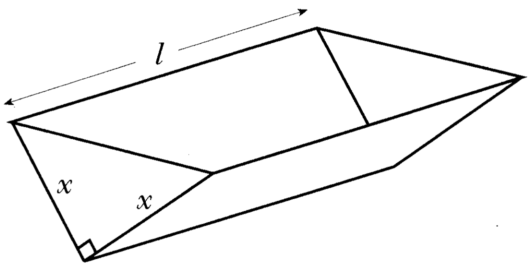
(*b*) Find the coordinates of the minimum turning point P. **3**



8. An open water tank, in the shape of a triangular prism, has a capacity of 108 litres. The tank is to be lined on the inside in order to make it watertight.



The triangular cross-section of the tank is right-angled and isosceles, with equal sides of length x cm. The tank has a length of l cm.

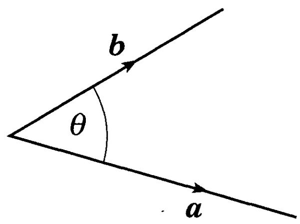


(*a*) Show that the surface area to be lined, A cm^2, is given by $A(x) = x^2 + \dfrac{432000}{x}$. **3**

(*b*) Find the value of x which minimises this surface area. **5**

Marks

9. The diagram shows vectors $\boldsymbol{a}$ and $\boldsymbol{b}$.
If $|\boldsymbol{a}| = 5$, $|\boldsymbol{b}| = 4$ and $\boldsymbol{a}.(\boldsymbol{a} + \boldsymbol{b}) = 36$, find the size of the acute angle θ between $\boldsymbol{a}$ and $\boldsymbol{b}$. **4**



10. Solve the equation $3\cos(2x) + 10\cos(x) - 1 = 0$ for $0 \le x \le \pi$, correct to 2 decimal places. **5**

11. (*a*) (i) Sketch the graph of $y = a^x + 1$, $a > 2$.

(ii) On the same diagram, sketch the graph of $y = a^{x+1}$, $a > 2$. **2**

(*b*) Prove that the graphs intersect at a point where the x-coordinate is $\log_a\left(\frac{1}{a-1}\right)$. **3**

[*END OF QUESTION PAPER*]

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QUALIFICATIONS
AUTHORITY

2003 Mathematics

Higher

Finalised Marking Instructions

NB **In and after the 2004 diet of examinations, the total number of marks for the Higher Mathematics examination will increase from 110 to 130. There will be <u>NO</u> other changes to the format of the examination.**

To provide guidance to Centres on how the 20 additional marks will be allocated, additional pages have been added to the following 2003 Marking Instructions to show how an additional 20 marks could have been allocated to the 2003 examination.

Notes to the marking scheme for Higher Mathematics 2003

1. Illustrations where additional marks could be added to bring the overall total up to 130 are shown as follows:

   Paper 1 extra marks are shown on pages 21-22 of the paper 1 m/s.
   Paper 2 extra marks are shown on pages 21-22 of the paper 2 m/s.

2. Legend for the coding at the beginning of each marking scheme:

1	**2.1.1, 2.1.3**	**CN**	**C**	**03/101**
question	syllabus code(s)	calculator neutral	level	catalogue no.
		NC non-calculator		
		C calculator required		

1. Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than marks deducted for what is wrong.
2. Award one mark for each 'bullet' point. Each error should be underlined in RED at the point in the working where it first occurs, and not at any subsequent stage of the working.
3. The working subsequent to an error must be followed through by the marker with possible full marks for the subsequent working, provided that the difficulty involved is approximately similar. Where, subsequent to an error, the working is eased, a deduction(s) of mark(s) should be made.
   This may happen where a question is divided into parts. In fact, failure to even answer an earlier section does not preclude a candidate from assuming the result of that section and obtaining full marks for a later section.

4. Correct working should be ticked (✓).This is essential for later stages of the SQA procedures. Where working subsequent to an error(s) is correct and scores marks, it should be marked with a crossed tick (✗). In appropriate cases attention may be directed to work which is not quite correct (e.g. bad form) but which has not been penalised, by underlining with a dotted or wavy line.
   Work which is correct but inadequate to score any marks should be corrected with a double cross tick (✗).

5. • The total mark for each section of a question should be entered in red in the **outer** right hand margin, opposite the end of the working concerned.
   • Only the mark should be written, **not** a fraction of the possible marks.
   • These marks should correspond to those on the question paper and these instructions.

6. It is of great importance that the utmost care should be exercised in adding up the marks. Where appropriate, all summations for totals and grand totals must be carefully checked.
   Where a candidate has scored zero marks for any question attempted, "0" should be shown against the answer.

7. As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Accept answers arrived at by inspection or mentally where it is possible for the answer so to have been obtained. Situations where you may accept such working will be indicated in the marking instructions.

cont/

8. Do not penalise:
   - working subsequent to a correct answer
   - omission of units
   - bad form
   - legitimate variations in numerical answers
   - correct working in the "wrong" part of a question

9. No piece of work should be scored through - even where a fundamental misunderstanding is apparent early in the answer. Reference should always be made to the marking scheme - answers which are widely off-beam are unlikely to include anything of relevance but in the vast majority of cases candidates still have the opportunity of gaining the odd mark or two provided it satisfies the criteria for the mark(s).

10. If in doubt between two marks, give an intermediate mark, but without fractions. When in doubt between consecutive numbers, give the higher mark.

11. In cases of difficulty covered neither in detail nor in principle in the Instructions, attention may be directed to the assessment of particular answers by making a referal to the P.A. Please see the general instructions for P.A. referrals.

12. No marks should be deducted at this stage for careless or badly arranged work. In cases where the writing or arrangement is very bad, a note may be made on the upper left-hand corner of the front cover of the script.

13 **Do not write any comments on the scripts**. A summary of acceptable notation is given on page 4.

Summary

Throughout the examination procedures many scripts are remarked. It is essential that markers follow common procedures:

1 **Tick** correct working.
2 Put a mark in the **right-hand margin to match the marks allocations on the question paper.**
3 Do **not** write marks as fractions.
4 Put each mark **at the end** of the candidate's response to the question.
5 **Follow through** errors to see if candidates can score marks subsequent to the error.
6 Do **not** write any comments on the scripts.

Higher Mathematics : A Guide to Standard Signs and Abbreviations

Remember - No comments on the scripts. Please use the following and nothing else.

Signs

✓ The tick. You are not expected to tick every line but of course you must check through the whole of a response.

___ ✗ The cross and underline. Underline an error and place a cross at the end of the line.

✓✗ The tick-cross. Use this to show correct work where you are **following through** subsequent to an error.

✗✓✗ The double cross-tick. Use this to show correct work but which is inadequate to score any marks.

∧ The roof. Use this to show something is missing such as a crucial step in a proof or a 'condition' etc.

~ The tilde. Use this to indicate a minor transgression which is not being penalised (such as bad form).

E Eased. Where working is found correct whilst following through subsequent to an error, the working has been eased sufficiently for a mark not to be awarded.

BOD Benefit of Doubt. Use this where you have to decide between two consecutive marks and award the higher.

Marks being allotted e.g. (•) would not normally be shown on scripts

		margins
$\frac{dy}{dx}=4x-7$	✓ •	
$4x-7=\underline{0}$	✗	
$x=\frac{7}{4}$		
$y=3\frac{7}{8}$	✓✗ •	2
$C=\underline{(1,-1)}$	✗	
$m=\frac{3-(-1)}{4-1}$		
$m_{rad}=\frac{4}{3}$	✓✗ • follow through	
$m_{tgt}=\frac{-1}{\frac{4}{3}}$		
$m_{tgt}=-\frac{3}{4}$	✓✗ •	
$y-3=-\frac{3}{4}(x-2)$	✓✗ •	3
$x^2-3x=28$	✓ •	
$x=7$ ∧	✗✓✗	1
$\sin(x)=0.75=inv\sin(0.75)=48.6°$	✓ •	1
$\log_3(x-\underline{2)=1}$	✗	
$(x-2)=3^1$	✓✗ •	
$x-2=3$		
$x=5$	✗✓✗ E	1

All of these are to help us be more consistent and accurate.

It goes without saying that however accurate you are in marking, it is to no avail unless you have added the marks up correctly. Please double check totals!!

	Give 1 mark for each •	Illustrations for awarding each •
1	Find the equation of the line which passes through the point (–1, 3) and is perpendicular to the line with equation $4x+y-1=0$.	3
1	**1.1.9, 1.1.7 CN C 03/1** **ans:** $x-4y+13=0$ **3 marks** •1 ic : interpret gradient from linear equ. •2 ic : find perp. gradient •3 ic : state equation of line	•1 $m=-4$ **stated or implied by •2** •2 $m_{perp}=\frac{1}{4}$ •3 $y-3=\frac{1}{4}(x-(-1))$

Notes
1 •3 is only available following an attempt to find the perpendicular gradient.
2 Wrong answer with no working gains no marks.

Example 1

$m=4$ •1 ✗
$m_{perp}=-\frac{1}{4}$ •2 ✓✗ f.t.
$y-3=-\frac{1}{4}(x--1)$ •3 ✓✗ f.t.
2 marks given

Example 2

$m=-4$ •1 ✓
•2 ✗
$y-3=-4(x--1)$ •3 ✗ no perp. grad.
1 mark given

Example 3

$y=-4x-1$ ignore the error (of –1)
$m=-4$ •1 ✓
etc

Example 4

$y=-4x+1$ •1 ✗
$m=4$ •2 ✗
$y-3=4(x--1)$ •3 ✓✗ f.t. BOD
1 mark awarded

Example 5

$m=4$
$y-3=4(x--1)$ 0 marks given

Example 6

$y=-4x+1$
$\frac{dy}{dx}=-4$ is acceptable for •1
$m=-4$

Example 7

$y-3=\frac{1}{4}(x--1)$ may be awarded 2 marks,
or 1 mark being lost through
$x-4y=-13$ lack of communication

Example 8

$m=-4$ •1 ✓
$m=\frac{1}{4}$ •2 ✓
$y=\frac{1}{4}x+c$
$3=\frac{1}{4}\times(-1)+c$
$c=3+\frac{1}{4}$ •3 ✓
3 marks given

Example 9

$m=\frac{1}{4}$
and nothing else 0 marks given

Example 10

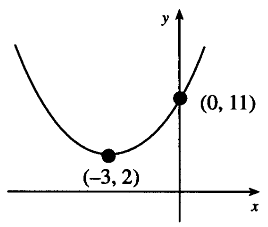
$m=-\frac{1}{4}$ •1 ✗
$y-3=-\frac{1}{4}(x--1)$ •2 ✗
•3 ✓✗ **b.o.d for perp gr.**
1 mark awarded

	Give 1 mark for each •	Illustrations for awarding each •

2 (*a*) Write $f(x) = x^2 + 6x + 11$ in the form $(x+a)^2 + b$. **2**

(*b*) Hence or otherwise sketch the graph of $y = f(x)$. **2**

2	**1.2.8 +** **CN** **C** **03/new** **(a) ans:** $(x+3)^2+2$ **2 marks** **(b) ans: sketch** **2 marks**	
	•1 ic : start to complete square •2 pd : finish completing the square	•1 $(x+3)^2$ •2 $+2$
	•3 ic : sketch •4 ic : sketch	•3 *any two from* * *U – parabola* * *minimum at* $(-3,2)$ * ***intercept on y – axis at*** $(0,11)$ •4 ***all three of the above facts***



Notes
1 2 marks may be awarded for stating a = 3 and b = 2
2 accept the information about (–3, 2) and (0, 11) in the body of the working provided the sketch shows a parabola in a consistent position.

(a)
$(x+3)^2+2$ •1 ✓ •2 ✓
(b)
For a U parabola through (0, 11) and (3, 2) •3 ✓ •4 ✗
3 mark given

(a)
$(x-3)^2+2$ •1 ✗ •2 ✓
(b)
For a U parabola through (0, 11) and (3, 2) •3 ✓✗ •4 ✓✗
3 marks given

(a)
$$(x+a)^2 + b = x^2 + 2ax + a^2 + b$$
$$2a = 6 \;......\; a = 3 \qquad \bullet 1 ✓$$
$$a^2 + b = 11 \;......\; b = 2 \qquad \bullet 2 ✓$$

(a)
$(x+3)^2+20$ •1 ✓ •2 ✗
(b)
For a U parabola through (0, 11) and the minimum in the right position but marked as (-3, 20) •3 ✓✗ •4 ✗
2 marks given

	Give 1 mark for each •	Illustrations for awarding each •

3 Vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ are defined by $\boldsymbol{u} = 3\boldsymbol{i} + 2\boldsymbol{j}$ and $\boldsymbol{v} = 2\boldsymbol{i} - 3\boldsymbol{j} + 4\boldsymbol{k}$.
Determine whether or not $\boldsymbol{u}$ and $\boldsymbol{v}$ are perpendicular to each other. **2**

3	**3.1.1/ .9/ .10 CN C 03/53** **ans : vectors are perpendicular** **2 marks** •1 ss : use scalar product and get zero •2 pd : process	•1 for perpendicularity $\boldsymbol{u}.\boldsymbol{v} = 0$ •2 $\begin{pmatrix}3\\2\\0\end{pmatrix}.\begin{pmatrix}2\\-3\\4\end{pmatrix} = 6 - 6 + 0 = 0$

Example 1

•1 $\begin{pmatrix}3\\2\\0\end{pmatrix}.\begin{pmatrix}2\\3\\4\end{pmatrix}$ •1 ✗

•2 $6 + 6 + 0 = 12$
so $\boldsymbol{u}$ and $\boldsymbol{v}$ not perp. •2 ✓✗ f.t.

1 mark given

Example 2

$\cos(\theta) = a_1b_1 + a_2b_2 + a_3b_3$ •1 ✗
$= 6 - 6 + 0$
$= 0$
$\theta = 90$ •2 ✓✗ f.t.

1 mark given

Example 3

$\cos(\theta) = \dfrac{6 - 6 + 0}{|u|\,|v|}$ •2 ✓
$= 0$
$\theta = 90$ •1 ✓

2 marks given

Example 4

$(3i + 2j).(2i - 3j + 4k)$
$= 6i.i - 9i.j + 12i.k + 4j.i - 6j.j + 8j.k$ •2 ✓
$= 6 - 6$
$= 0$
so u, v *perpendicular* •1 ✓

2 marks given

Notes
1 Accept correct use of the cosine rule
2 Treat $\begin{pmatrix}3i\\2j\\0k\end{pmatrix}$ as bad form.

Cosine rule

•1 $\cos(A) = \dfrac{b^2 + c^2 - a^2}{2bc}$,
and $a = \begin{pmatrix}1\\5\\-4\end{pmatrix}$ *or* $\begin{pmatrix}-1\\-5\\4\end{pmatrix}$

•2 13, 29 *and* 42 and complete

Converse of Pythagoras

•1 *length of third side* $= \sqrt{42}$
•2 $(\sqrt{13})^2 + (\sqrt{29})^2 = 13 + 29$
$= 42$
$= (\sqrt{42})^2$

	Give 1 mark for each •	Illustrations for awarding each •

4 A recurrence relation is defined by $u_{n+1} = pu_n + q$ where $-1 < p < 1$ and $u_0 = 12$.

(*a*) If $u_1 = 15$ and $u_2 = 16$, find the values of p and q. **2**

(*b*) Find the limit of this recurrence relation as $n \to \infty$. **2**

4	**1.4.3, 1.4.4** **CN** **CB 03/90** **ans: (a)** $p=\frac{1}{3}, q=11$ **2 marks** **(b)** $16\frac{1}{2}$ **2 marks** •1 ss : e.g. form two equations in p and q •2 pd : process •3 ss : algebraic strategy for limit •4 pd : process limit	•1 $15 = 12p + q, 16 = 15p + q$ •2 $p = \frac{1}{3}, q = 11$ •3 *e.g.* $L = \frac{1}{3}L + 11$ •4 $L = 16\frac{1}{2}$

Example 1

$12 = 16p + q$	•1 ✗
$15 = 15p + q$	
$p = -3, q = 60$	•2 ✓✗ f.t.
no limit exists since p outside range -1 to 1	•3 ✓✗ f.t.
	•4 **not available**
	2 marks given

Example 2

$12 = 16p + q$	•1 ✗
$15 = 15p + q$	
$p = -3, q = 60$	•2 ✓✗ f.t.
$L = \frac{60}{1-(-3)}$	•3 ✓✗
$L = 15$	•4 ✓✗
	1 mark given

Notes

1 for •1
either two equations explicitly stated
or
a trial and improvement approach **checking** in particular that u_1 does in fact equal 15 and u_2 does in fact equal 16

2 for (a) correct answers with no working may only earn •2 (one mark being lost through lack of communication)

3 for (a) trial and improvement leading to answers other than the correct ones earn no marks

4 for any rounding eg p =0.3 or 0.33 **instead** of $p = \frac{1}{3}$ in (a) or (b) the candidate loses •2 or •4
BUT candidates may not lose both •2 and •4

6 other acceptable strategies for the limit at •3 are

- $L = \frac{q}{1-p}$
- "lost part" = "add on" i.e. $\frac{2}{3}L = 11$

7 if p has been incorrectly valued ≥ 1 or ≤ −1, •3 may still be awarded for a statement that the limit does not exist but •4 is not available.

8 candidates who choose values for $p$and q *ex nihilo* may still earn •3 and •4

9 •4 is lost if answers are left like $\frac{11}{\frac{2}{3}}$ but uncancelled fractions e,g $\frac{66}{4}$, are acceptable

	Give 1 mark for each •	Illustrations for awarding each •
5	Given that $f(x) = \sqrt{x} + \frac{2}{x^2}$, find $f'(4)$.	5
5	**1.3.2, 1.3.4 CN C 03/19** **ans : $\frac{3}{16}$ 5 marks**	
	•1 pd : express in standard form	•1 $x^{\frac{1}{2}}$ stated or implied by •3
	•2 pd : express in standard form	•2 $2x^{-2}$ stated or implied by •4
	•3 pd : differentiate fractional index	•3 $\frac{1}{2}x^{-\frac{1}{2}}$
	•4 pd : differentiate negative index	•4 $-4x^{-3}$
	•5 pd : evaluation	•5 $\frac{3}{16}$

Notes

1 if incorrectly expressed in standard form, follow throughs must match the mark descriptors.

2 •5 can only be awarded on a follow through provided the evaluation involves a fractional index and a negative index.

3 for •5 accept $\frac{12}{64}$.

4 no marks can be gained for finding $f(4)$

Example 1

$f(x) = x^{\frac{1}{2}} + 2x^{-\frac{1}{2}}$ •1 ✓

•2 ✗

$f'(x) = \frac{1}{2}x^{-\frac{1}{2}} - x^{-\frac{3}{2}}$ •3 ✓✗ f.t.

•4 ✓✗ f.t.

$f'(4) = \frac{1}{4} - \frac{1}{8} = \frac{1}{8}$ •5 ✓✗ f.t.

4 marks given

Example 2

$f(x) = x^{-2} + 2x^{-2}$ •1 ✗

•2 ✓

$f'(x) = -x^{-3} - 4x^{-3}$ •3 ✓✗ no fractional index

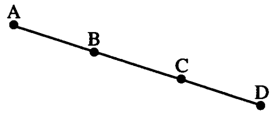
•4 ✓

$f'(4) = -\frac{1}{64} - \frac{1}{16} = -\frac{5}{64}$ •5 ✓✗ f.t. eased

2 marks given

	Give 1 mark for each •	Illustrations for awarding each •

6 A and B are the points (–1, –3, 2) and (2, –1, 1) respectively.
B and C are the points of trisection of AD, that is AB = BC = CD.
Find the coordinates of D. **3**



6	**3.1.6, 3.1.2** **CN** **C** **03/48** **ans : (8, 3, –1)** **3 marks** •1 ss : e.g. use a vector approach •2 ic : interpret trisection •3 pd : process coordinates	•1 $\vec{AB}=\begin{pmatrix}3\\2\\-1\end{pmatrix}$ may be stated or implied by •2 •2 $\vec{AD}=3\vec{AB}=\begin{pmatrix}9\\6\\-3\end{pmatrix}$ may be stated or implied by •3 but not as well as the above! •3 $D=(8,3,-1)$

Alternative 1

•1 $\vec{AB}=\begin{pmatrix}3\\2\\-1\end{pmatrix}$
•2 $C=(5,1,0)$
•3 $D=(8,3,-1)$

Alternative 2

•1 $\vec{AB}=\begin{pmatrix}3\\2\\-1\end{pmatrix}$
•2 $\vec{BD}=2\vec{AB}=\begin{pmatrix}6\\4\\-2\end{pmatrix}$
•3 $D=(8,3,-1)$

Alternative 3
one of many forms of the section formula

•1 $b=\frac{2}{3}a+\frac{1}{3}d$
•2 *substitution*
•3 $d=\begin{pmatrix}8\\3\\-1\end{pmatrix}$

Notes

1 Treat as bad form expressions such as

$$D=\begin{pmatrix}8\\3\\-1\end{pmatrix} \text{ or } \vec{BD}=(6,4,-2)$$

2 D= (8, 3, –1) with no working may be awarded 2 marks, 1 mark being lost for poor communication

3 A wrong answer with no working earns no marks

4 If A is taken as (2, –1, 1) and B as (–1, –3, 2) then work leading to D(–7, –7, 4) may be awarded 2 marks.

Example 1

C = (5, 1, 0) •1 ✓ •2 ✓
D = (8, 3, –1) •3 ✓

3 marks given

	Give 1 mark for each •	Illustrations for awarding each •

7 Show that the line with equation $y = 2x + 1$ does not intersect the parabola with equation $y = x^2 + 3x + 4$. **5**

7	2.1.8, 2.1.6 CN B 03/27 ans : proof 5 marks	
	•[1] ss : substitute linear into quadratic	•[1] $x^2 + 3x + 4 = 2x + 1$
	•[2] pd : express in standard form	•[2] $x^2 + x + 3 = 0$ the zero explicitly stated
	•[3] ss : e.g. use discimnant	•[3] $b^2 - 4ac = 1^2 \ldots\ldots$
	•[4] pd : evaluate discriminant	•[4] $b^2 - 4ac = -11$
	•[5] ic : complete proof	•[5] $b^2 - 4ac < 0$ $\therefore$ no intersection

Alternatives for marks •3 and •4

•[3] $a = 1, b = 1, c = 3$
•[4] $b^2 - 4ac = 1 - 4 \times 1 \times 3 < 0$

•[3] $roots = \dfrac{-1 \pm \sqrt{1^2 - 4 \times 1 \times 3}}{2}$
•[4] $\dfrac{-1 \pm \sqrt{-11}}{2}$

Notes

1 Use of the "alternative" discriminant $b^2 + 4ac$: lose •3 and follow through.
All other versions lose •3, •4 and •5.

•[3] $\left(x + \frac{1}{2}\right)^2 + \frac{11}{4}$
•[4] $so\ x^2 + x + 3\ is\ \cup,\ min\ at\ \left(-\frac{1}{2}, \frac{11}{4}\right)$

•[3] $\frac{dy}{dx} = 2x + 1 = 0 \Rightarrow x = -\frac{1}{2},\ y = \frac{11}{4}$
•[4] $so\ x^2 + x + 3\ is\ \cup,\ min\ at\ \left(-\frac{1}{2}, \frac{11}{4}\right)$

•[3] $\left(x + \frac{1}{2}\right)^2 + \frac{11}{4}$
•[4] $which\ is\ \geq \frac{11}{4}$

Example 1

$x^2 + 3x + 4 = 0$
$b^2 - 4ac = 9 - 16$ no marks awarded
etc

Example 2

$x^2 + 3x + 4 \neq 2x + 1$ lose •2 for using the "not equals" sign.
etc Treat the rest as bad form

Example 3

$y = (2x + 1)^2 + 3(2x + 1) + 4 = 0$	•1 ✗
$4x^2 + 10x + 8 = 0$	•2 ✗
$b^2 - 4ac = 100 - 128 = -28$	•3 ✓✗
so no intersection	•4 ✓✗
	•5 ✓✗

3 marks given

Example 4

$x^2 + 3x + 4 = 2x + 1$	•1 ✓
$x^2 + x + 3 = 0$	•2 ✓
$b^2 - 4ac < 0$	•3 ✗
so no intersection	•4 ✗
	•5 ✓

3 marks given

Example 5

$x^2 + 3x + 4 = 2x + 1$	•1 ✓
$x^2 + x + 3 = 0$	•2 ✓
$b^2 - 4ac < 0$	•3 ✗
so no real roots	•4 ✗
	•5 ✗

2 marks given

	Give 1 mark for each •	Illustrations for awarding each •
8	Find $\int_0^1 \frac{dx}{(3x+1)^{\frac{1}{2}}}$.	4
8	**3.2.3 CN CA 03/55** **ans : $\frac{2}{3}$ 4 marks** •1 pd : express in standard form •2 pd : integrate •3 pd : integrate •4 pd : evaluate using limits	•1 $(3x+1)^{-\frac{1}{2}}$ •2 $\frac{1}{\frac{1}{2}}(3x+1)^{\frac{1}{2}}$ •3 $\ldots\times\frac{1}{3}$ •4 $\frac{2}{3}$

Notes

1 Treat $\frac{2}{3}+c$ as bad form

2 $\frac{1}{1.5}$ does not gain •4

3 •4 is only available after an attempt has been made to integrate

4 •4 is only available if the evaluation involves a fractional power.

Example 1

$$\left[\frac{\frac{1}{(3x+1)^{\frac{3}{2}}}}{\frac{3}{2}\times 3}\right]_0^1$$
$$=\frac{9}{2}\left(\frac{1}{8}-1\right)$$
$$=-\frac{63}{16}$$

•1 ✗
•2 ✗
•3 ✓✗ f.t.
•4 ✓✗ f.t.

2 marks given

Example 2

$$\left[\frac{1}{\frac{3}{2}(3x+1)^{-\frac{1}{2}}}\right]_0^1$$
$$= \ldots$$
$$=\frac{2}{3}$$

•1 ✗
•2 ✗
•3 ✗
•4 ✓✗ f.t.

1 mark given

Example 3

$$\left[-\tfrac{3}{2}(3x+1)^{\frac{3}{2}}\right]_0^1$$
$$= \ldots$$
$$=\frac{21}{16}$$

•1 ✗
•2 ✗
•3 ✗
•4 ✓✗ f.t.

1 mark given

Example 4

$$\left[2(3x+1)^{\frac{1}{2}}\right]_0^1$$
$$=2\times 4^{\frac{1}{2}}-2\times 1^{\frac{1}{1}}$$
$$=2$$

•1 ✓
•2 ✓
•3 ✗
•4 ✓✗ f.t.

3 marks given

Example 5

$$\left[2(3x+1)^{\frac{1}{2}}\right]_0^1$$
$$=\left[(6x+2)^{\frac{1}{2}}\right]_0^1$$
$$= \ldots$$
$$=\sqrt{2}$$

•1 ✓
•2 ✓
•3 ✗
•4 ✗

2 marks given

	Give 1 mark for each •	Illustrations for awarding each •

9 Functions $f(x)=\frac{1}{x-4}$ and $g(x)=2x+3$ are defined on suitable domains.

(*a*) Find an expression for $h(x)$ where $h(x)=f(g(x))$. **2**

(*b*) Write down any restriction on the domain of *h*. **1**

9	**1.2.1, 1.2.6** **CN** **CA 03/5**	
	(a) ans : $\frac{1}{2x-1}$ **2 marks**	
	(b) ans : $x\neq\frac{1}{2}$ **1 mark**	
	•1 ic : start composite function	•1 $f(2x+3)$ **stated or implied by •2**
	•2 ic : complete composite function	•2 $\frac{1}{2x+3-4}$
	•3 ic : interpret denominator	•3 $x\neq\frac{1}{2}$

Example 1

$\ldots\left(\frac{1}{x-4}\right)$	•1 ✗
$\frac{2}{x-4}+3$	•2 ✓✗ f.t.
$x\neq 4$	•3 ✓✗ f.t.
	2 marks given

Notes

1 Use example 1 if candidate finds g(f(x)) [which they may call f(g(x))!!

2 •3 is only available for working containing an algebraic fraction e.g. $\frac{a}{x+c}$ or harder.

3 for •3 accept any statement involving a $\frac{1}{2}$ e.g.

a $x\neq\frac{1}{2}$ (the actual restriction)

b $x=\frac{1}{2}$ (the value to be restricted from the domain)

c $x>\frac{1}{2}$ (part of the restricted domain)

d $x<\frac{1}{2}$ (also part of the restricted domain)

In each case the candidate has identified the value of *x* which makes the denominator zero (which was the point of (b)).

4 for (b) do not accept unsimplified forms such as $2x-1=0$.

4 for •3, treat $h\neq\frac{1}{2}$ as bad form.

	Give 1 mark for each •	Illustrations for awarding each •

10 A is the point (8, 4). The line OA is inclined at an angle p radians to the x-axis.

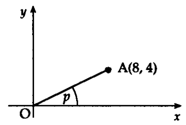
(*a*) Find the exact values of

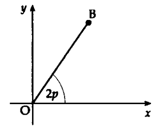
(i) $\sin(2p)$

(ii) $\cos(2p)$. **5**

The line OB is inclined at an angle $2p$ radians to the x-axis.

(*b*) Write down the exact value of the gradient of OB. **1**





	Give 1 mark for each •	Illustrations for awarding each •
10	**2.3.3, 1.1.6 NC CB 03/34**	
	(a) ans : $\frac{4}{5}$, $\frac{3}{5}$ 5 marks	
	(b) ans : $\frac{4}{3}$ 1 mark	
	•1 pd : calculate hypotenuse	•1 hypot $=\sqrt{80}$
	•2 pd : calculate sinp and cosp	•2 $\sin(p)=\frac{4}{\sqrt{80}}$ *and* $\cos(p)=\frac{8}{\sqrt{80}}$
	•3 ss : use double angle formula	•3 $\sin(2p)=2\sin(p)\cos(p)$
	•4 pd : process sin2p	•4 $\sin(2p)=\frac{4}{5}$
	•5 pd : process cos2p	•5 $\cos(2p)=\frac{3}{5}$
	•6 pd: relate gradient and tan	•6 $\frac{4}{3}$

Example 1

$\tan(p)=\frac{1}{2}$ •1 ✗

$p=30°$ •2 ✗

$\sin(p)=\frac{1}{2}$, $\cos(p)=\frac{\sqrt{3}}{2}$ •3 ✓ f.t.

$\sin(2p)=2\sin(p)\cos(p)$

$=2\times\frac{1}{2}\times\frac{\sqrt{3}}{2}$

$=\frac{\sqrt{3}}{2}$ •4 ✓ f.t.

$\cos(2p)=2\cos^2(p)-1$

$=2\left(\frac{\sqrt{3}}{2}\right)^2-1$

$=\frac{1}{2}$ •5 ✓ f.t.

$\tan(2p)=\frac{\frac{\sqrt{3}}{2}}{\frac{1}{2}}=\sqrt{3}$ •6 ✓ f.t.

3,1 marks given

Example 2

$\tan(p)=\frac{1}{2}$ •1 ✗

$p=30°$ •2 ✗

$\sin(2p)=\sin 60$ •3 ✗

$=\frac{\sqrt{3}}{2}$ •4 ✗

$\cos(2p)=\cos 60$ •5 ✗

$=\frac{1}{2}$ •6 ✗

$\tan(2p)=\tan 60=\sqrt{3}$ 0 marks given

Notes

1 accept uncancelled fractions for •4, •5 and •6.

e.g. $\frac{64}{80}$, $\frac{48}{80}$ *and* $\frac{64}{48}$ are common

2 marks 4-6 are not available to candidates who base their answers on the assumption that p = 30°, 45 etc so that sin(2p) = sin(60) etc. See examples 1 & 2.

Example 3

a wrong hypotenuse leading to •1 ✗

$hyp=\sqrt{32}$ •2 ✓

$\sin(p)=\frac{4}{\sqrt{32}}$, $\cos(p)=\frac{8}{\sqrt{32}}$ •3 ✓

$\sin(2p)=2\sin(p)\cos(p)$

$=2\times\frac{4}{\sqrt{32}}\times\frac{8}{\sqrt{32}}$ •4 ✗

$=2$

$\cos(2p)=2\cos^2(p)-1$

$=2\left(\frac{8}{\sqrt{32}}\right)^2-1$

$=\frac{3}{2}$ •5 ✓

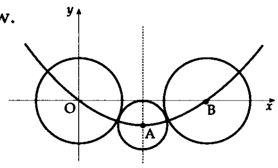
$\tan(2p)=\frac{2}{\frac{3}{2}}=\frac{4}{3}$ •6 ✓

3,1 marks given

	Give 1 mark for each •	Illustrations for awarding each •

11

- O, A and B are the centres of the three circles shown in the diagram below.
- The two outer circles are congruent and each touches the smallest circle.
- Circle centre A has equation $(x-12)^2+(y+5)^2=25$.
- The three centres lie on a parabola whose axis of symmetry is shown by the broken line through A.



(*a*) (i) State the coordinates of A and find the length of the line OA. **2**

(ii) Hence find the equation of the circle with centre B. **3**

(*b*) The equation of the parabola can be written in the form $y = px(x+q)$.
Find the values of p and q. **2**

11 **2.4.1, 2.1.10 CN CBA 03/40**

(a) ans : A(12, –5), OA = 13

$(x-24)^2+y^2=64$ **5 marks**

(b) ans : $p=\frac{5}{144}$, $q=-24$ **2 marks**

	Give 1 mark for each •	Illustrations for awarding each •
•1	ic : interpret centre	$A=(12,-5)$
•2	pd : use Pythagoras	$OA=13$ accept $\sqrt{169}$
•3	ic : interpret radius	$r_B=8$ **stated or implied by •5**
•4	ic : interpret centre	$B=(24,0)$ **stated or implied by •5**
•5	ic : state equ of circle	$(x-24)^2+y^2=64$
•6	pd : process	$p=\frac{5}{144}$
•7	pd : process	$q=-24$

Notes

1 Take care with the implications at •3 and •4. Only the correct values for *r* and B can be implied by •5. Incorrect values of *r* and/or B must be stated before the equation of the circle is given in order that •5 can be awarded as a follow-through mark.

	Give 1 mark for each •	Illustrations for awarding each •

12 Simplify $3\log_e(2e) - 2\log_e(3e)$, expressing your answer in the form $A + \log_e(B) - \log_e(C)$ where A, B and C are whole numbers. **4**

12	**3.3.6, 3.3.2** **CN** **BA 03/43** **ans : 1 + ln(8) - ln(9)** **4 marks** $\bullet^1$ pd : use log laws $\bullet^2$ pd : use log laws $\bullet^3$ pd : process $\bullet^4$ pd : use log laws	$\bullet^1$ $\ln(2e)^3 - \ln(3e)^2$ $\bullet^2$ $\ln\left(\frac{(2e)^3}{(3e)^2}\right)$ $\bullet^3$ $\ln(\frac{8e}{9})$ $\bullet^4$ $1 + \ln(8) - \ln(9)$

Alternative 1

$\bullet^1$ $3[\ln(2) + \ln(e)]$
$\bullet^2$ $-2[\ln(3) + \ln(e)]$
$\bullet^3$ $3\ln(2) + 3 - 2\ln(3) - 2$
$\bullet^4$ $1 + \ln(8) - \ln(9)$

Notes

1 $\ln 2e^3 - \ln 3e^2$ will not gain •1 unless you see an '8' and a '9' appearing in subsequent work, in which case you can treat it as bad form.

Example 1

$\ln(2e)^3 - \ln(3e)^2$	•1 ✓
$\ln\left(\frac{2e^3}{3e^2}\right)$	•2 ✗
	•3 ✓✗
$\ln(\frac{2e}{3})$	•4 ✓✗
$1 + \ln(2) - \ln(3)$	3 marks given

Example 2

$\ln(2e)^3 - \ln(3e)^2$	•1 ✓
$\ln(8e) - \ln(9e)$	•3 ✗
$\ln\left(\frac{8e}{9e}\right)$	•2 ✓✗
	•4 ✓✗ **Eased**
$\ln(8) - \ln(9)$	2 marks given

Example 3

$\ln(2e)^3 - \ln(3e)^2$	•1 ✓
$\ln\left(\frac{2e^3}{3e^2}\right)$	•2 ✓ ev. line 3
	•3 ✗
$\ln(\frac{8}{9})$	•4 ✓✗ **Eased**
$\ln(8) - \ln(9)$	2 marks given

Example 4

$\ln 2e^3 - \ln 3e^2$	•1 ✓ see line 2
$\ln\left(\frac{8e^3}{9e^2}\right)$	•2 ✓
	•3 ✓ ev in line 4
	•4 ✓✗
$\ln(8) - \ln(9) + \ln(e)$	3 marks given

Example 5

$\ln(8e) - \ln(9e)$	•1 ✗
$\ln\left(\frac{8e}{9}\right)$	•2 ✗
	•3 ✗
	•4 ✓✗
$1 + \ln(8) - \ln(9)$	1 mark given

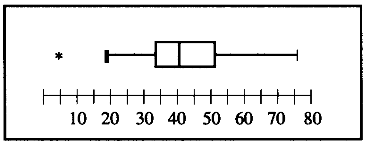
Example 6

$\ln(8e) - \ln(9e)$	•1 ✗
$\ln\left(\frac{8e}{9e}\right)$	•2 ✓✗
	•3 ✓✗ **Eased**
$\ln(8) - \ln(9)$	•4 ✓✗ **Eased**
	1 mark given

Example 7

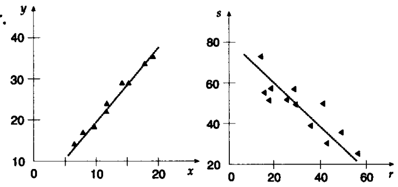
$3\ln(2) + \ln(e) - 2\ln(3) + \ln(e)$	•1 ✗
$\ln(8) + 1 - \ln(9) + 1$	•2 ✗
$2 + \ln(8) - \ln(9)$	•3 ✓✗
	•4 ✓✗
	2 marks given

	Give 1 mark for each •	Illustrations for awarding each •
1	The times taken by a group of students to complete a statistical project are given in the stem-and-leaf diagram. Identify any outliers and illustrate the data with a box-plot.	Time taken to the nearest minute 0 5 1 9 2 7 7 8 3 0 2 4 5 5 6 8 9 4 0 0 1 2 2 3 6 8 8 9 5 1 3 3 4 8 8 6 0 7 6 n = 31 5 | 3 represents 53 minutes **5**
S1	**4.1.2, 4.1.4 CN C 03/64** **ans : 5 is outlier, boxplot 5 marks** •1 pd : calculate quartiles •2 ss : know how to caculate fence •3 ic : determine upper fence/outliers •4 ic : determine lower fence/outliers •5 ic : determine mean/draw box-plot	•1 34 & 51 •2 $LF = Q_1 - \frac{3}{2}(Q_3 - Q_1)$ •3 $76\cdot5$ & no outliers •4 8.5 & 5 min. is outlier •5 41 & box – plot



	Give 1 mark for each •	Illustrations for awarding each •

2 The diagrams below show the scattergraphs of y on x and s on r.



The equation of the least squares regression line of y on x is $y = 1{\cdot}7x + 2$.

The equation of the least squares regression line of s on r is $s = 81 - 1{\cdot}05r$.

(*a*) Predict the expected value of (i) y when $x = 10$
(ii) s when $r = 20$. **2**

(*b*) Which prediction is more reliable? Give a reason for your answer. **1**

	Give 1 mark for each •	Illustrations for awarding each •
S2	**4.4.3 NC C 03/59** **(a) ans : 19, 60 2 marks** **(b) ans : y on x more reliable** •1 pd : use regression equation •2 pd : use regresion equation •3 ic : interpret scatter diagram	•1 19 •2 60 •3 y on x more reliable as diagram shows stronger relationship

	Give 1 mark for each •	Illustrations for awarding each •

3 A farmer sells eggs in boxes of 6. The discrete random variable X represents the number of brown eggs in a box.

X has the following probability distribution:

$$P(X = x) = \begin{cases} \frac{1}{3}k(7-x) & \text{for } x = 0,1,2,3,4,5 \text{ and } 6 \\ 0 & \text{otherwise} \end{cases}$$

where k is a constant.

(*a*) Find the value of k. **2**

(*b*) Find the expected value and variance of X, the number of brown eggs in a box. **3**

S3 **4.2.11, 4.2.12** **CN** **C 03/68**

(a) ans : $\frac{3}{28}$ **2 marks**

(b) ans : 2, 3 **3 marks**

Give 1 mark for each •	Illustrations for awarding each •
•[1] ss : use $\Sigma P(X) = 1$	•[1] $P(X)\ \frac{7k}{3}, \frac{6k}{3}, \frac{5k}{3}, \frac{4k}{3}, \frac{3k}{3}, \frac{2k}{3}, \frac{k}{3}$
•[2] pd : evaluate k	•[2] $\Sigma P(X) = 1 \Rightarrow k = \frac{3}{28}$
•[3] pd : calculate expected value	•[3] $E(X) = 2$
•[4] pd : calculate $E(X^2)$	•[4] $E(X^2) = 7$
•[5] pd : calculate variance	•[5] $V(X) = 3$

x	0	1	2	3	4	5	6		
$P(x)$	$\frac{7k}{3}$	$\frac{6k}{32}$	$\frac{5k}{3}$	$\frac{4k}{3}$	$\frac{3k}{3}$	$\frac{2k}{3}$	$\frac{k}{3}$		$\Sigma = \frac{28k}{3} = 1 \quad k = \frac{3}{28}$
$P(x)$	7	6	5	4	3	2	1	$\times\frac{1}{28}$	
$xP(x)$	0	6	10	12	12	10	6	$\times\frac{1}{28}$	$\Sigma = \frac{56}{28} = 2$
$x^2P(x)$	0	6	20	36	48	50	36	$\times\frac{1}{28}$	$\Sigma = \frac{196}{28} = 7$
									$\text{var} = 7 - 2^2 = 3$

Give 1 mark for each •	Illustrations for awarding each •
Additional marks in Paper 1	
Question 1 +1	
•[1] ic : rearrange in standard form	•[1] $y=-4x+1$
•[2] ic : interpret gradient from linear equ.	•[2] $m=-4$
•[3] ic : find perp. gradient	•[3] $m_{perp}=\frac{1}{4}$
•[4] ic : state equation of line	•[4] $y-3=\frac{1}{4}(x-(-1))$
Question 2 +1	
•[1] ic : start to complete square	•[1] $(x+3)^2$
•[2] pd : finish completing the square	•[2] $+2$
•[3] ic : sketch	•[3] *U-shaped parabola*
•[4] ic : sketch	•[4] *minimum at* $(-3,2)$
•[5] ic : sketch	•[5] *intercept on y-axis at* $(0,11)$
Question 3 +1	
•[1] ic : interpret unit vectors	•[1] $\begin{pmatrix}3\\2\\0\end{pmatrix}$ *and* $\begin{pmatrix}2\\-3\\4\end{pmatrix}$
•[2] ss : know to use scalar product and get zero	•[2] for perpendicularity "$u.v$"$=0$
•[3] pd : process	•[3] $\begin{pmatrix}3\\2\\0\end{pmatrix}.\begin{pmatrix}2\\-3\\4\end{pmatrix}=6-6+0=0$
Question 4 +1	
•[1] ss : e.g. form two equations in p and q	•[1] $15=12p+q, 16=15p+q$
•[2] pd : process	•[2] $p=\frac{1}{3}, q=11$
•[3] ic : state the condition for limit to exist	•[3] since $-1<\frac{1}{3}<1$, limit exists
•[4] ss : algebraic strategy for limit	•[4] *e.g.* $L=\frac{1}{3}L+11$
•[5] pd : process limit	•[5] $L=16\frac{1}{2}$
Question 5 +1	
•[1] pd : express in standard form	•[1] $x^{\frac{1}{2}}$
•[2] pd : express in standard form	•[2] $2x^{-2}$
•[3] pd : differentiate fractional index	•[3] $\frac{1}{2}x^{-\frac{1}{2}}$
•[4] pd : differentiate negative index	•[4] $-4x^{-3}$
•[5] pd : evaluation	•[5] $\frac{1}{2}\times4^{-\frac{1}{2}}=\frac{1}{4}$ *or* $-4\times4^{-3}=-\frac{1}{16}$
•[6] pd : evaluation	•[6] $\frac{3}{16}$

Give 1 mark for each •	Illustrations for awarding each •
Question 8 +1	
•1 pd : express in standard form	•1 $(3x+1)^{-\frac{1}{2}}$
•2 pd : integrate	•2 $\frac{1}{\frac{1}{2}}(3x+1)^{\frac{1}{2}}$
•3 pd : integrate	•3 $\ldots\times\frac{1}{3}$
•4 ic : substitute the limits	•4 $\left[\frac{2}{3}(3\times1+1)^{\frac{1}{2}}\right]-\left[\frac{2}{3}(3\times0+1)^{\frac{1}{2}}\right]$
•5 pd : evaluate	•5 $\frac{2}{3}$
Question 10 +2	
•1 pd : calculate hypotenuse	•1 hypot $=\sqrt{80}$
•2 pd : calculate sinp and cosp	•2 $\sin(p)=\frac{4}{\sqrt{80}}$ *and* $\cos(p)=\frac{8}{\sqrt{80}}$
•3 ss : use double angle formula	•3 $\sin(2p)=2\sin(p)\cos(p)$
•4 pd : process sin2p	•4 $\sin(2p)=\frac{4}{5}$
•5 ss : use double formula	•5 $\cos(2p)=2\cos^2(p)-1$
•6 pd : process cos2p	•6 $\cos(2p)=\frac{3}{5}$
•7 ic: relate gradient and tan	•7 gradient $=\tan(2p)$
•8 pd : process	•8 $\frac{4}{3}$
Question 11 +1	
•1 ic : interpret centre	•1 $A=(12,-5)$
•2 pd : use Pythagoras	•2 $OA=13$
•3 ic : interpret radius	•3 $r_B=8$
•4 ic : interpret centre	•4 $B=(24,0)$
•5 ic : state equ of circle	•5 $(x-24)^2+y^2=64$
•6 ic : interpret B and q	•6 $q=-24$
•7 ss : strategy for p	•7 substitute $(12,-5)$
•8 pd : process	•8 $p=\frac{5}{144}$

Increase in marks for Paper 1 = 9
Increase in marks for Paper 2 = 11
Total increase in marks = 20.

For 2004 the marks will allocated as follows:
Paper 1 60
Paper 2 70
Total 130

[END OF MARKING INSTRUCTIONS]

1. Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than marks deducted for what is wrong.
2. Award one mark for each 'bullet' point. Each error should be underlined in RED at the point in the working where it first occurs, and not at any subsequent stage of the working.
3. The working subsequent to an error must be followed through by the marker with possible full marks for the subsequent working, provided that the difficulty involved is approximately similar. Where, subsequent to an error, the working is eased, a deduction(s) of mark(s) should be made.
   This may happen where a question is divided into parts. In fact, failure to even answer an earlier section does not preclude a candidate from assuming the result of that section and obtaining full marks for a later section.

4. Correct working should be ticked (✓).This is essential for later stages of the SQA procedures. Where working subsequent to an error(s) is correct and scores marks, it should be marked with a crossed tick (✓̸). In appropriate cases attention may be directed to work which is not quite correct (e.g. bad form) but which has not been penalised, by underlining with a dotted or wavy line.
   Work which is correct but inadequate to score any marks should be corrected with a double cross tick (✓̸̸).

5. • The total mark for each section of a question should be entered in red in the **outer** right hand margin, opposite the end of the working concerned.
   • Only the mark should be written, **not** a fraction of the possible marks.
   • These marks should correspond to those on the question paper and these instructions.

6. It is of great importance that the utmost care should be exercised in adding up the marks. Where appropriate, all summations for totals and grand totals must be carefully checked.
   Where a candidate has scored zero marks for any question attempted, "0" should be shown against the answer.

7. As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Accept answers arrived at by inspection or mentally where it is possible for the answer so to have been obtained. Situations where you may accept such working will be indicated in the marking instructions.

cont/

8. Do not penalise:
   - working subsequent to a correct answer
   - omission of units
   - bad form
   - legitimate variations in numerical answers
   - correct working in the "wrong" part of a question

9. No piece of work should be scored through - even where a fundamental misunderstanding is apparent early in the answer. Reference should always be made to the marking scheme - answers which are widely off-beam are unlikely to include anything of relevance but in the vast majority of cases candidates still have the opportunity of gaining the odd mark or two provided it satisfies the criteria for the mark(s).

10. If in doubt between two marks, give an intermediate mark, but without fractions. When in doubt between consecutive numbers, give the higher mark.

11. In cases of difficulty covered neither in detail nor in principle in the Instructions, attention may be directed to the assessment of particular answers by making a referal to the P.A. Please see the general instructions for P.A. referrals.

12. No marks should be deducted at this stage for careless or badly arranged work. In cases where the writing or arrangement is very bad, a note may be made on the upper left-hand corner of the front cover of the script.

13 **Do not write any comments on the scripts**. A summary of acceptable notation is given on page 4.

Summary

Throughout the examination procedures many scripts are remarked. It is essential that markers follow common procedures:

1 **Tick** correct working.
2 Put a mark in the **right-hand margin to match the marks allocations on the question paper.**
3 Do **not** write marks as fractions.
4 Put each mark **at the end** of the candidate's response to the question.
5 **Follow through** errors to see if candidates can score marks subsequent to the error.
6 Do **not** write any comments on the scripts.

Higher Mathematics : A Guide to Standard Signs and Abbreviations

Remember - No comments on the scripts. Please use the following and nothing else.

Signs

✓ The tick. You are not expected to tick every line but of course you must check through the whole of a response.

___ ✗ The cross and underline. Underline an error and place a cross at the end of the line.

✓✗ The tick-cross. Use this to show correct work where you are **following through** subsequent to an error.

✗✓ The double cross-tick. Use this to show correct work but which is inadequate to score any marks.

∧ The roof. Use this to show something is missing such as a crucial step in a proof or a 'condition' etc.

~~~~~~

The tilde. Use this to indicate a minor transgression which is not being penalised (such as bad form).

E Eased. Where working is found correct whilst following through subsequent to an error, the working has been eased sufficiently for a mark not to be awarded.

BOD Benefit of Doubt. Use this where you have to decide between two consecutive marks and award the higher.

Marks being allotted e.g. (•) would not normally be shown on scripts

		margins
$\frac{dy}{dx} = 4x - 7$	✓ •	
$4x - 7 = \underline{0}$	✗	
$x = \frac{7}{4}$		
$y = 3\frac{7}{8}$	✓✗ •	2
$C = \underline{(1, -1)}$	✗	
$m = \frac{3-(-1)}{4-1}$		
$m_{rad} = \frac{4}{3}$	✓✗ • follow through	
$m_{tgt} = \frac{-1}{\frac{4}{3}}$		
$m_{tgt} = -\frac{3}{4}$	✓✗ •	
$y - 3 = -\frac{3}{4}(x - 2)$	✓✗ •	3
$x^2 - 3x = 28$	✓ •	
$x = 7$ ∧	✗✓	1
$\sin(x) = 0.75 = inv\sin(0.75) = 48.6°$	✓ •	1
$\log_3(x - \underline{2) = 1}$	✗	
$(x - 2) = 3^1$	✓✗ •	
$x - 2 = 3$		
$x = 5$	✗✓ E	1

All of these are to help us be more consistent and accurate.

It goes without saying that however accurate you are in marking, it is to no avail unless you have added the marks up correctly. Please double check totals!!
~~~~~~

	Give 1 mark for each •	Illustrations for awarding each •

1 $f(x) = 6x^3 - 5x^2 - 17x + 6$.

(*a*) Show that $(x - 2)$ is a factor of $f(x)$.

(*b*) Express $f(x)$ in its fully factorised form. **4**

1	**2.1.1, 2.1.3 CN C 03/101** **ans: proof and** $(x-2)(2x+3)(3x-1)$ **4 marks** •[1] ss : synthetic division, long division or evaluation •[2] ic : complete proof •[3] ic : state quadratic factor •[4] pd : factorise fully	•[1] $\begin{array}{r\|rrrr} 2 & 6 & -5 & -17 & 6 \\ & & 12 & & \\ \hline & 6 & & & \end{array}$ •[2] $\begin{array}{r\|rrrr} 2 & 6 & -5 & -17 & 6 \\ & & 12 & 14 & -6 \\ \hline & 6 & 7 & -3 & 0 \end{array}$ •[3] $6x^2 + 7x - 3$ •[4] $(x-2)(2x+3)(3x-1)$ **stated explicitly**

Alternative 1

•[1] $f(2) = 6 \times 2^3 \ldots\ldots$

•[2] $f(2) = 48 - 20 - 34 + 6 = 0$

•[3] $6x^2 + 7x - 3$

•[4] $(x-2)(2x+3)(3x-1)$

Alternative 2

•1

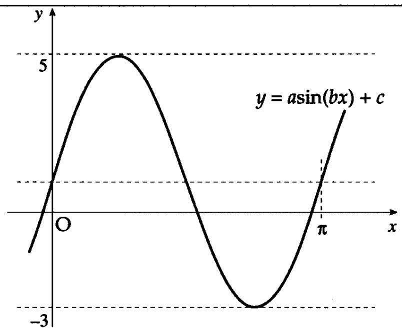
$$\begin{array}{r|rrrr} & 6x^2 & & & \\ \hline x-2 & 6x^3 & -5x^2 & -17x & +6 \\ & 6x^3 & -12x^2 & & \end{array}$$

•2

$$\begin{array}{r|rrrr} & 6x^2 & +7x & -3 & \\ \hline x-2 & 6x^3 & -5x^2 & -17x & +6 \\ & \underline{6x^3} & \underline{-12x^2} & & \\ & & 7x^2 & -17x & \\ & & \underline{7x^2} & \underline{-14x} & \\ & & & -3x & +6 \\ & & & \underline{-3x} & \underline{+6} \\ & & & 0 & 0 \end{array}$$

Notes

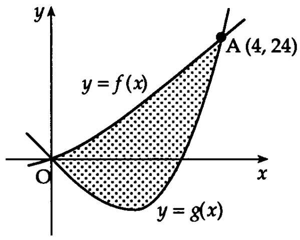
1 See page 16 for advice on solutions obtained via a graphics calculator

	Give 1 mark for each •	Illustrations for awarding each •
2	The diagram shows a sketch of part of the graph of a trigonometric function whose equation is of the form $y = a\sin(bx) + c$. Determine the values of a, b and c.	 3
2	**1.2.3, 2.3.3 CN C 03/new** **ans: a = 4, b = 2, c = 1 3 marks** •1 ic : interpret amplitude •2 ic : interpret period •3 ic : interpret vertical displacement	•1 $a = 4$ •2 $b = 2$ •3 $c = 1$
		Notes 1 Accept $4\sin(2x) + 1$ for 3 marks.

	Give 1 mark for each •	Illustrations for awarding each •

3 The incomplete graphs of $f(x) = x^2 + 2x$ and $g(x) = x^3 - x^2 - 6x$ are shown in the diagram. The graphs intersect at A (4, 24) and the origin.

Find the shaded area enclosed between the curves. **5**



3

2.1.2, 2.2.7 **CN** **CB 03/109**

ans: $42\frac{2}{3}$ **5 marks**

- •1 ss : area= $\int$ upper function – lower function
- •2 ic : interpret limits
- •3 pd : simplify prior to integration
- •4 pd : integrate
- •5 pd : evaluate using limits

•1 $\int\left(\left(x^2+2x\right)-\left(x^3-x^2-6x\right)\right)dx$ **stated, or implied by •3**

•2 $\int_0^4 \ldots\ldots$

•3 $\int\left(8x+2x^2-x^3\right)dx$

•4 $\left[4x^2+\frac{2}{3}x^3-\frac{1}{4}x^4\right]_0^4$

•5 $42\frac{2}{3}$

Alternative 1

•1 $\int\left(x^2+2x\right)-\left(x^3-x^2-6x\right)dx$

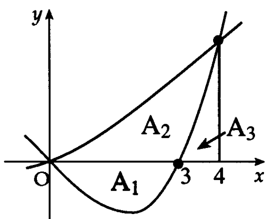
•2 $\int_0^4 \ldots\ldots$

•3 $\left[\frac{1}{3}x^3+x^2\right]_0^4$

•4 $\left[\frac{1}{4}x^4-\frac{1}{3}x^3-3x^2\right]_0^4$

•5 $42\frac{2}{3}$

Alternative 2



•1 $\int_0^3\left(x^3-x^2-6x\right)dx$

•2 $A_2+A_3=\int_0^4\left(x^2+2x\right)dx$

•3 $A_3=\int_3^4\left(x^3-x^2-6x\right)dx$

•4 $15\frac{3}{4}$ *or* $37\frac{1}{3}$ *or* $10\frac{5}{12}$

•5 $15\frac{3}{4}+37\frac{1}{3}-10\frac{5}{12}=42\frac{2}{3}$

Notes

1. •1 is lost for subtracting the wrong way round.
   •5 will also be lost for statements such as
   $-42\frac{2}{3}=42\frac{2}{3}$,
   $-42\frac{2}{3}$ so ignore the –ve,
   $-42\frac{2}{3}=42\frac{2}{3}$sq units

   •5 may still be gained for statements such as
   $\ldots -42\frac{2}{3}$ and so the area $=42\frac{2}{3}$.
2. For candidates who split up the area into three integrals, see model in Alternative 2
3. Do not penalise decimal approximations
4. Differentiation loses •4 and •5
5. $\int_4^0 (f(x)-g(x))dx$ loses •2 and possibly •5
6. $\int_4^0 (g(x)-f(x))dx$ is technically correct and hence all 5 marks are available
7. Accept at •3, $8x+2x^2-x^3$ appearing from solving "upper" = "lower"
8. using $f(x)+g(x)$ leading to 32 gains •2,•4 and •5

Example 1

$\int\left(x^2+2x-x^3-x^2-6x\right)dx$	•1 ✓ bad form
$\left[-\frac{1}{4}x^4-2x^2\right]_0^4$	•2 ✓
	•3 ✗
-96	•4 ✓✗ Eased
$\therefore$ *area* $=$ 96	•5 ✓✗
	3 marks given

	Give 1 mark for each •	Illustrations for awarding each •

4 (*a*) Find the equation of the tangent to the curve with equation $y = x^3 + 2x^2 - 3x + 2$ at the point where $x = 1$. **5**

(*b*) Show that this line is also a tangent to the circle with equation $x^2 + y^2 - 12x - 10y + 44 = 0$ and state the coordinates of the point of contact. **6**

4	**1.3.9, 2.4.4** **CN** **CB 03/104**	
	(a) ans: $y = 4x - 2$ **5 marks**	
	(b) proof & (2, 6) **6 marks**	
	•1 ss : know to differentiate and start	•1 $\frac{dy}{dx} = 3x^2 \ldots$
	•2 pd : differentiate	•2 $\frac{dy}{dx} = 3x^2 + 4x - 3$
	•3 pd : evaluate gradient	•3 $m = \frac{dy}{dx}_{x=1} = 4$ **gradient stated or implied by •5**
	•4 pd : evaluate y-coordinate	•4 $y_{x=1} = 2$
	•5 ic : state equation of line	•5 $y - 2 = 4(x - 1)$
	•6 ss : prepare for substitution	•6 $y = 4x - 2$
	•7 ss : substitute	•7 $x^2 + (4x-2)^2 - 12x - 10(4x-2) + 44 = 0$
	•8 pd : express in standard form	•8 $17x^2 - 68x + 68 = 0$
	•9 ss : know how to solve	•9 $17(x-2)(x-2) = 0$
	•10 ic : complete proof	•10 equal roots $\Rightarrow$ tangent
	•11 pd : determine coordinates	•11 pt of contact = (2,6)

Alternative 1

•6 $y = 4x - 2$
•7 $C = (6,5)$ *and* $m_{radius} = -\frac{1}{4}$
•8 $y - 5 = -\frac{1}{4}(x - 6)$
•9 start to solve sim. equations
•10 $x = 2, y = 6$
•11 check that (2,6) lies on the circle

Example 1

$3x^2 + 4x - 3$	•1 ✓
$3 \times 1^2 + 4 \times 1 - 3 = 4$	•2 ✓
$1^3 + 2 - 3 + 2 = 2$	•3 ✗
$y - 4 = 2(x - 1)$	•4 ✗
	•5 ✓✗
	3 marks given

Cave

Look out for candidates who use $y = -\frac{1}{4}x + \frac{9}{4}$ instead of $y = 4x - 2$ leading to point of contact of (5, 1). This is worth 5 marks.

Notes

1 •5 is only available after an attempt has been made to find the gradient from differentiation

2 alternatives for •9

•9 $(x-2)(17x-34) = 0$

3 alternatives for •10

•10 $x = 2,2 \Rightarrow$ tangent

or •10 $x = 2$ only $\Rightarrow$ tangent

4 alternative for •9 and •10

•9 $b^2 - 4ac = 68^2 - 4 \times 17 \times 68$

•10 $= 68^2 - 68^2 = 0 \Rightarrow$ tangent

5 alternative for •10 and •11

•10 $x = 2, y = 6$ *and* $m_{radius} = -\frac{1}{4}$

•11 $m_1 m_2 = 4 \times -\frac{1}{4} = -1 \Rightarrow$ line is tangent

6 For notes 2, 3 and 4 it is acceptable to deal with the reduced quadratic $x^2 - 4x + 4 = 0$.

7 an "= 0" must occur somewhere between •7 and •9

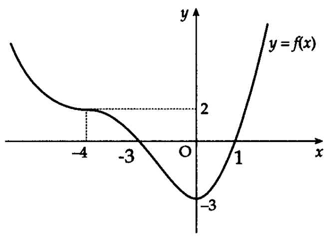
	Give 1 mark for each •	Illustrations for awarding each •

5 The diagram shows part of the sketch of a function f.

f has a minimum turning point at $(0, -3)$ and a point of inflexion at $(-4, 2)$.

(*a*) Sketch the graph of $y = f(-x)$. **2**

(*b*) On the same diagram sketch the graph of $y = 2f(-x)$. **2**

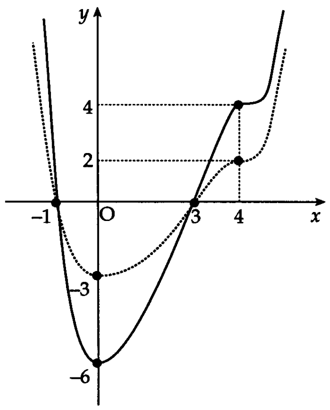


5	**1.2.4**	**CN**	**CA 03/84r**
	(a) ans: sketch		**2 marks**
	(b) ans: sketch		**2 marks**

•[1] ic : interpret $f(-x)$
•[2] ic : communication
•[3] ic : interpret $2f$
•[4] ic : communication

•[1] refl. in y – axis & (0,-3)
•[2] annotate (4,2),(3,0),(–1,0)

•[3] a scaling & (3,0),(-1,0)
•[4] annotate (0,–6), (4,4)



Alternative 1

•[1] (–1,0) and (3,0)
•[2] minimum at (0,–3) and p / i at (4,2)
•[3] (–1,0) *and* (3,0)
•[4] minimum at (0,–6) and p / i at (4,4)

Notes

1 **Ignore**
poor drawing skills
no labels on graphs
using separate diagrams

2 For (a)
reflection in x-axis scores 1 out of 2
reflection in $x = -3$ scores 1 out of 2
reflection in (0, 0) scores 1 out of 2

3 For (b)
sketching $2f(2x)$ etc scores no marks
sketching $f\left(\frac{1}{2}x\right)$ etc scores no marks

4 for •3, any scaling parallel to the y-axis is acceptable.

	Give 1 mark for each •	Illustrations for awarding each •
6	If $f(x)=\cos(2x)-3\sin(4x)$, find the exact value of $f'(\frac{\pi}{6})$.	**4**
6	**3.2.2, 3.2.1, 1.2.11 NC** **BA 03/42** **ans :** $6-\sqrt{3}$ **4 marks** •1 pd : differentiate compound trig •2 pd : differentiate compound trig •3 ic : interpret •4 pd : evaluate derivative	•1 $f'(x)=-2\sin(2x)+\ldots$ •2 $\ldots\ldots-12\cos(4x)$ •3 $f'(\frac{\pi}{6})=-2\sin(\frac{2\pi}{6})-12\cos(\frac{4\pi}{6})$ •4 $6-\sqrt{3}$

Alternative 1

- •1 $f'(x)=-2\sin(2x)+\ldots$
- •2 $\ldots\ldots-12\cos(4x)$
- •3 $-2\sin(\frac{2\pi}{6})=-\sqrt{3}$
- •4 $-12\cos(\frac{4\pi}{6})$

Example 1

$f'(x)=-\sin(2x)-3\cos(4x)$

$-\sin(\frac{2\pi}{6})=-\frac{\sqrt{3}}{2}$

$-3\cos(\frac{4\pi}{6})=+\frac{3}{2}$

•1 ✗
•2 ✓✗
•3 ✓✗
•4 ✓✗

3 marks given

Example 2

$f'(x)=-\frac{1}{2}\sin(2x)-\frac{3}{4}\cos(4x)$

$-\frac{1}{2}\sin(\frac{2\pi}{6})=-\frac{\sqrt{3}}{4}$

$-\frac{3}{4}\cos(\frac{4\pi}{6})=+\frac{3}{8}$

•1 ✗
•2 ✓✗
•3 ✓✗
•4 ✓✗

3 marks given

Notes

1 Evidence for •3:

$-2\sin 2\left(\frac{\pi}{6}\right)-12\cos 4\left(\frac{\pi}{6}\right)$	
$-2\sin\left(\frac{\pi}{3}\right)-12\cos\left(\frac{2\pi}{3}\right)$	•3 ✓
or $-2\sin\left(\frac{2\pi}{6}\right)-12\cos\left(\frac{4\pi}{6}\right)$	•3 ✓
or $-2\times\frac{\sqrt{3}}{2}-12\times\left(-\frac{1}{2}\right)$	•3 ✓
or $-1.732+6$	•3 ✓
but	
$-2\times 2\times\frac{1}{2}-12\times 4\times\frac{\sqrt{3}}{2}$	•3 ✗

2 Do not penalise the use of 30° at •3

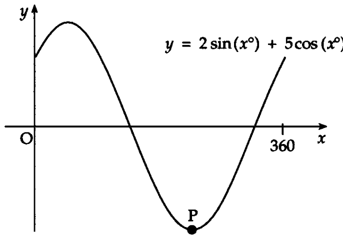
3 Do not penalise $\frac{-4\sqrt{3}}{4}$ instead of $-\sqrt{3}$

	Give 1 mark for each •	Illustrations for awarding each •

7 Part of the graph of $y = 2\sin(x^\circ) + 5\cos(x^\circ)$ is shown in the diagram.

(*a*) Express $y = 2\sin(x^\circ) + 5\cos(x^\circ)$ in the form $k\sin(x^\circ + a^\circ)$ where $k > 0$ and $0 \le a < 360$. **4**

(*b*) Find the coordinates of the minimum turning point P. **3**



7 **3.4.1, 3.4.3 Ca BA 03/118**

(a) ans : $\sqrt{29}\sin(x + 68.2)^\circ$ **4 marks**

(b) ans : $(201.8^\circ, -\sqrt{29})$ **3 marks**

•1 ic : expand	•1 $k\sin(x^\circ)\cos(a^\circ) + k\cos(x^\circ)\sin(a^\circ)$	**stated explicitly**
•2 ic : compare coefficients	•2 $k\cos(a^\circ) = 2,\ k\sin(a^\circ) = 5$	**stated explicitly**
•3 pd : process k	•3 $k = \sqrt{29}$ (5.4...)	
•4 pd : process angle	•4 $a = 68\cdot2^\circ$	
•5 ic : interpret minimum	•5 $\sqrt{29}\sin(x + 68.2)^\circ = -\sqrt{29}$	
•6 pd : process	•6 $x_P = 201\cdot8^\circ$	
•7 ic : interpret y-coordinate	•7 $y_P = -\sqrt{29}$	

Notes

1 Candidates may use any form eg $k\sin(x - a)$ as long as the final answer is in the form $k\sin(x + a)$. If not they lose •4

2 For •1 treat $k\sin x\cos a + \cos x\sin a$ as bad form provided you see $k\cos(a^\circ)$ *and* $k\sin(a^\circ)$ appearing for •2.

3 For •1 accept $k(\sin x\cos a + \cos x\sin a)$.

4 For •4 accept any answer which rounds to 68

5 The following are acceptable for •5

•5 $\sin(x + 68.2)^\circ = -1$

or

•5 $x + 68.2 = 270$

6 candidates who use differentiation for (b) will most likely lose 1 mark for omitting the factor $\frac{\pi}{180}$

7 $(201.8^\circ, -\sqrt{29})$ with no working at all may earn marks •6 and •7.

8 $(-\sqrt{29}, 201.8^\circ)$ with no working at all may earn mark •7.

9 See page 16 for advice on solutions obtained via a graphics calculator

Example 1
(b)

$\sqrt{29}\sin(x + 68.2)^\circ = -1$

$x = 123,\ 281$ award 1 mark

$(281, -1)$ *or* $(281, -\sqrt{29})$

Example 2
(b)

at P, m = 0

$2\cos(x) + 5(-\sin(x)) = 0$ •5 ✗ note 6

$\tan(x) = \frac{2}{5}$

$x = 21.8,\ 201.8$ •6 ✓

$x = 201.8$ at minimum •7 ✓

$(201.8, -\sqrt{29})$

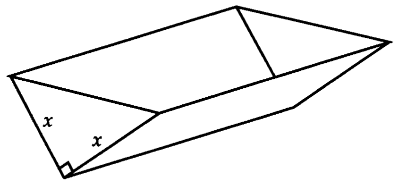
2 marks given

Notes cont

10 If the •4 answer is in radians, the mark is lost
If the •6 answer is in radians, the mark is lost
If both answers at •4 and •6 are in radians, only penalise once.

	Give 1 mark for each •	Illustrations for awarding each •

8 An open water tank, in the shape of a triangular prism, has a capacity of 108 litres. The tank is to be lined on the inside in order to make it watertight.
The triangular cross-section of the tank is right-angled and isosceles, with equal sides of length x cm.
The tank has a length of l cm.



(a) Show that the surface area to be lined, A cm^2, is given by $A(x) = x^2 + \frac{432000}{x}$. **3**

(b) Find the value of x which minimises this surface area. **5**

8 **1.3.15** **CN** **CBA** **03/97**

(a) ans : proof **3 marks**
(b)ans : 60 **5 marks**

•1	ss : identify crucial aspect	$length = \frac{108000}{\frac{1}{2}x^2}$
•2	ic : start proof	$SA = 2 \times \frac{1}{2}x^2 + 2x \times length$
•3	ic : complete proof	$\ldots SA = x^2 + \frac{432000}{x}$
•4	ss : know to set derivative to zero	$\frac{dA}{dx} = \ldots = 0$
•5	pd : express in standard form	$432000x^{-1}$
•6	pd : differentiate	$2x - 432000x^{-2}$
•7	pd : solve	$x = 60$
•8	ic : justify minimum	*e.g.* nature table

Notes
1 Evidence of the nature table should take the form

x	60^-	60	60^+
$\frac{dA}{dx}$	$-ve$	0	$+ve$
	\	—	/
		minimum	

2 For •8, the second derivative is acceptable

$\frac{d^2A}{dx^2} = 2 + 864000x^{-3}$

$\left.\frac{d^2A}{dx^2}\right|_{x=60} = 2 + 4 > 0$

so minimum at x = 60

3 A trial and error approach earns no marks

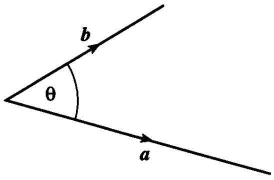
4 The " = 0" shown at •4 must appear somewhere for •4 to be awarded (but not necessarily at •4 stage)

5 in (b) if a candidate uses an incorrect formula then only •4, •5 and •6 are available i.e. maximum score would be 3 marks.To score 3 marks, working has to be of a similar difficulty.

Notes cont
6 For •8, a sketch of the graph would an acceptable alternative. At least 3 point should be shown (eg at x = 59, x=60 and x=61).

	Give 1 mark for each •	Illustrations for awarding each •

9 The diagram shows vectors $\boldsymbol{a}$ and $\boldsymbol{b}$.
If $|\boldsymbol{a}| = 5$, $|\boldsymbol{b}| = 4$ and $\boldsymbol{a}.(\boldsymbol{a}+\boldsymbol{b}) = 36$, find the size of the acute angle θ between $\boldsymbol{a}$ and $\boldsymbol{b}$. **4**



9 **3.1.9** **Ca** **BA 03/115**
ans: 56·6° **4 marks**

- •1 ss : use distributive law
- •2 pd : expand scalar product
- •3 pd : expand scalar product
- •4 pd ; complete calculations

- •1 $\boldsymbol{a}.(\boldsymbol{a}+\boldsymbol{b}) = \boldsymbol{a}.\boldsymbol{a} + \boldsymbol{a}.\boldsymbol{b}$
- •2 $\boldsymbol{a}.\boldsymbol{b} = 5\times 4\cos(\theta)$
- •3 $\boldsymbol{a}.\boldsymbol{a} = 5^2$
- •4 $[\cos(\theta) = 0.55] \Rightarrow \theta = 56.6°$ 0.99 radians

Example 1

$\boldsymbol{a}.(\boldsymbol{a}+\boldsymbol{b}) = \boldsymbol{a}^2 + \boldsymbol{ab}$	•1 ✓ bad form
$\boldsymbol{ab} = 5\times 4\cos(\theta)$	•2 ✓ bad form
$\boldsymbol{a}^2 = 5^2$	•3 ✓ bad form
$\theta = 56.6°$	•4 ✓
	4 marks given

Example 2

$a.a + a.b = 36$	•1 ✓
$25\cos(\theta) + 20\cos(\theta) = 36$	•2 ✓
$\cos(\theta) = \frac{36}{45}$	•3 ✗
$\theta = 36.9$	•4 ✗
	2 marks given

Example 3 CAVE

$$\cos(\theta) = \frac{|a|\,|b|}{a.b} = \frac{20}{36}$$
$\theta = 56.3$

0 marks given

Notes

1 Using "$a.b = |a|\,|b|\sin(\theta)$" loses 1 mark

	Give 1 mark for each •	Illustrations for awarding each •
10	Solve the equation $3\cos(2x)+10\cos(x)-1=0$ for $0 \le x \le \pi$, correct to 2 decimal places.	5
10	**2.3.1** **Ca** **BA 03/106** **ans: 1.23 radians** **5 marks** •1 ss : know to use double angle formula •2 pd : arrange in standard form •3 ss : know how to solve •4 pd : solve •5 pd : solve	•1 $3\left(2\cos^2(x)-1\right)......$ •2 $6\cos^2(x)+10\cos(x)-4=0$ •3 $2(3\cos(x)-1)(\cos(x)+2)$ •4 $\cos(x)=\frac{1}{3}$ ***and*** $\cos(x)=-2$ •5 $x=1.23$ ***and*** no solution

Example 1

$6\cos^2(x)+10\cos(x)-2=0$
leading to
$\cos(x)=0.180$ *or* $\cos(x)=-1.84$
$x=1.39$ *radians* *no solution*

•1 ✗
•2 √✗
•3 √✗
•4 √✗
•5 √✗

4 marks given

Notes

1 alternative for •3

•3 $\cos(x)=\dfrac{-10\pm\sqrt{10^2-4\times6\times(-4)}}{2\times6}$

2 •5 must include some indication that $\cos(x)=-2$ has no solutions.

3 in the event of other substitutions being used for $\cos(2x)$, no credit can be given until the equation reduces to a quadratic in $\cos(x)$.

4 •4 and •5 are only available as a consequence of solving a quadratic equation.

5 •4 and •5 may also be marked as follows

•4 $\cos(x)=\frac{1}{3}$ ***and*** $x=1.23$

•5 $\cos(x)=-2$ ***and*** no solution

6 For •5, accept $\frac{70.5\pi}{180}$ in lieu of 1.23

7 If an answer starts

$3\times2\cos^2(x)-1+10\cos(x)-1=0$
$6\cos^2(x)+10\cos(x)-4=0$

then treat the first line as bad form.

If an answer starts

$3\times2\cos^2(x)-1+10\cos(x)-1=0$
$6\cos^2(x)+10\cos(x)-2=0$

then use Example 1.

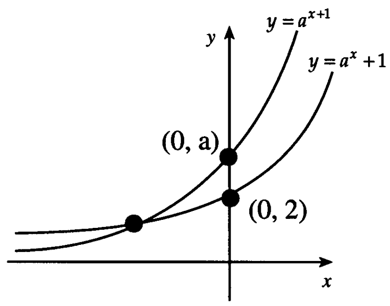
	Give 1 mark for each •	Illustrations for awarding each •

11 (*a*) (i) Sketch the graph of $y = a^x + 1, a > 2$.

(ii) On the same diagram sketch the graph of $y = a^{x+1}$, $a > 2$. **2**

(*b*) Prove that the graphs intersect at a point where the x-coordinate is $\log_a\left(\frac{1}{a-1}\right)$. **3**

11	**3.3.7, 3.3.4 CN A 03/120** **(a) ans: sketch 2 marks** **(b) ans : proof 3 marks** •1 ic : sketch expo function •2 ic : sketch expo function •3 ss : know to equate for intersection •4 pd : start to solve - crucial step •5 ic : complete proof	•1 expo sketch thr (0,2) •2 expo sketch thr $(0,a)$ •3 $a^{x+1} = a^x + 1$ •4 $a \times a^x - a^x = 1$ •5 $(a-1)\times a^x = 1$ ...& complete



Notes

1. For •2, the second graph must cut the y-axis above (0, 2) and must finish up between the first graph and the x-axis as x tends to minus infinity. '2' and 'a' must be marked on the y-axis.
2. Both graphs correct but with no annotation may be awarded 1 mark.

Alternative 1

Let $x = \log_a\left(\frac{1}{a-1}\right)$

Then $\frac{1}{a-1} = a^x$ •1 ✓

$y = a^x + 1 = \frac{1}{a-1} + 1 = \frac{a}{a-1}$ $\quad$ $y = a^{x+1} = a^x \times a = \frac{1}{a-1} \times a = \frac{a}{a-1}$ •2 ✓

$\therefore$ curves intersect at $\left(\log_a\left(\frac{1}{a-1}\right), \frac{a}{a-1}\right)$ •3 ✓

3 marks given

Alternative 1

Let $x = \log_a\left(\frac{1}{a-1}\right)$

Then $\frac{1}{a-1} = a^x$ •1 ✓

$1 = a^x(a-1)$ •2 ✓

$1 = a^{x+1} - a^x$

$a^x + 1 = a^{x+1}$

Hence the graphs intersect •3 ✓

3 marks given

	Give 1 mark for each •	Illustrations for awarding each •

Solutions obtained by employing the facilities on a graphics calculator

1

(a) •1 $f(2) = 6 \times 2^3 \ldots\ldots$

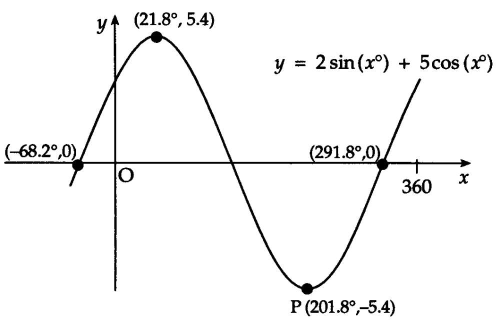
•2 $f(2) = 48 - 20 - 34 + 6 = 0$

(b) •3 for a sketch of the cubic with the zeroes indicated at 2, $\frac{1}{3}$ and $-\frac{3}{2}$ and the statement : the roots are 2, $\frac{1}{3}$ and $-\frac{3}{2}$.

•4 so $f(x) = k(x-2)(x+\frac{3}{2})(x-\frac{1}{3})$
by comparing the leading terms, for example, of
$f(x) = k(x-2)(x+\frac{3}{2})(x-\frac{1}{3})$
and $f(x) = 6x^3 - 5x^2 - 17x + 6$
we have $k = 6$
and so $f(x) = (x-2)(2x+3)(3x-1)$ **explicitly stated**

7

The graphics calculator plot shows the following



(a) •1 *annotated on diagram*
max at (21.8,5.4) ***and*** min at (201.8,–5.4)

•2 *annotated on diagram*
(–68.2,0) ***or*** (291.8,0)

•3 "from the amplitude $k = 5.4$"

•4 "from the left shift $a = 68.2$"

(b)

•5,•6 $P = (201.8,\ -5.4)$)

•7 The last mark has to be awarded for some communication about the minimum e.g. the minimum should occur at 270 shifted left by 68.2

	Give 1 mark for each •	Illustrations for awarding each •

1 After a leaflet drop advertising a new garden centre, a random sample of households were surveyed. The results are summarised in the following table.

	Read the leaflet	Did not read the leaflet
Visited the centre	80	20
Did not visit the centre	60	40

(a) Find (i) P(leaflet read) **2**

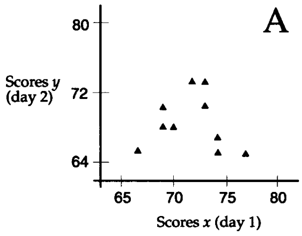
(ii) P(leaflet read and garden centre visited).

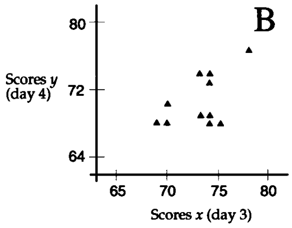
(*b*) Comment on whether the proportion who had visited the garden centre was the same whether or not they had read the leaflet. **3**

S1	**4.1.1, 4.1.3 CN CA 03/new** **(a) ans: $\frac{140}{200}$, $\frac{80}{200}$ 2 marks** **(b) ans : comment 3 marks** •1 ic : interpret table •2 ic : interpret table •3 ic : interpret sample •4 ic : interpret sample •5 ic : comment	•1 $\frac{140}{200}$ •2 $\frac{80}{200}$ •3 $\frac{80}{140} = 0.57$ •4 $\frac{20}{60} = 0.33$ & not the same •5 seems that the leaflet had some effect

	Give 1 mark for each •	Illustrations for awarding each •

2 The scatter diagrams A and B show the scores of ten players in an open golf championship.





(*a*) In diagram A, $\Sigma x = 716,\ \Sigma x^2 = 51338,\ \Sigma y = 689,\ \Sigma y^2 = 47521,\ \Sigma xy = 49351$.
Calculate the correlation coefficient. **3**

(*b*) In diagram B, the correlation coefficient is 0·694. Comment on the relationship between
(i) the scores on days 1 and 2
(ii) the scores on days 3 and 4. **2**

S2	**4.4.4** **Ca** **C 03/130** **(a) ans: r = 0.3126** **3 marks** **(b) ans : comment** **2 marks** •1 pd : process one S •2 pd : process other two Ss •3 pd : process correlation coefficient •4 ic : interpret results •5 ic : interpret results	•1 determine any one from $S_{xx} = 72\cdot4,\ S_{yy} = 48\cdot9,\ S_{xy} = 18\cdot6$ •2 determine remaining two •3 $r = 0\cdot3126$ •4 no linear relationship •5 moderate linear relationship

	Give 1 mark for each •	Illustrations for awarding each •

3 The regulations for a charity state that the Board of Trustees must consist of 6 people.

(*a*) How many ways are there of choosing 6 members from 10 nominations? **1**

Ideally the Board should consist of four employees of the charity and two persons not employed by the charity (i.e. volunteers).

Ten people have been nominated for the Board. Six are employees and four are volunteers.

(*b*) If each nominee has an equally likely chance of being selected, what is the probability that the six members elected will form the ideal choice, that is four employees and two volunteers? **3**

S3	**4.2.5, 4.2.10** **CN** **B** **03/128** **(a) ans: 210** **1 mark** **(b) ans : $\frac{3}{7}$** **3 marks**	
	•1 ss : know to use nCr	•1 $^{10}C_6 = 210$
	•2 pd : process	•2 4 workers: $^6C_4 = 15$
	•3 pd : process	•3 2 co-opts: $^4C_2 = 6$
	•4 ss : know how to determine probability	•4 $P(\text{ideal}) = \frac{15 \times 6}{210} = \frac{3}{7}$

	Give 1 mark for each •	Illustrations for awarding each •

4 The cumulative distribution function for a continuous random variable X is given by

$$F(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{4}x^2 & 0 \le x \le 2 \\ 1 & x > 2 \end{cases}$$

(*a*) Calculate the exact value of the median. **2**

(*b*) Determine the probability density function $f(x)$. **2**

(*c*) Calculate the mean of X. **2**

	Give 1 mark for each •	Illustrations for awarding each •
S4	**4.3.1** **CN** **CA 03/new**	
	(a) ans : $+\sqrt{2}$ **2 marks**	
	(b) ans : $f(x) = \frac{1}{2}x$ *for* $0 \le x \le 2$ $f(x) = 0$ *otherwise* **2 marks**	
	(c) ans : $\frac{4}{3}$ **2 marks**	
	•1 ic : interpret cdf and median	•1 $F(m) = \frac{1}{2}$
	•2 pd : process	•2 $m = +\sqrt{2}$
	•3 ss : know $pdf = \frac{d}{dx}cdf$	•3 $f(x) = \frac{d}{dx}F(x)$
	•4 pd : process	•4 $f(x) = \frac{1}{2}x$ *for* $0 \le x \le 2$ $f(x) = 0$ *otherwise*
	•5 ss : know $mean = \int xf(x)\ dx$	•5 $\mu = \int_0^2 \frac{1}{2}x^2 dx$
	•6 pd : process	•6 $\mu = \frac{4}{3}$

Give 1 mark for each •	Illustrations for awarding each •
Additional marks in Paper 2	
Question 1 +1	
•1 ss : know to evaluate $f(2)$	•1 $f(2)=6\times 2^3 \ldots\ldots$
•2 pd : evaluate $f(2)$ and complete proof	•2 $f(2)=48-20-34+6=0$ so $(x-2)$ is factor
•3 ss : synthetic division or long division	•3 $\begin{array}{c\|cccc} 2 & 6 & -5 & -17 & 6 \\ & & 12 & 14 & -6 \\ \hline & 6 & 7 & -3 & 0 \end{array}$
•4 ic : state quadratic factor	•4 $6x^2+7x-3$
•5 pd : factorise fully	•5 $(x-2)(2x+3)(3x-1)$
Question 2 +3	
•1 ic : interpret amplitude	•1 $a=4$
•2 ic : explanation	•2 half the vertical distance between max and min
•5 ic : interpret period	•3 $b=2$
•4 ic : explanation	•4 graph completes 2 cycles between 0 and 2π
•5 ic : interpret vertical displacement	•5 $c=1$
•6 ic : explanation	•6 half way between y = 5 and y = –3
Question 3 +1	
•1 ss : area= $\int$ upper function – lower function	•1 $\int\left(\left(x^2+2x\right)-\left(x^3-x^2-6x\right)\right)dx$ **stated, or implied by •3**
•2 ic : interpret diagram for limits	•2 $\int_0^4 \ldots\ldots$
•3 pd : simplify prior to integration	•3 $\int\left(8x+2x^2-x^3\right)dx$
•4 pd : integrate	•4 $\left[4x^2+\frac{2}{3}x^3-\frac{1}{4}x^4\right]_0^4$
•5 ic : interpret the limits	•5 $\left(4\times 4^2+\frac{2}{3}\times 4^3-\frac{1}{4}\times 4^4\right)-0$
•6 pd : evaluate using limits	•6 $42\frac{2}{3}$
Question 4 +1	
•1 ss : know to differentiate	•1 $\frac{dy}{dx}=$
•2 pd : differentiate	•2 *any 2 terms from* $3x^2+4x-3$
•3 pd : differentiate	•3 $\frac{dy}{dx}=3x^2+4x-3$
•4 pd : evaluate gradient	•4 $m=\frac{dy}{dx}_{x=1}=4$ **gradient stated or implied by •6**
•5 pd : evaluate y-coordinate	•5 $y_{x=1}=2$
•6 ic : state equation of line	•6 $y-2=4(x-1)$
Question 5 +1	
•1 ic : interpret $f(-x)$	•1 refl. in y – axis
•2 ic : communication	•2 annotate any two from (0,-3),(4,2),(3,0),(–1,0)
•3 ic : communication	•3 annotate remaining two
•4 ic : interpret $2f$	•4 a scaling & (3,0),(-1,0)
•5 ic : communication	•5 annotate (0,–6), (4,4)

Give 1 mark for each •	Illustrations for awarding each •
Question 6 +1	
•1 pd : differentiate compound trig	•1 $f'(x) = -2\sin(2x) + \ldots$
•2 pd : differentiate compound trig	•2 $\ldots\ldots -12\cos(4x)$
•3 ic : interpret	•3 $f'(\frac{\pi}{6}) = -2\sin(\frac{2\pi}{6}) - 12\cos(\frac{4\pi}{6})$
•4 pd : evaluate derivative	•4 $-2\sin(\frac{2\pi}{6}) = -\sqrt{3}$
•5 pd : evaluate derivative	•5 $-12\cos(\frac{4\pi}{6}) = 6$
Question 8 +2	
•1 ss : identify crucial aspect	•1 $length = \frac{108000}{\frac{1}{2}x^2}$
•2 ic : start proof	•2 $SA = 2\times\frac{1}{2}x^2 + 2x\times length$
•3 ic : complete proof	•3 $\ldots SA = x^2 + \frac{432000}{x}$
•4 ss : know to differentiate	•4 $\frac{dA}{dx} = \ldots$
•5 ss : know to set derivative to zero	•5 $\frac{dA}{dx} = 0$
•6 pd : express in standard form	•6 $432000x^{-1}$
•7 pd : differentiate	•7 $2x - 432000x^{-2}$
•8 pd : start to solve	•8 $2x = \frac{432000}{x^2}$
•9 pd : solve	•9 $x = 60$
•10 ic : justify minimum	•10 *e.g.* nature table
Question 9 +1	
•1 ss : use distributive law	•1 $\boldsymbol{a.(a+b) = a.a + a.b}$
•2 pd : expand scalar product	•2 $\boldsymbol{a.b} = 5\times 4\cos(\theta)$
•3 pd : expand scalar product	•3 $\boldsymbol{a.a} = 5^2$
•4 ic : substitution	•4 $20\cos(\theta) = 11$
•5 pd : complete calculations	•5 $\theta = 56.6°$

Increase in marks for Paper 1 = 9
Increase in marks for Paper 2 = 11
Total increase in marks = 20.

For 2004 the marks will allocated as follows:
Paper 1 60
Paper 2 70
Total 130

[END OF MARKING INSTRUCTIONS]