X100/301

NATIONAL QUALIFICATIONS 2005 FRIDAY, 20 MAY 9.00 AM - 10.10 AM MATHEMATICS HIGHER Units 1, 2 and 3 Paper 1 (Non-calculator)

Read Carefully

- 1 Calculators may NOT be used in this paper.
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.



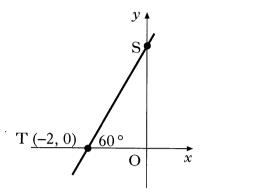


ALL questions should be attempted.

Marks

3

1. Find the equation of the line ST, where T is the point (-2, 0) and angle STO is 60°.

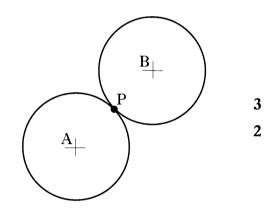


2. Two congruent circles, with centres A and B, touch at P.

Relative to suitable axes, their equations are

$$x^{2} + y^{2} + 6x + 4y - 12 = 0$$
 and
 $x^{2} + y^{2} - 6x - 12y + 20 = 0$.

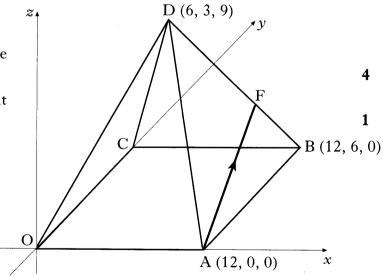
- (a) Find the coordinates of P.
- (b) Find the length of AB.



3. D,OABC is a pyramid. A is the point (12, 0, 0), B is (12, 6, 0) and D is (6, 3, 9).

F divides DB in the ratio 2:1.

- (a) Find the coordinates of the point F.
- (b) Express \overrightarrow{AF} in component form.



[Turn over

- Functions f(x) = 3x 1 and $g(x) = x^2 + 7$ are defined on the set of real numbers.
 - (a) Find h(x) where h(x) = g(f(x)).

2

- (i) Write down the coordinates of the minimum turning point of v = h(x).
 - Hence state the range of the function h.

2

Differentiate $(1 + 2 \sin x)^4$ with respect to x.

2

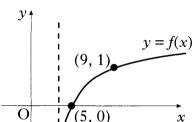
- (a) The terms of a sequence satisfy $u_{n+1} = ku_n + 5$. Find the value of k which produces a sequence with a limit of 4.
- 2
- (b) A sequence satisfies the recurrence relation $u_{n+1} = mu_n + 5$, $u_0 = 3$.
 - (i) Express u_1 and u_2 in terms of m.
 - (ii) Given that $u_2 = 7$, find the value of m which produces a sequence with no limit.

5

2

1

The function f is of the form $f(x) = \log_b (x - a)$. The graph of y = f(x) is shown in the diagram.



- (a) Write down the values of a and b.
- (b) State the domain of f.

A function f is defined by the formula $f(x) = 2x^3 - 7x^2 + 9$ where x is a real number.

(a) Show that (x-3) is a factor of f(x), and hence factorise f(x) fully.

5

(b) Find the coordinates of the points where the curve with equation y = f(x)crosses the x- and y-axes.

2

(c) Find the greatest and least values of f in the interval $-2 \le x \le 2$.

5

9. If $\cos 2x = \frac{7}{25}$ and $0 < x < \frac{\pi}{2}$, find the exact values of $\cos x$ and $\sin x$.

Marks

- 10. (a) Express $\sin x \sqrt{3} \cos x$ in the form $k \sin (x a)$ where k > 0 and $0 \le a \le 2\pi$.
- 4
- (b) Hence, or otherwise, sketch the curve with equation $y = 3 + \sin x \sqrt{3} \cos x$ in the interval $0 \le x \le 2\pi$.

5

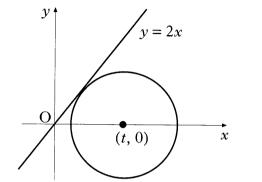
1

5

11. (a) A circle has centre (t, 0), t > 0, and radius 2 units.

Write down the equation of the circle.

(b) Find the exact value of t such that the line y = 2x is a tangent to the circle.



 $[END\ OF\ QUESTION\ PAPER]$

X100/303

NATIONAL QUALIFICATIONS 2005 FRIDAY, 20 MAY 10.30 AM - 12.00 NOON MATHEMATICS HIGHER Units 1, 2 and 3 Paper 2

Read Carefully

- 1 Calculators may be used in this paper.
- 2 Full credit will be given only where the solution contains appropriate working.
- 3 Answers obtained by readings from scale drawings will not receive any credit.





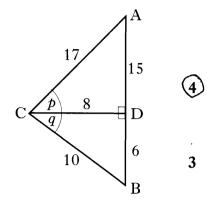
ALL questions should be attempted.

Marks

1. Find
$$\int \frac{4x^3 - 1}{x^2} dx$$
, $x \neq 0$.

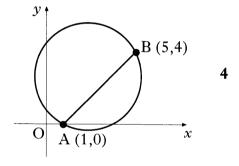


- **2.** Triangles ACD and BCD are right-angled at D with angles p and q and lengths as shown in the diagram.
 - (a) Show that the exact value of $\sin(p+q)$ is $\frac{84}{85}$.
 - (b) Calculate the exact values of:
 - (i) $\cos(p+q)$;
 - (ii) tan(p+q).



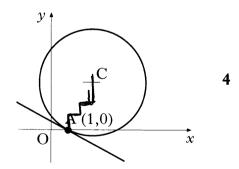
3. (a) A chord joins the points A(1,0) and B(5,4) on the circle as shown in the diagram.

Show that the equation of the perpendicular bisector of chord AB is x + y = 5.



(b) The point C is the centre of this circle. The tangent at the point A on the circle has equation x + 3y = 1.

Find the equation of the radius CA.



- (c) (i) Determine the coordinates of the point C.
 - (ii) Find the equation of the circle.

4

[Turn over

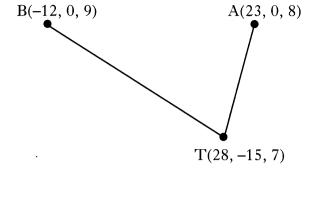
4. The sketch shows the positions of Andrew(A), Bob(B) and Tracy(T) on three hill-tops.

Relative to a suitable origin, the coordinates (in hundreds of metres) of the three people are A(23, 0, 8), B(-12, 0, 9) and T(28, -15, 7).

In the dark, Andrew and Bob locate Tracy using heat-seeking beams.



- (a) Express the vectors TA and TB in component form.
- (b) Calculate the angle between these two beams.

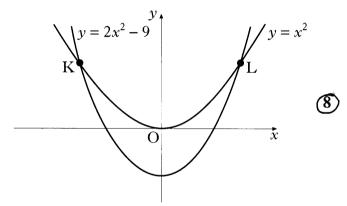


(2)

(5)

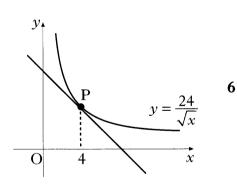
5. The curves with equations $y = x^2$ and $y = 2x^2 - 9$ intersect at K and L as shown.

Calculate the area enclosed between the curves.



6. The diagram shows the graph of $y = \frac{24}{\sqrt{x}}$, x > 0.

Find the equation of the tangent at P, where x = 4.



7. Solve the equation $\log_4(5-x) - \log_4(3-x) = 2$, x < 3.

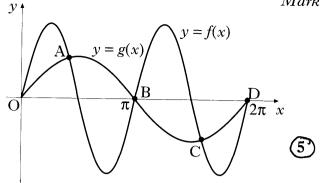
4

Marks

8. Two functions, f and g, are defined by $f(x) = k\sin 2x$ and $g(x) = \sin x$ where k > 1.

The diagram shows the graphs of y = f(x) and y = g(x) intersecting at O, A, B, C and D.

Show that, at A and C, $\cos x = \frac{1}{2k}$.



- 9. The value V (in £ million) of a cruise ship t years after launch is given by the formula $V = 252e^{-0.06335t}$.
 - (a) What was its value when launched?

1

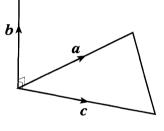
(b) The owners decide to sell the ship once its value falls below £20 million. After how many years will it be sold?

4

10. Vectors **a** and **c** are represented by two sides of an equilateral triangle with sides of length 3 units, as shown in the diagram.

Vector b is 2 units long and b is perpendicular to both a and c.

Evaluate the scalar product $a \cdot (a + b + c)$.



4

- 11. (a) Show that x = -1 is a solution of the cubic equation $x^3 + px^2 + px + 1 = 0$.
 - (b) Hence find the range of values of p for which all the roots of the cubic equation are real.

7

1

[END OF QUESTION PAPER]



2005 Mathematics

Higher

Finalised Marking Instructions

These Marking Instructions have been prepared by Examination Teams for use by SQA Appointed Markers when marking External Course Assessments.

Mathematics Higher

Instructions to Markers

- 1. Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than marks deducted for what is wrong.
- 2. Award one mark for each 'bullet' point. Each error should be underlined in RED at the point in the working where it first occurs, and not at any subsequent stage of the working.
- 3. The working subsequent to an error must be followed through by the marker with possible full marks for the subsequent working, provided that the difficulty involved is approximately similar. Where, subsequent to an error, the working is eased, a deduction(s) of marks(s) should be made.

This may happen where a question is divided into parts. In fact, failure to even answer an earlier section does not preclude a candidate from assuming the result of that section and obtaining full marks for a later section.

4. Correct working should be ticked (✓). This is essential for later stages of the SQA procedures. Where working subsequent to an error(s) is correct and scores marks, it should be marked with a crossed tick (✗ or ✗ ✓). In appropriate cases attention may be directed to work which is not quite correct (eg bad form) but which has not been penalised, by underlining with a dotted or wavy line.

Work which is correct but inadequate to score any marks should be corrected with a double cross tick (X).

- 5. The total mark for each section of a question should be entered in red in the **outer** right hand margin, opposite the end of the working concerned.
 - Only the mark should be written, **not** a fraction of the possible marks.
 - These marks should correspond to those on the question paper and these instructions.
- 6. It is of great importance that the utmost care should be exercised in adding up the marks. Where appropriate, all summations for totals and grand totals must be carefully checked.

Where a candidate has scored zero marks for any question attempted, "0" should be shown against the answer.

7. As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Accept answers arrived at by inspection or mentally where it is possible for the answer so to have been obtained. Situations where you may accept such working will normally be indicated in the marking instructions.

- 8. Do not penalise:
 - working subsequent to a correct answer
 - legitimate variations in numerical answers
 - correct working in the "wrong" part of the question
- omission of units
- bad form
- 9. No piece of work should be scored through without careful checking even where a fundamental misunderstanding is apparent early in the answer. Reference should always be made to the marking scheme answers which are widely off-beam are unlikely to include anything of relevance but in the vast majority of cases candidates still have the opportunity of gaining the odd mark or two provided it satisfies the criteria for the mark(s).
- 10. If in doubt between two marks, give an intermediate mark, but without fractions. When in doubt between consecutive numbers, give the higher mark.
- 11. In cases of difficulty covered neither in detail nor in principle in the Instructions, attention may be directed to the assessment of particular answers by making a referral to the PA. Please see the general instructions for PA referrals.
- 12. No marks should be deducted at this stange for careless or badly arranged work. In cases where the writing or arrangement is very bad, a note may be made on the upper left-hand corner of the front cover of the script.
- 13. Transcription errors: In general, as a consequence of a transcription error, candidates lose the opportunity of gaining either the first ic mark or the first pd mark.
- 14. Casual errors: In general, as a consequence of a casual error, candidates lose the opportunity of gaining either the first ic mark or the first pd mark.
- 15. **Do not write any comments on the scripts.** A **revised** summary of acceptable notation is given on page 4.

Summary

Throughout the examination procedures many scripts are remarked. It is essential that markers follow common procedures:

- 1. **Tick** correct working.
- 2. Put a mark in the right-hand margin to match the marks allocations on the question paper.
- 3. Do **not** write marks as fractions.
- 4. Put each mark **at the end** of the candidate's response to the question.
- 5. **Follow through** errors to see if candidates can score marks subsequent to the error.
- 6. Do **not** write any comments on the scripts.

Higher Mathematics: A Guide to Standard Signs and Abbreviations

Remember – No comments on the scripts. Please use the following and nothing else.

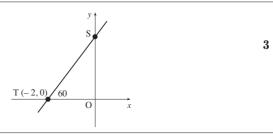
Signs √ X	The tick. You are not expected to tick every line but of course you must check through the whole of a response. The cross and underline. Underline an error and place a cross at the end of the line.	Bullets showing where marks have allotted may be shown on scripts $\frac{dy}{dx} = 4x - 7$ $4x - 7 = 0$ $x = \frac{7}{4}$	been
X or X ✓	The tick-cross. Use this to show correct work where you are following through subsequent to an error.	$y = 3\frac{7}{8}$ $C = (1,-1)$ $m = \frac{3 - (-1)}{4 - 1}$ X	2
		$m_{rad} = \frac{4}{3}$ $m_{tgt} = \frac{-1}{\frac{4}{3}}$	
		$m_{tgt} = -\frac{3}{4} \qquad \qquad \mathbf{X} \bullet$ $y - 3 = -\frac{3}{4}(x - 2) \qquad \qquad \mathbf{X} \bullet$	3
\wedge	The roof. Use this to show something is missing such as a crucial step in a proof of a 'condition' etc.	$x^{2} - 3x = 28$ $x = 7$	1
	The tilde. Use this to indicate a minor transgression which is not being penalised (such as bad form).	$\sin(x) = 0.75 = inv\sin(0.75) = 48.6^{\circ}$	1
*	The double-cross tick. Use this to show correct work but which is inadequate to score any marks. This may happen when working has been eased.		

Remember – No comments on the scripts. No abbreviations. No new signs. Please use the above and nothing else.

All of these are to help us be more consistent and accurate.

Note: There is no such thing as a transcription error, a trivial error, a casual error or an insignificant error. These are all mistakes and as a consequence a mark is lost.

Find the equation of the line ST, where T is the 1 point (-2, 0) and angle STO is 60.



Qu. part marks Grade Syllabus Code Calculator class Source NC G2, G3 05/6 1 3 C

The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- use $m = \tan \theta$
- pd use exact value
- interpret result

Primary Method : Give 1 mark for each •

- $m = \tan(60^{\circ})$ stated or implied by 2

3 marks

Notes

A candidate who states $\,m\,$ = $\, an(heta^{
m o})$, and does not go on to use it earns no marks.

Incompletion 1

$$m = \tan(60^{\circ})$$

 $y - 0 = \tan(60^{\circ})(x - (-2))$

award 2 marks

Common Error 1

$$m = \sin(60^{\circ})$$

 $y - 0 = \frac{\sqrt{3}}{2}(x - (-2))$

award 2 marks

Alternative Method 1

• OS =
$$2\tan(60^\circ) = 2\sqrt{3}$$

$$\bullet^2 \quad m = \frac{2\sqrt{3}}{2} = \sqrt{3}$$

$$\left(cf\ y = mx + c\right)$$

$$\bullet^3 \quad y = \sqrt{3}x + 2\sqrt{3}$$

Alternative Method 2

•
$$\cos(60^\circ) = \frac{2}{ST}$$
 leading to

$$ST = 4$$
 and $OS = \sqrt{12}$

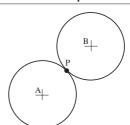
•
$$m = \frac{\sqrt{12}}{2}$$

•
$$y - 0 = \frac{\sqrt{12}}{2} (x - (-2))$$

2 Two congruent circles, with centres A and B, touch at P. Relative to suitable axes, their equations are

$$x^2 + y^2 + 6x + 4y - 12 = 0 \text{ and } x^2 + y^2 - 6x - 12y + 20 = 0 \; .$$

- (a) Find the coordinates of P.
- (b) Find the length of AB.



3 2

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
2	a	3	С	G9, G6	CN	05/18
	b	2	С	G9	CN	

The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- •¹ ic interpret equ. of circle
- 2 ic interpret equ. of circle
- ³ pd process midpoint
- ss know how to find length
- ⁵ pd process

Primary Method : Give 1 mark for each •

- centre A = (-3, -2) [Note 1]
- \bullet^2 centre B = (3,6)

3 marks

- 3 P = (0,2)
- 4 AB² = $(3 (-3))^{2} + (6 (-2))^{2}$ [CE 1]
 - AB = 10 [Note 2] 2 marks

Notes

1 at •1, •2

Each of the following may be awarded 1 mark from the first two marks

$$A = (6,4)$$
 and $B = (-6,-12)$

$$A = (-6, -4)$$
 and $B = (6, 12)$

$$A = (3,2)$$
 and $B = (-3,-6)$

At •5 stage, some errors lead to unsimplified surds. DO NOT accept unsimplified square roots of perfect squares (up to 100). e.g. √100 would not gain •5.

Alternative Method 1 for marks 1,2,3

$$\boldsymbol{p} = \frac{1}{2}(\boldsymbol{b} + \boldsymbol{a})$$

- $\bullet^1 \quad \boldsymbol{b} = \begin{array}{c} 3 \\ 6 \end{array}$
- $\bullet^2 \quad a = \begin{array}{c} -3 \\ -2 \end{array}$
- 3 P = (0,2) [Note 1]

Notes

1 Treat $P = \frac{0}{2}$ as bad form.

Common Error 1 for (b)

$$AB^2 = (3 + (-3))^2 + (6 + (-2))^2$$

$$AB = 4$$

$$\bullet^5 \times \sqrt{}$$

award 1 mark for (b)

Alternative Method 2 for marks 4,5

•⁴
$$r^2 = 3^2 + 2^2 - (-12)$$

or $r^2 = (-3)^2 + (-6)^2 - 20$

•
5
 AB = $2r = 10$

Alternative Method 3 for marks 4,5

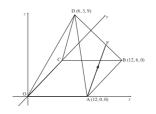
$$\bullet^4 \quad \overrightarrow{AB} = \begin{array}{c} 6 \\ 8 \end{array}$$

$$\bullet^5$$
 AB = 10

- $\mathbf{3}$ D,OABC is a pyramid. A is the point (12, 0,
 - 0), B is (12, 6, 0) and D is (6, 3, 9).

F divides DB in the ratio 2:1.

- (a) Find the coordinates of the point F.
- (b) Express AF in component form.



4

1

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
3	а	4	С	G25	CN	05/24
	b	1	C	G17	CN	

The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- •¹ ss know to find DB
- ic interpret ratio
- 3 pd process scalar times vector
- 4 ic interpret vector and end points
- ic interpret coordinates to vector

Primary Method: Give 1 mark for each •

$$\overrightarrow{DB} = 6 - 3$$

$$\bullet^2 \quad \overrightarrow{DF} = \frac{2}{2}\overrightarrow{DB}$$

$$\bullet^3 \quad \overrightarrow{DF} = \frac{2}{3} \quad 3 \quad = \quad 2$$

$$D = (6,3,9) \text{ so } F = (10,5,3)$$

4 marks

$$\bullet^5 \quad \overrightarrow{AF} = \begin{array}{c} -2 \\ 5 \\ 3 \end{array}$$

1 mark

Notes

- Do not penalise candidates who write the coordinates of F as a column vector (treat as bad form).
- 2 A correct answer to (a) with no working may be awarded one mark only.
- 3 For guessing the coordinates of F, no marks should be awarded in (a).
 - 1 mark is still available in (b) provided the guess in (a) is geographically compatible with the diagram

$$0 \le z \le 9$$

- 4 In (a)
 Where the ratio has been reversed (ie 1:2) leading to
 F=(8, 4, 6) then 3 marks may be awarded (•1, •3, •4).
- 5 In (b)

Accept AF = -2i + 5j + 3k for •5.

Alternative Method 1 [Marks 1-4]

$$\overrightarrow{DF} = 2\overrightarrow{FB}$$
 s/i by •2

$$\bullet^2 \quad \mathbf{f} - \mathbf{d} = 2\mathbf{b} - 2\mathbf{f}$$

•
$$3f = 2 \ 6 + 3$$
0 9

•
4
 F = $(10,5,3)$ [Note 1]

Alternative Method 3 [Marks 1-5]

$$\bullet^1 \quad \overrightarrow{AF} = \overrightarrow{AB} + \overrightarrow{BF}$$

$$\bullet^2 \quad \overrightarrow{AF} = \overrightarrow{AB} + \frac{1}{3}\overrightarrow{BD}$$

$$\bullet^3 \quad \overrightarrow{AF} = \begin{array}{c} 0 & 6 & 12 \\ 6 & +\frac{1}{3} & 3 & -6 \\ 0 & 0 & 0 \end{array}$$

$$\bullet^4 \quad \overrightarrow{AF} = 5$$
3

•
$$(A = (12,0,0 \text{ so}) \text{ F} = (10,5,3)$$

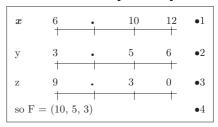
Alternative Method 2 [Marks 1-4]

$$\bullet^1 \quad f = \frac{mb + nd}{m + n} \quad \text{s/i by } \cdot 3$$

•
$$m = 2, n = 1$$
 s/i by •3

•
4
 F = $(10,5,3)$ [Note 1]

Alternative Method 4 [Marks 1-4]



- Functions f(x) = 3x 1 and $g(x) = x^2 + 7$ are defined on the set of real numbers.
 - (a) Find h(x) where h(x) = g(f(x)).

2

- (b) (i) Write down the coordinates of the minimum turning point of y = h(x).
 - (ii) Hence state the range of the function h.

2

Qu.partmarksGradeSyllabus CodeCalculator class4a2CA4NCb2CA1NC	s Source 05/7
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The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- •¹ ic interpret comp. function build-up
- •² ic interpret comp. function build-up
- ic interpret function
- ic interpret function

Primary Method: Give 1 mark for each •

 \bullet^1 g(3x-1)

stated or implied by ·2

 $\bullet^2 \left(3x-1\right)^2 + 7$

2 marks

• $\frac{1}{3}$, $\frac{7}{3}$

[Note 1]

- $\bullet^4 \quad y \ge 7$
- [Note 2]
- 2 marks

Notes

1 For •3

No justification is required for •3. Candidates may choose to dfferentiate etc but may still only earn one mark for a correct answer.

2 For •4

Accept $y>7,\ h\geq 7,\ h>7,\ h(x)>7,\ h(x)\geq 7$ Do not accept $x\geq 7,\ x>7$

Common Error No.1

$$\bullet^1 \times f(x^2 + 7)$$

•
$$^2 \times J = 3x^2 + 20$$

$$\bullet^3 \times \checkmark$$
 $(0,20)$

$$\bullet^4 \times \checkmark \qquad y \ge 20$$

award 3 marks

Notes 1 & 2 apply.

Differentiate $(1+2\sin(x))^4$ with respect to x. $\mathbf{5}$

 $\mathbf{2}$

Qu. part marks 5

Grade

Syllabus Code C20, C21

Calculator class CN

Source 05/28

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- pd start differentiation process
- pd use the chain rule

Primary Method: Give 1 mark for each .

- $4(1 + 2\sin(x))^3$
- $\dots \times 2\cos(x)$

2 marks

Common Error 1

$$\bullet^1$$
 x

$$1 + 2\sin^4(x)$$

$$\bullet^2 \times \bullet$$

$$8\sin^3(x) \times \cos(x)$$

award 1 mark

Common Error 2

$$\bullet^1$$
 ×

$$1 + 16\sin^4(x)$$

$$\bullet^2 \times J$$

$$e^2 \times \checkmark \qquad 64\sin^3(x) \times \cos(x)$$

award 1 mark

Common Error 3

[mixture of differentiating and integrating]

$$\frac{1}{4}\Big(1+2\sin(x)\Big)^3$$

$$\times \frac{1}{2}\cos(x)$$

 $award\ 0\ marks$

Common Error 4

$$\bullet^1$$
 :

$$4(1+2\sin(x))^5$$

$$\bullet^2$$
 ×

$$\times 2\cos(x)$$

award 1 mark

- 6 The terms of a sequence satisfy $u_{n+1} = ku_n + 5$. Find the value of k which produces a sequence (a) with a limit of 4.
- 2

5

- A sequence satisfies the recurrence relation $u_{n+1} = mu_n + 5$, $u_0 = 3$.
 - Express u_1 and u_2 in terms of m.
 - Given that $u_2 = 7$, find the value of m which produces a sequence with no limit. (ii)

Qu. 6	part marks Grade Syllabus Code a 2 C A13 b 5 B A11, A13
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The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- know how to find limit
- pd process
- ic interpret rec. relation
- interpret rec. relation
- pd arrange in standard form
- pd process a quadratic
- use limit condition

Primary Method: Give 1 mark for each .

 $e.g. 4 = k \times 4 + 5$

 $u_1 = 3m + 5$

[Notes 1,2,3]

2 marks

- $u_2 = m(3m + 5) + 5$ [Note 4] $\left(m(3m+5)+5=7\right)$
- $3m^2 + 5m 2 = 0$ [Note 5]
- (3m-1)(m+2)=0
- m = -2

5 marks

Notes

for (a)

Guess and Check

Guessing k = -0.25 and checking algebraically or iteratively that this does yield a limit of 4 may be awarded 1

No working

Simply stating that k = -0.25 earns no marks.

Wrong formula

Work using an incorrect 'formula' leading to a valid value of k (ie |k| < 1) may be awarded 1 mark.

for (b)

- If $\boldsymbol{u_2}$ is not a quadratic, then no further marks are available.
- 5 An "=0" must appear at least once in working at the •5/•6 stage.
- For candidates who make errors leading to no values outside the range -1 < m < 1, or to two values outside the range, then they must say why they are accepting or rejecting in order to gain •7
- For •7, either crossing out the "1/3" or underlining the "-2" is the absolute minimum communication required for this i/c mark. [A statement would be preferable]

Alternative Method 1 for (a)

Using
$$L = \frac{b}{1-a}$$

- $\bullet^1 \quad 4 = \frac{5}{1 k}$

Alternative Method 2 for (a)

$$L = kL + 5$$

$$kL = L - 5$$

$$\bullet^1$$
 $k = \frac{L-5}{I}$

$$\bullet^2 \quad k = \frac{4-5}{4} = -\frac{1}{4}$$

Common Error 1

$$\bullet^1 \times 4 = \frac{5}{1-a}$$

 $\times \sqrt{a} = -\frac{1}{4}$

award 1 mark

 $\bullet^3 \quad \checkmark \quad u_{_1} = 3m + 5$

 $\bullet^4 \quad \times \quad u_2 = 3m^2 + 5$

 $\bullet^5 \times 3m^2 = 2$

or equivalent

 $\bullet^6 \times m = \sqrt{\frac{2}{3}}$ (eased)

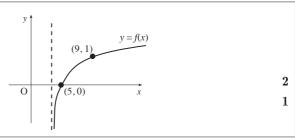
 $\times \checkmark$ there are no values which do not yield a limit

award 2 marks

7 The function f is of the form $f(x) = \log_b(x - a)$.

The graph of y = f(x) is shown in the diagram.

- (a) Write down the values of a and b.
- (b) State the domain of f.



Qu. 7	part a b	marks 2 1	Grade C C	Syllabus Code A7 A1	Calculator class NC NC	Source 05/9	
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The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- ic interpret the translation
- ic interpret the base
- ic interpret diagram

Prim	ary Method : Give 1 m	ark for each •	
•1 •2	a = 4 $b = 5$ domain is $x > a$	[Note 1]	2 marks 1 mark

Notes

 No justification is required for marks 1 and 2. BUT simply stating

$$0 = \log_b \left(5 - a \right) \ and \ 1 = \log_b \left(9 - a \right)$$

with no further work earns no marks.

However

$$1 = \log_b(9 - a)$$
 and $b = 9 - a$

may be awarded 1 mark.

Of course to gain the other mark, both values would need to be stated.

2 Clearly x > 4 is correct

but **do not** accept a domain of $x \ge 4$.

Higher Mathematics 2005 Paper 1: Marking Scheme Version 4

 $\mathbf{5}$

2

 $\mathbf{5}$

- A function f is defined by the formula $f(x) = 2x^3 7x^2 + 9$ where x is a real number.
 - Show that (x-3) is a factor of f(x), and hence factorise f(x) fully.
 - Find the coordinates of the points where the curve with equation y = f(x) crosses (b) the x- and y-axes.
 - Find the greatest and least values of f in the interval $-2 \le x \le 2$. (c)

|--|

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- know to use x = 3
- pd complete strategy
- \bullet^3 ic interpret zero remainder
- interpret quadratic factor
- pd complete factorising

Primary Method: Give 1 mark for each •

- -7 0
- 3 9 6 -9 0
- remainder is zero so (x 3) is a factor [Note 1]
- $2x^2 x 3$
- (x-3)(2x-3)(x+1) stated explicitly 5 marks

Notes

In the Primary method, (a)

- 1 Candidates must show some acknowledgement of the result of the synthetic division. Although a statement w.r.t. the zero is preferable, accept something as simple as "underlining" the zero.
- 2 Candidates may use a second synthetic division to complete the factorisation. •4 and •5 are available.

Alternative method 1 (marks 1-5) (linear factor by substitution)

- f(3) = ...
- $f(3) = 2 \times 3^3 7 \times 3^2 + 9 = 54 63 + 9 = 0$
- $eg \ 3 \ 2 \ -7 \ 0 \ 9$ -1 -3 0
- $2x^2 x 3$
- (x 3)(2x 3)(x + 1)

Alternative method 3 (marks 1-5) (quad factor by inspection)

- f(3) = ...
- $f(3) = 2 \times 3^3 7 \times 3^2 + 9 = 54 63 + 9 = 0$
- \bullet^3 $(x-3)(2x^2 \dots)$
- $\bullet^4 (x-3)(2x^2-x-3)$
- \bullet^5 (x-3)(2x-3)(x+1)

Alternative method 2 (marks 1-5) (long division)

- \bullet^4 $(x-3)(2x^2-x-3)$
- \bullet^5 (x-3)(2x-3)(x+1)

- A function f is defined by the formula $f(x) = 2x^3 7x^2 + 9$ where x is a real number.
 - (a) Show that (x-3) is a factor of f(x), and hence factorise f(x) fully.
 - (b) Find the coordinates of the points where the curve with equation y = f(x) crosses the x- and y-axes.
 - (c) Find the greatest and least values of f in the interval $-2 \le x \le 2$.
 - \bullet^6 ic interpret y-intercept
 - \bullet^7 ic interpret x-intercepts
 - •⁸ ss set derivative to zero
 - 9 pd solve
 - •¹⁰ ss evaluate function at an end point
 - •¹¹ ic interpret results
 - •¹² ic interpret results

Primary Method: Give 1 mark for each •

- \bullet^6 (0,9)
- \bullet^7 $(-1,0),(\frac{3}{2},0),(3,0)$
- [Note 3] 2 marks

- $6x^2 14x = 0$
- $\bullet^9 \quad x = 0 \text{ or } x = \frac{14}{6}$
- [Note 6]
- 10 $f(-2) = -35 \ OR \ f(2) = -3$
- \bullet^{11} greatest value = 9
- 12 least value = -35
- [Note 7]

5 marks

 $\mathbf{5}$

2

 $\mathbf{5}$

Notes

In the Primary method (b)

- 3 Only coordinates are acceptable for full marks. Simply stating the values at which it cuts the x- and yaxes may be awarded 1 mark (out of 2).
- 4 If all the coordinates are "round the wrong way" award 1 mark.
- 5 If the brackets are missing, treat as bad form.

In the Primary method (c)

- 6 Ignore any attempt to evaluate function at x = 7/3.
- 7 •11 and •12 are not available unless both end points and the st. points have been considered.

- In the Alt.5 method (c)
- •12 is not available unless both end points have been considered.

In (c)

9 Some candidates simply draw up a table using integer values from -2 to 2 and make conclusions from it. This earns •9 (Primary) ONLY, provided that one of the end points is correct.

Alternative method 5 (marks 8-12) (nature table)

- $\bullet^8 \quad 6x^2 14x = 0$
- $\bullet^9 \quad x = 0 \text{ or } x = \frac{14}{6}$
- [Note 6]
- nature table showing x = 0 is max. tp and the greatest (maximum) value is 9
- \bullet^{11} $f(-2) = -35 \ \mathbf{OR} \ f(2) = -3$
- 12 least value = -35
- [Note 8]

If $\cos(2x) = \frac{7}{25}$ and $0 < x < \frac{\pi}{2}$, find the exact values of $\cos(x)$ and $\sin(x)$.

Qu. part marks Grade Syllabus Code Calculator class Source 05/16

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- use double angle formula
- pd process
- process
- pd process

 $2\cos^2(x) - 1 = \frac{7}{25}$

Primary Method: Give 1 mark for each •

- $\cos^2(x) = \frac{32}{50}$
- $\cos(x) = \frac{4}{5}$
- $\sin(x) = \frac{3}{5}$

4 marks

b

Notes

- In the event of $\cos^2(x) \sin^2(x)$ being used, no marks are available until the equation reduces to a quadratic in either $\cos(x)$ or $\sin(x)$.
- $\cos(x) = \pm \frac{4}{5}, \sin(x) = \pm \frac{3}{5} \text{ loses } \cdot 3.$
- ·3 and ·4 are only available as a consequence of 3 attempting to apply the double angle formula. (This note does note apply to alt. method 2)
- Guess and Check.

For guessing that $\cos(x) = \frac{4}{5}$ and $\sin(x) = \frac{3}{5}$,

substituting them into any valid expression for $\cos(2x)$ and getting 7/25, award 1 mark only.

Alternative Method 1

- e^{1} 1 $2\sin^{2}(x) = \frac{7}{25}$
- $e^2 \sin^2(x) = \frac{18}{50}$
- $\sin(x) = \frac{3}{5}$
- $\cos(x) = \frac{4}{5}$

Alternative Method 2

- (7,24,25) triangle a + b = 24and angle bisector $\frac{a}{b} = \frac{7}{25}$
- $a + \frac{25}{7}a = 24$ $a = \frac{21}{4}$
- (21,28,35) triangle $t = \frac{35}{4}$
- $\cos(x) = \frac{4}{5}$ and $\sin(x) = \frac{3}{5}$

Common Error 1

Common Incompletion 1

- $\int 2\cos^2(x) 1 = \frac{7}{25}$
- $\bullet^3 \quad \times \quad \cos(x) = \sqrt{\frac{32}{50}}$
- $\bullet^4 \times \sqrt{\sin(x)} = \sqrt{\frac{18}{50}}$

award 3 marks

- 10 (a) Express $\sin(x) \sqrt{3}\cos(x)$ in the form $k\sin(x-a)$ where k>0 and $0 \le a \le 2\pi$.
 - (b) Hence, or otherwise, sketch the curve with equation $y = 3 + \sin(x) \sqrt{3}\cos(x)$ in the interval $0 \le x \le 2\pi$.

Qu. 10

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- \bullet^1 ic expand
- \bullet^2 ic compare coefficients
- \bullet^3 pd process k
- 4 pd process angle
- \bullet^5 ic state equation
- ic completing graph
- •⁷ ic completing graph
- •⁸ ic completing graph
- ⁹ ic completing graph

Notes

In the whole question

Do not penalise more than once for not using radians.

In (a)

- 1 $k(\sin(x)\cos(a) \cos(x)\sin(a))$ is acceptable for •1
- 2 No justification is required for •3
- 3 $^{\circ 3}$ is not available for an unsimplified $\sqrt{4}$

$$2\left(\sin(x)\cos(a) - \cos(x)\sin(a)\right)$$

or $2\sin(x)\cos(a) - 2\cos(x)\sin(a)$ is acceptable for•1 and •3

- Candidates may use any form of the wave equation to start with as long as their final answer is in the form $k\sin(x-a)$. If it is not, then 4 is not available.
- 6 •4 is only available for an answer in radians.
- 7 Treat $k\sin(x)\cos(a) \cos(x)\sin(a)$ as bad form only if •2 is gained.

In (b)

- 8 The **correct** sketch need not include annotation of max, min or intercept for •5 to be awarded but you would need to see the graph lying between y=1 and y=5.
- 9 •6 is available for one cycle of any sinusoidal curve of period 2π except $y=\sin(x)$. Some evidence of a scale is required.
- 10 For •7, accept 1·3 in lieu of $3 \sqrt{3}$
- 11 Do not penalise graphs which go beyond the interval $0...2\pi$.

Primary Method: Give 1 mark for each •

- \bullet^1 $k\sin(x)\cos(a) k\cos(x)\sin(a)$
- $\bullet^2 \quad k\cos(a) = 1, k\sin(a) = \sqrt{3}$
- \bullet^3 k=2
- $\bullet^4 \quad a = \frac{\pi}{3}$

STATED EXPLICITLY
STATED EXPLICITLY

5

- [Notes 1-7]
 - 4 marks
- $y = 3 + 2\sin x \frac{\pi}{3}$
- stated or implied by a correct sketch [Note 8]
- a sketch showing
- [Notes 9,10]
- a sinusoidal curve
- 7 y-intercept at $(0, 3 \sqrt{3})$ and no x-intercepts
- max at $\frac{5\pi}{6}$, 5

5 marks

• min at $\frac{11\pi}{6}$, 1

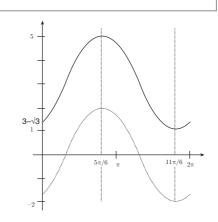
Alternative marking for ·8 and ·9

- •8 max at $x = \frac{5\pi}{6}$ and min at $x = \frac{11\pi}{6}$
- graph lies between y = 1 and y = 5

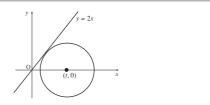
Alternative method for .5 to .9 (Calculus)

- $\bullet^5 \quad \frac{dy}{dx} = \cos(x) + \sqrt{3}\sin(x) = 0$
- $\bullet^6 \quad \tan(x) = -\frac{1}{\sqrt{3}}$
- $\bullet^7 \quad \max at\left(\frac{5\pi}{6},5\right)$
- $\bullet^8 \quad \min at \left(\frac{11\pi}{6},1\right)$
- $\bullet^9 \quad x = 0 \quad y = 3 \sqrt{3}$

and annotated sketch.



- A circle has centre (t, 0), t > 0, and radius 2 units. 11 Write down the equation of the circle.
 - Find the exact value of t such that the line y = 2x is (*b*) a tangent to the circle.



Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
11	a	1	С	G10	CN	05/28
	b	4	Α	G13	CN	

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- ic state equ. of circle
- ss substitute
- pd rearrange in standard form.
- know to use "discriminant = 0"
- identify "a","b" and "c"
- pd process

Primary Method : Give 1 mark for each •

$$\bullet^1$$
 $(x-t)^2 + (y-0)^2 = 2^2$

1 mark

1

 $\mathbf{5}$

$$\bullet^2 (x-t)^2 + (2x)^2 = 4$$

$$-3$$
 $5x^2 - 2tx + t^2 - 4 = 0$

$$\bullet^4 \quad "b^2 - 4ac" = 0$$

[Note 1]

$$\bullet^5$$
 $a = 5, b = -2t, c = t^2 - 4$

$$\bullet^6$$
 $4t^2 - 20(t^2 - 4) = 0$

and
$$t = \sqrt{5}$$

[Note 2]

5 marks

Notes

- Subsequent to trying to use an expression masquerading as the discriminant e.g. $a^2 - 4bc = 0$, only •5 (from the last two marks) is still available.
- Treat $t = \pm \sqrt{5}$ as bad form.

Common Error No. 1

$$\bullet^5 \times a = 5, b = -2, c = t^2 - 4$$

$$\bullet^6$$
 4 - 20(t^2 - 4) = 0

$$20t^2 = 84$$

$$\times \checkmark t = \sqrt{\frac{21}{5}} \text{ or } \sqrt{4.2}$$

Alternative Method 1 (for (b))

Let P be point of contact, C the centre of the circle. Consider triangle OPC.

- OPC = 90 (tg
 PC = 2 (radius) (tgt/radius)
- CP/OP = tan(COP) = 2 (gradient of tgt)
- Hence OP = 1
- and, by Pythagoras, $t = OC = \sqrt{(2^2 + 1^2)} = \sqrt{5}$.

Alternative Method 2 (for (b))

$$y = 2x$$
 $m_{tgt} = 2$ and $m_{rad} = -\frac{1}{2}$

• equ of radius is x + 2y = t

$$ie \ x - t = -2y$$

$$\bullet^3 \quad \left(-2y\right)^2 + y^2 = 4$$

$$\bullet^4 \quad y = \frac{2}{\sqrt{5}}$$

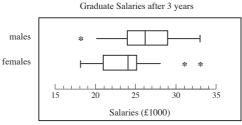
$$\bullet^5 \quad x = \frac{1}{2}y \quad x = \frac{1}{\sqrt{5}}$$

$$\bullet^6 \quad t = x + 2y \quad t = \sqrt{5}$$

Higher Mathematics 2005 Paper 1: Marking Scheme Version 4

- S1The boxplot shows the salaries of male and
- female graduates working for a large [3] company at the end of their third year of employment.

Compare the salaries of these males and females.



Salaries (x £1000)

irce 70	culator class	Syllabus Code 4.1.3/4	Grade C	marks 3	part	Qu. S1
------------	---------------	--------------------------	------------	------------	------	-----------

- ic comment
- comment
- comment

- one comment from list
- one comment from list
- one comment from list males had higher salaries on average by £2000

range of salaries is broadly similar

only 2 females achieved same salary as top 25% males majority of males earned more than the average female any other reasonable comment

- S2A bag contains 4 blue and 2 red counters. 2 counters are drawn at random without replacement.
- The random variable X is the number of blue counters drawn. [5]
 - Find the probability distribution for X.
 - (b) Find E(X).

marks Grade Syllabus Code Calculator class Source 05/22 C 4.2.11/12 NC

- know to find P(X=0) etc
- process

part

Qu.

S2

- process
- process
- choose correct form
- process

- $5t^k$ 0 < t < 1S3A continuous random variable T has probability density function f(t) =otherwise. [10]
 - (a) Find the value of k.
- Calculate $P(0 < T < \frac{1}{2})$.

4,3

4 marks

2 marks

3

3 marks

4 $\mathbf{2}$

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
S3		6	В	4.3.2	CN	05/65

- know
- pd process
- process
- process
- use $\int 5t^{\kappa}dt$
- pd integrate
- pd process limits

- f(t)dt = 1

- - 4 marks
- - 3 marks

Mathematics Higher

Instructions to Markers

- 1. Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than marks deducted for what is wrong.
- 2. Award one mark for each 'bullet' point. Each error should be underlined in RED at the point in the working where it first occurs, and not at any subsequent stage of the working.
- 3. The working subsequent to an error must be followed through by the marker with possible full marks for the subsequent working, provided that the difficulty involved is approximately similar. Where, subsequent to an error, the working is eased, a deduction(s) of marks(s) should be made.

This may happen where a question is divided into parts. In fact, failure to even answer an earlier section does not preclude a candidate from assuming the result of that section and obtaining full marks for a later section.

4. Correct working should be ticked (✓). This is essential for later stages of the SQA procedures. Where working subsequent to an error(s) is correct and scores marks, it should be marked with a crossed tick (✗ or ✗ ✓). In appropriate cases attention may be directed to work which is not quite correct (eg bad form) but which has not been penalised, by underlining with a dotted or wavy line.

Work which is correct but inadequate to score any marks should be corrected with a double cross tick (X).

- 5. The total mark for each section of a question should be entered in red in the **outer** right hand margin, opposite the end of the working concerned.
 - Only the mark should be written, **not** a fraction of the possible marks.
 - These marks should correspond to those on the question paper and these instructions.
- 6. It is of great importance that the utmost care should be exercised in adding up the marks. Where appropriate, all summations for totals and grand totals must be carefully checked.

Where a candidate has scored zero marks for any question attempted, "0" should be shown against the answer.

7. As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Accept answers arrived at by inspection or mentally where it is possible for the answer so to have been obtained. Situations where you may accept such working will normally be indicated in the marking instructions.

- 8. Do not penalise:
 - working subsequent to a correct answer
 - legitimate variations in numerical answers
 - correct working in the "wrong" part of the question
- omission of units
- bad form
- 9. No piece of work should be scored through without careful checking even where a fundamental misunderstanding is apparent early in the answer. Reference should always be made to the marking scheme answers which are widely off-beam are unlikely to include anything of relevance but in the vast majority of cases candidates still have the opportunity of gaining the odd mark or two provided it satisfies the criteria for the mark(s).
- 10. If in doubt between two marks, give an intermediate mark, but without fractions. When in doubt between consecutive numbers, give the higher mark.
- 11. In cases of difficulty covered neither in detail nor in principle in the Instructions, attention may be directed to the assessment of particular answers by making a referral to the PA. Please see the general instructions for PA referrals.
- 12. No marks should be deducted at this stange for careless or badly arranged work. In cases where the writing or arrangement is very bad, a note may be made on the upper left-hand corner of the front cover of the script.
- 13. Transcription errors: In general, as a consequence of a transcription error, candidates lose the opportunity of gaining either the first ic mark or the first pd mark.
- 14. Casual errors: In general, as a consequence of a casual error, candidates lose the opportunity of gaining either the first ic mark or the first pd mark.
- 15. **Do not write any comments on the scripts.** A **revised** summary of acceptable notation is given on page 4.

Summary

Throughout the examination procedures many scripts are remarked. It is essential that markers follow common procedures:

- 1. **Tick** correct working.
- 2. Put a mark in the right-hand margin to match the marks allocations on the question paper.
- 3. Do **not** write marks as fractions.
- 4. Put each mark **at the end** of the candidate's response to the question.
- 5. **Follow through** errors to see if candidates can score marks subsequent to the error.
- 6. Do **not** write any comments on the scripts.

Higher Mathematics: A Guide to Standard Signs and Abbreviations

Remember – No comments on the scripts. Please use the following and nothing else.

Signs √ X	The tick. You are not expected to tick every line but of course you must check through the whole of a response. The cross and underline. Underline an error and place a cross at the end of the line.	Bullets showing where marks have allotted may be shown on scripts $\frac{dy}{dx} = 4x - 7$ $4x - 7 = 0$ $x = \frac{7}{4}$	been
X or X ✓	The tick-cross. Use this to show correct work where you are following through subsequent to an error.	$y = 3\frac{7}{8}$ $C = (1,-1)$ $m = \frac{3 - (-1)}{4 - 1}$ X	2
		$m_{rad} = \frac{4}{3}$ $m_{tgt} = \frac{-1}{\frac{4}{3}}$	
		$m_{tgt} = -\frac{3}{4} \qquad \qquad \mathbf{X} \bullet$ $y - 3 = -\frac{3}{4}(x - 2) \qquad \qquad \mathbf{X} \bullet$	3
\wedge	The roof. Use this to show something is missing such as a crucial step in a proof of a 'condition' etc.	$x^{2} - 3x = 28$ $x = 7$	1
	The tilde. Use this to indicate a minor transgression which is not being penalised (such as bad form).	$\sin(x) = 0.75 = inv\sin(0.75) = 48.6^{\circ}$	1
*	The double-cross tick. Use this to show correct work but which is inadequate to score any marks. This may happen when working has been eased.		

Remember – No comments on the scripts. No abbreviations. No new signs. Please use the above and nothing else.

All of these are to help us be more consistent and accurate.

Note: There is no such thing as a transcription error, a trivial error, a casual error or an insignificant error. These are all mistakes and as a consequence a mark is lost.

Find $\int \frac{4x^3-1}{x^2} dx$, $x \neq 0$.

4

- Qu.
- part marks
- Grade
- Syllabus Code C14, C13
- Calculator class
- Source 05/20

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- arrange in integrable form
- pd: integrate positive index
- pd: integrate negative index
- complete including const. of int.

Primary Method : Give 1 mark for each •

- - [Note 3]

4 marks

Notes

- If incorrectly expressed in integrable form, follow throughs must match the generic marking scheme.
- •3 can only be awarded on follow through provided the integral involves a negative index.
- ·4 can only be awarded if the constant of integration appears somewhere in the working.
- ·4 can only be awarded as a result of at least one valid integration at the •2 or •3 stage.

Common Error 1

- 4x-1
- $[see Generic \bullet^3]$
- ward 2 mar

Common Error 2

- 3 mar

Common Error 3

- $4x^3 1 + x^{-2}$

- award m a

Common Error 4

$$\frac{x^4 - x}{\frac{1}{2}x^2} + c$$

ward

marks

Common Error 5

$$\left(x - 1 \right) x^{-2}$$

$$\left(2x^2 - x \right) \left(-x^{-1} \right) + c$$

award

marks

Common Error 6

$$(4x^3 - 1)x^{-2}$$

 $(x^4 - x)(-x^{-1}) + c$

k & ward I marks

Throughout this paper, unless specifically mentioned, a correct answer with no working receives no credit.

- Triangles ACD and BCD are right-angled at D with angles p and q and lengths as shown in the diagram.
 - Show that the exact value of $\sin(p+q)$ is $\frac{84}{8}$.
 - (b) Calculate the exact values of
 - (i)
- $\cos(p+q)$ (ii) $\tan(p+q)$.

rapei 2 . Maik	ing Scheme versi
	A
17	15
$C \stackrel{p}{{}{}} 8$	D
10	6
10	В

Qu. 2	part a b	marks 4 3	Grade C C	Syllabus Code T9 T9	Calculator class CN CN	Source 05/41	
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The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- ic: interpret diagram
- ic: interpret diagram
- ss: expand $\sin(A+B)$
- pd: sub. and complete
- ss: expand $\cos(A+B)$
- pd: sub. and complete
- use tan(x) = sin(x) / cos(x)

Primary Method: Give 1 mark for each .

- •¹ $\cos(p) = \frac{8}{17}, \sin(p) = \frac{15}{17}$ [Note 1]
 •² $\cos(q) = \frac{8}{10}, \sin(q) = \frac{6}{10}$ stated or implied by •4 when written in the same order as •3
- $\bullet^3 \quad \sin(p)\cos(q) + \cos(p)\sin(q) \qquad \text{explicitly stated}$ $\bullet^4 \quad \frac{15}{17} \times \frac{8}{10} + \frac{8}{17} \times \frac{6}{10} = \& \ complete \qquad \qquad 4$ 4 marks

- $\begin{array}{ll} \bullet^5 & \cos(p)\cos(q) \sin(p)\sin(q) \\ \bullet^6 & -\frac{13}{85} \ or \ equivalent \ fraction \\ \bullet^7 & -\frac{84}{13} \ or \ equivalent \ fraction \left(eg \frac{7140}{1105}\right) \end{array}$
 - 3 marks

4

3

Notes

- ·1 and ·2 may, if necessary, be awarded as follows
 - $\bullet^1 \quad \sin(p) = \frac{15}{17}, \sin(q) = \frac{6}{10}$
 - \bullet^2 $\cos(p) = \frac{8}{17}, \cos(q) = \frac{8}{10}$
- For •4 There has to be some working to show the completion.

- Calculating approx angles using invsin and invcos can gain no credit at any point.
- 4 Any attempt to use $\sin(p+q) = \sin(p) + \sin(q)$ loses ·3

Any attempt to use $\cos(p+q) = \cos(p) + \cos(q)$ loses •5 and ·6.

This second option must not be treated as a repeated error.

Alternative 1 (for marks 3 & 4)

- $\bullet^3 \quad \frac{21}{\sin(p+q)} = \frac{10}{\frac{8}{2}}$
- •⁴ $10\sin(p+q) = \frac{168}{17}$ and complete

Alternative 2 (for marks 5 & 6)

- $\cos(p+q) = \frac{17^2 + 10^2 21^2}{21710}$

Alternative 3 (for marks 5 & 6)

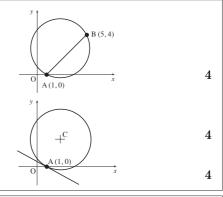
- $\cos^2(p+q) = 1 \left(\frac{84}{85}\right)^2$
- •6 $\cos(p+q) = -\frac{13}{85}$ with justification of the choice of negative sign e.g. $(15+6)^2$ (= 441) > $17^2 + 10^2$ (= 389)

or using the cosine rule

3 (a) A chord joins the points A(1, 0) and B(5, 4) on the circle as shown in the diagram.

Show that the equation of the perpendicular bisector of chord AB is x+y=5.

- (b) The point C is the centre of this circle. The tangent at the point A on the circle has equation x+3y=1. Find the equation of the radius CA.
- (c) (i) Determine the coordinates of the point C.
 - (ii) Find the equation of the circle.



Qu. 3	part a b c	marks 4 4 4	Grade C C C	Syllabus Code G7 G15 G10	Calculator class CN CN CN	Source 05/44	
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- •¹ ss: find perp. bisector
- ² pd: calc. perp. gradient
- ss: find approp. mid-point
- ic: complete proof
- ss: compare with y = mx + c
- ic: state gradient
- ss: find gradient of radius
- ic: state equation of line
- \bullet^9 ss: solve sim. equations
- ¹⁰ pd: solve sim. equations
- •¹¹ ic: state equation of circle
- 12 pd: calculate radius

Notes

1 To gain •4 some evidence of completion needs to be shown

eg
$$y-2=-1(x-3)$$

$$y-2=-x+3$$

$$y+x=5$$

- 2 •4 is only available if an attempt has been made to find and use both a perpendicular gradient and a midpoint.
- 3 •8 is only available if an attempt has been made to find and use a perpendicular gradient.
- 4 At the •9, •10 stage
 Guessing (2,3) (from stepping) and checking it lies on perp.
 bisector of AB may be awarded •9 and •10

bisector of AB may be awarded •9 and •10
Guessing (2,3) (with or without reason) and with no check
gains neither •9 nor •10

- 5 Solving y = 3x 3 and x + 3y = 1 leading to (1,0) will lose **·9 and ·10**.
- 6 to gain •12 some evidence of use of the distance formula needs to be shown.
- 7 At the •11 and •12 stage Subsequent to a guess for the coordinates of C, •11 and •12 are only available if the guess is such that 0<x<5 and 0<y<4.</p>

Primary Method : Give 1 mark for each •

- \bullet^1 $m_{AB} = 1$
- \bullet^2 $m_{\perp} = -1$
- \bullet^3 midpoint = (3,2)
- $ullet^4 \quad y-2=-1(x-3)$ a roonsplete [Notes 1,2] 4 marks
- $ullet^5 \quad y = -rac{1}{2}x.....$ stated/implied by •6
- \bullet^6 $m_{tgt} = -\frac{1}{3}$
- $ullet^7 \quad m_{_{rad}}=3$ stated/implied by •8
 - y 0 = 3(x 1) [Note 3]

4 marks

4 marks

- $use \ x + y = 5$ and $use \ x + y = 3$ [Notes 4,5]
- x = 2, y = 3
- $\bullet^{11} (x-2)^2 + (y-3)^2 = r^2$
 - $r^2 = 10$ [Note 6]

Alternative 1 [for ·9 and ·10]

- D=(3,6) where D is intersection of the perp. to AB through B and the circle.
- • 10 C = midpoint of AD = (2,3)

Common Error 1 [for ·5 to ·8]

$$3y = -x + 1$$

$$m = -1$$

$$m_{rad} = 1$$

$$y - 0 = 1(x - 1)$$
•5 × •6 × •7 × eased •8 × $\sqrt{}$
a ward 1 mark

Common Error 2 [for ·5 to ·8]

$$x+3y=1 \ so \ m=3$$

$$y-0=3(x-1)$$
 a ward 0 marks 7

The sketch shows the positions of Andrew(A), Bob (B) and Tracy(T) on three hill-tops.

Relative to a suitable origin, the coordinates (in hundreds of metres) of the three people are A(23, 0, 8), B(-12, 0, 9) and T(28, -15, 7). In the dark, Andrew and Bob locate Tracy using heat-seeking beams.



- Express the vectors TA and TB in component form. (a)
- (b) Calculate the angle between these two beams.

	2
	5

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
4	a	2	С	G17	CN	05/55
	b	5	С	G28	Ca	

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- \bullet^1 state vector components
- state vector components
- \bullet^3 pd: find length of vector
- pd: find length of vector
- \bullet^5 pd: find scalar product
- use scalar product
- pd: evaluate angle

Notes In (a)

- For calculating \overrightarrow{AT} and \overrightarrow{BT} award 1 mark out of 2.
- Treat column vectors written like (-40, 15, 2) as bad form. In (b)
- For candidates who do not attempt •7, the formula quoted at •6 must relate to the labelling in the question for •6 to be awarded.
- Do not penalise premature rounding.
- The use of $\tan(A\,\hat{T}B)=\frac{TA.TB}{\overrightarrow{}$ loses •6
- 6 The use of $\cos(A\,\hat{T}B)=\frac{TA.TB}{^{\rightarrow}}$ means that only •5 and
 - •7 are available.

Primary Method: Give 1 mark for each •

$$\bullet^1 \quad \stackrel{\rightarrow}{TA} = \begin{pmatrix} -5\\15\\1 \end{pmatrix}$$

$$\bullet^2 \quad \stackrel{\rightarrow}{TB} = \begin{pmatrix} -40\\15\\2 \end{pmatrix}$$

[Notes 1,2]

2 marks

- TA.TB = 427

$$\begin{array}{ccc}
\bullet^{6} & \cos(A\,\hat{T}B) = \frac{\overrightarrow{TA}.\overrightarrow{TB}}{\overrightarrow{TA} | TB} \\
& | TA | | TB |
\end{array}$$

stated or implied by •7

[Note 3]

5 marks

 $A\hat{T}B = 50 \cdot 9^{\circ} OR 0.889^{c}$ [Note 4] OR 56.6 grads

Alternative 1 for •3 to •7 (Cosine Rule)

$$\bullet^3 \quad \left| \overrightarrow{TA} \right| = \sqrt{251}$$

$$\bullet^4 \quad \begin{vmatrix} \overrightarrow{TB} \end{vmatrix} = \sqrt{1829}$$

$$\bullet^5 \quad \begin{vmatrix} \overrightarrow{AB} \end{vmatrix} = \sqrt{1226}$$

•
$$\begin{vmatrix} AB \end{vmatrix} = \sqrt{1220}$$

• 6 $\cos(A\hat{T}B) = \frac{1829 + 251 - 1226}{2\sqrt{1829}\sqrt{251}}$

stated or implied by •7

 $A \hat{T}B = 50 \cdot 9^{\circ}$

Common Error No.1

$$\bullet^{1} \quad \times \quad \overrightarrow{TA} = t - a = \begin{pmatrix} 5 \\ -15 \\ -1 \end{pmatrix}$$

awwd 1 mark

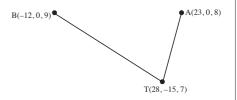
Common Error No.2

•
$$\overrightarrow{TA} = t + a = \begin{pmatrix} 51 \\ -15 \\ 15 \end{pmatrix}$$
• $\times \sqrt{TB} = t + b = \begin{pmatrix} 16 \\ -15 \\ 16 \end{pmatrix}$

Further common errors overleaf.

4 The sketch shows the positions of Andrew(A), Bob (B) and Tracy(T) on three hill-tops.

Relative to a suitable origin, the coordinates (in hundreds of metres) of the three people are A(23, 0, 8), B(-12, 0, 9) and T(28, -15, 7). In the dark, Andrew and Bob locate Tracy using heat-seeking beams.



- Express the vectors TA and TB in component form. (a)
- Calculate the angle between these two beams. (b)

2 $\mathbf{5}$

Common Error 1: Finding angle BOA

$$\begin{array}{cccc} \mathbf{u} & \mathbf{s} & \overset{\rightarrow}{OB} \mathbf{n} \begin{pmatrix} -12 \\ \mathbf{0} \\ 9 \end{pmatrix} and & \overset{\rightarrow}{OA} = \begin{pmatrix} 23 \\ 0 \\ 8 \end{pmatrix} \end{array}$$

- $|\overrightarrow{OB}| = \sqrt{225}$ and $|\overrightarrow{OA}| = \sqrt{593}$
- $\cos(B\hat{O}A) = \frac{\stackrel{\rightarrow}{OB.OA}}{\stackrel{\rightarrow}{|OB||OA|}}$
- $B\hat{O}A = 124.0^{\circ} OR \ 2.163^{\circ}$
- vnd a 1 rm a r k

bullet

per

b

Common Error 2: Finding angle BOT

$$\begin{array}{ccc} \mathbf{u} & \mathbf{s} & \overset{\rightarrow}{OB} & \mathbf{n} \begin{pmatrix} -12 \\ \mathbf{0} \\ 9 \end{pmatrix} and & \overset{\rightarrow}{OT} = \begin{pmatrix} 28 \\ -15 \\ 7 \end{pmatrix} \end{array}$$

- $|OB| = \sqrt{225}$ and $|OT| = \sqrt{1058}$
- OB.OT = -273

$$\bullet \quad \left\langle \begin{array}{c} \cos(B\hat{O}T) = \frac{\stackrel{\rightarrow}{OB.OT}}{\stackrel{\rightarrow}{OB} \mid OT \mid} \\ |OB| \mid OT \mid \\ |B\hat{O}T = 1240^{\circ} \mid OR \mid 2163^{c} \end{array} \right\rangle$$

awwd 1 mark per

Common Error 3: Finding angle AOT

- $|OA| = \sqrt{593}$ and $|OT| = \sqrt{1058}$

$$\bullet \quad \left\langle \begin{array}{c} \overrightarrow{\cos(A\hat{O}T)} = \overrightarrow{OA.OT} \\ \overrightarrow{OA \mid OT \mid} \\ |OA \mid |OT \mid \\ |A\hat{O}T = 27.9^{\circ} \quad OR \quad 0.487^{c} \end{array} \right\rangle$$

ua wde1tmopaerk bulle t

Common Error 4: Finding angle ABT

- $|\overrightarrow{BA}| = \sqrt{1226}$ and $|\overrightarrow{BT}| = \sqrt{1829}$
- BA.BT = 1402

$$\begin{pmatrix}
\cos(\hat{ABT}) = \frac{\vec{OA} \cdot \vec{OT}}{\vec{OA} \cdot \vec{OT}} \\
| OA || OT | \\
A\hat{BT} = 20.6^{\circ} \quad OR \quad 0.359^{c}
\end{pmatrix}$$

award 1 mark

Common Error 5 : Finding angle BAT

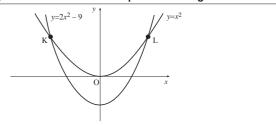
- $|\overrightarrow{AB}| = \sqrt{1226}$ and $|\overrightarrow{AT}| = \sqrt{251}$
- AB.AT = -176

$$\begin{pmatrix}
\cos(B\hat{A}T) = \frac{\overrightarrow{AB} \cdot \overrightarrow{AT}}{\overrightarrow{AB} \mid |AT|} \\
B\hat{A}T = 108.5^{\circ} \quad OR \quad 1.894^{c}
\end{pmatrix}$$

pe nawbodu 11 mhpae ertrk bulle t g

5 The curves with equations $y = x^2$ and $y = 2x^2 - 9$ intersect at K and L as shown.

Calculate the area enclosed between the curves.



Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source	
5		8	С	C17	CN	05/49	

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- •¹ ss: find intersection
- ² pd: process quadratic equ.
- \bullet^3 ss: upper lower
- ic: interpret limits
- 5 pd: sub. & simplify Upper − Lower
- •⁶ pd: integrate
- ic: substitute limits
- 8 pd: evaluate and complete

Primary Method : Give 1 mark for each •

- $\bullet^1 \quad x^2 = 2x^2 9$
- \bullet^2 $x = \pm 3$
- $ullet^3 \int upper-lower$ [Notes 3,4] stated or implied by •5
- \bullet^4 $eg \int_0^3 \dots$
- $\bullet^5 \quad x^2 2x^2 + 9$
- $\bullet^6 \quad \left[-\frac{1}{3}x^3 + 9x \right]_0^3$
- $\bullet^7 \quad \left(-\frac{1}{3} \times 3^3 + 9 \times 3\right) 0$
- 8 2×18 = 36

[Note 3]

8 marks

8

Notes

- 1 There is no penalty for working with
 - $\frac{1}{3}x^3 \frac{2}{3}x^3 + 9x$ or $even \ \frac{1}{3}x^3 \left(\frac{2}{3}x^3 9x\right)$ but in the latter case, the minus signs need to be dealt with correctly at some point for •5 o be awarded.
- 2 Candidates who attempt to find a solution using a graphics calculator earn no marks. The only acceptable solution is via calculus.
- 3 •3 is lost for subtracting the wrong way round and subsequently •8 may be lost for such statements as
 - -36
 - -36 square units
 - -36 = 36
 - −36 so ignore the −ve
 - -36 = 36 square units
 - ·8 may be gained for statements such as

$$-36$$
 so the area $=36$

4 $\int_{3}^{-3} (lower - upper) \text{ or } \int_{3}^{0} (lower - upper)$

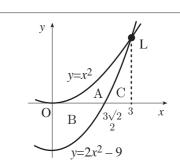
are technically correct and hence all 8 marks are available.

- 5 For $\int_{K}^{L} (upper lower)$, •3,•5,•6 and •7 are available
- 6 Differentiation loses ·6, ·7 and ·8.
- 7 Using $x^2 + 2x^2 9$ and $\int_{-3}^{3} (3x^2 9) dx$ leading to zero can only gain •4 and •6 from the last 6 marks.
- 8 Candidates may attempt to split the area up. In Alt.2, for candidates who treat "C" as a triangle, the last three marks are not available.

Alternative 1 for •4 to •8

- \bullet^4 $eg \int_{-3}^3 \dots$
- $\bullet^5 \quad x^2 2x^2 + 9$
- $\bullet^6 \left[-\frac{1}{3}x^3 + 9x \right]_{3}^{3}$
- $\bullet^7 \quad \left(-\frac{1}{3} \times 3^3 + 9 \times 3\right) \left(-\frac{1}{3} \times (-3)^3 + 9 \times (-3)\right)$
- •⁸ 36

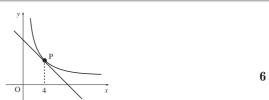
Alternative 2 for •3 to •8



- $\bullet^3 \quad x = \frac{3}{2}\sqrt{2}$
- $\int_{0}^{\frac{3}{2}\sqrt{2}} \left(9 2x^2\right) dx$ leading to B=9 $\sqrt{2}$ (12.7)
- $\int_0^3 \left(x^2\right) dx$ leading to A+C=9
- $\bullet^6 \int_{\frac{3}{2}\sqrt{2}}^{3} \left(2x^2 9\right) dx$ leading to C=9 $\sqrt{2} 9$ (3.7) [Note 8]
- 7 $A = 18 9\sqrt{2}$ (5.3)
- \bullet ⁸ Total area = 36

6 The diagram shows the graph of $y = \frac{24}{\sqrt{x}}$, x > 0.

Find the equation of the tangent at P, where x = 4.



Qu.partmarksGradeSyllabus CodeCalculator class6BC5, C3CN

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- •¹ ss : know to differentiate
- \bullet^2 ic : express in st. form
- ³ pd : differentiate –ve fractional index
- 4 pd : evaluate –ve fractional index
- \bullet ⁵ pd : evaluate *y*-coord
- 6 ic : state equ of tangent

Primary Method : Give 1 mark for each •

Source

05/43

- \bullet^1 $\frac{dy}{dx} = \dots$
- \bullet^2 $y = 24x^{-\frac{1}{2}}$
- \bullet^3 $\frac{dy}{dx} = -12x^{-\frac{3}{2}}$
- $y_{r-4} = 12$
- $y 12 = -\frac{3}{2}(x 4)$ [Notes 1,2,3]
- 6 marks

$$nr \quad [2y + 3x = 36]$$

 $nr = not \ required$

Notes

- 1 •4 and •6 are only available if an attempt to find the gradient is based on differential calculus.
- 2 •6 is not available to candidates who find and use a perpendicular gradient.
- 3 •6 is only available for a numerical value of *m*.

Common Error 1

- $\bullet^1 \quad \frac{dy}{dx} = \dots$
- $y = 24x^{-\frac{1}{2}}$
- $\bullet^3 \quad \frac{dy}{dx} = \frac{24x^{\frac{1}{2}}}{\frac{1}{2}}$
- $\bullet^4 \quad \frac{dy}{dx} = 96$
- $\bullet^5 \quad y_{x=4} = 12$
- y 12 = 96(x 4)
- •1 √
- •2 √
- •3 ×
- $\bullet 4 \times eased$
- •5 √
- •6 ×√
- award 4 ma

Common Error 2

- \bullet^1
- $u = 24x^{-}$
- $\int 24x^{-\frac{1}{2}}dx = \frac{24x^{\frac{1}{2}}}{\frac{1}{2}} + c$
- \bullet^4 gradient = 96
- $y_{x=4} = 12$
- y 12 = 96(x 4)
- •1 ×
- •2 √
- •3 ×
- $\bullet 4 \times Note 1$
- •5 v
- $\bullet 6 \times Note 1$
- rakwsard 2 marks

7 Solve the equation $\log_4(5-x) - \log_4(3-x) = 2$, x < 3.

4

Qu. 7

part marks 4

Grade A Syllabus Code

Calculator class

Source 0525

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- \bullet^1 ss: use the log laws
- ss: know to convert from log to expo
- d: process conversion
- ⁴ pd: find valid solution

Primary Method: Give 1 mark for each •

- \bullet^1 $\log_4\left(\frac{5-x}{3-x}\right)$
- $ullet^2 \quad use \ \log_a(b) = c \Leftrightarrow b = m{a}^c \qquad$ stated or implied by •3
- $\bullet^3 \quad \frac{5-x}{3-x} = 4^2$

See Cave

 $\bullet^4 \quad x = \frac{43}{15}$

4 marks

Notes

1 For •4

Accept answer as a decimal.

Common Error No.1

- •1 $\sqrt{\log_4\left(\frac{5-x}{3-x}\right)} = \log_4(8)$
- •2 ×
- •3 ×

$$\frac{5-x}{3-x} = 8$$

•4 $\times \sqrt{x} = \frac{19}{7}$

award 2 marks

$x = \frac{19}{1}$

Common Error No.2

- $\bullet 1 \quad \sqrt{\qquad} \log_4 \left(\frac{5-x}{3-x} \right) = 2$
- $\bullet 2 \times 4^{\frac{5-x}{3-x}} = 2$
- •3 ×

$$\frac{5-x}{3-x} = \frac{1}{2}$$

•4 $\times \sqrt{x} = 7$ which is not a valid sol.

award 2 marks

Common Error No.3

- $\bullet 1 \quad \sqrt{\quad \log_4 \left(\frac{5-x}{3-x} \right)} = 2$
- $\bullet 2 \times \log_4 \left(\frac{5-x}{3-x} \right) = \log_4 2$
- •3 $\times \frac{5-x}{2} = 2$
- $\bullet 4 \times \sqrt{r-1}$

award 2 mark\$

Alternative 1

- $\bullet^1 \quad \log_4\left(\frac{5-x}{3-x}\right)$
- \bullet^2 2 log 4

stated or implied by ·3

- $\bullet^3 \quad \left(\frac{5-x}{3-x}\right) = 4^2$
- $\bullet^4 \quad x = \frac{43}{15}$

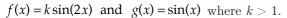
Cave

$\log_4\left(\frac{5-x}{3-x}\right)$		$\log_4\left(\frac{5-x}{3-x}\right)$
$\frac{5-x}{3-x} = 16$	BUT	$\frac{5-x}{3-x} = 2^4$
leading to		leading to
$x = \frac{43}{15}$		$x = \frac{43}{15}$
award	4 m	a $\bullet^1 \text{r} \sqrt{, \text{k}^2 \times \text{s} \bullet^3 \times, \bullet^4 \times }$
		award 2

12

m a

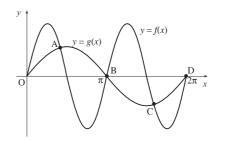
8 Two functions, f and g, are defined by



The diagram shows the graphs of

y = f(x) and y = g(x) intersecting at O, A, B, C

and D. Show that, at A and C, $cos(x) = \frac{1}{2k}$.



Qu. part mai

marks 5

Grade _A Syllabus Code Calcula

Calculator class CN

Source 05/47

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- •¹ ss: equate for intersection
- \bullet^2 ss: use double angle formula
- 3 pd: factorise
- ⁴ pd: process two solutions
- •⁵ ic: complete proof

Primary Method: Give 1 mark for each •

- $\bullet^1 \quad k\sin(2x) = \sin(x)$
- [Note 1]
- $\int \bullet^2 k \times 2\sin(x)\cos(x)$
- $\bullet^3 \quad \sin(x) \Big(2k\cos(x) 1 \Big) = 0$
- $\bullet^4 \quad \sin(x) = 0$
 - a $\operatorname{res}(x) = \frac{1}{2k}$
- $\sin(x) = 0 \Rightarrow x = 0, \pi, 2\pi$
 - i.e. at (O),B and D

[Note 2]

a $mos(\mathbf{Q}) = \frac{1}{2k}$ is for A and C.

5 marks

5

Notes

- 1 Only •1 is available for candidates who substitute a numerical value for k at the start.
- *5 is only available if a suitable comment regarding points (O), B and D is made.
- 3 If all the terms are transposed to one side, then an "=0" needs to appear at least once.
- 4 For Alternative 3
 - •4 and •5 are not available unless •3 has been awarded.

Common Error 1

- $\bullet^1 \quad \sqrt{\quad k \sin(2x)} = \sin(x)$
- •² $\sqrt{k \times 2\sin(x)\cos(x) \sin(x)} = 0$
- $^3 \quad \sqrt{\sin(x)(2k\cos(x)-1)}$
- $\bullet^4 \times 2k\cos(x) 1 = 0$
- •⁵ \times $\cos(x) = \frac{1}{2k}$ at A and C.
- aw 3armolar ks

Alternative 1 for ·4 and ·5

- at (O), B and D, $\sin(x) = 0$
- •5 so at A and C, $2k\cos(x) 1 = 0$ $\Rightarrow \cos(x) = \frac{1}{2k}.$

Alternative 2 for ·4 and ·5

- at A and C, $\sin(x) \neq 0$
- •5 so at A and C, $2k\cos(x) 1 = 0$ $\Rightarrow \cos(x) = \frac{1}{2k}.$

Common Error 2

- $\bullet^1 \quad \sqrt{\quad k \sin(2x)} = \sin(x)$
- 2 $\sqrt{k \times 2\sin(x)\cos(x)} = \sin(x)$
- $\bullet^3 \times k \times 2\cos(x) = 1$
- •4 ×
- •5 \times $\cos(x) = \frac{1}{2k}$ at A and C.
- a ward
- 2
- ma r
- k s

Alternative 3 for ·1 to ·5

- \bullet^1 $k\sin(2x) = \sin(x)$
- \bullet^2 $k \times 2\sin(x)\cos(x) = \sin(x)$
- at A and C, $\sin(x) \neq 0$
- so at A and C, $2k\cos(x) = 1$
- $\bullet^5 \quad \cos(x) = \frac{1}{2k}$

Higher Mathematics 2005 Paper 2 : Marking Scheme Version 4

- 9 The value V (in £ million) of a cruise ship t years after launch is given by the formula $V = 252e^{-0.06335t}$.
 - (a) What was its value when launched?
 - (b) The owners decide to sell the ship once its value falls below £20 million. After how many years will it be sold?

Qu. part marks Grade Syllabus Code Calculator class 9 a 1 B A34 CN b 4 A A34 Ca

The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- \bullet^1 pd: evaluate at t=0
- ic: substitute V = 20
- \bullet^3 pd: process
- ic: expo to log conversion
- bd: solve a logarithmic equation

Primary Method: Give 1 mark for each •

 $\bullet^1 \quad V_{t=0} = 252 \, (\pounds m)$

1 mark

1

4

- \bullet^2 252 $e^{-0.06335t} = 20$
- $\bullet^3 \quad e^{-0.06335t} = \frac{20}{252}$
- $\bullet^4 \quad -0.06335t = \ln\left(\frac{20}{252}\right)$
- t = 40
- [Note 1]
- 4 marks

Notes

in (b)

- 1 For •5 accept any correct answer which rounds to 40.
- 2 An answer obtained by trial and improvement which rounds to 40 may be awarded a max. of 1 mark (out of 4) but only if they have checked 39 as well.
- 3 In following through from an error, *5 is only available for a positive answer.

Common Error 1

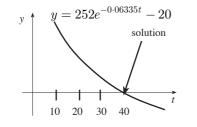
- $\bullet^2 \quad \sqrt{\quad \log \left(252e^{-0.06335t}\right)} = \log 20$
- $\bullet^3 \times -0.06335t \log 252 = \log 20$
- $\bullet^4 \times -0.06335t = \frac{\log 20}{\log 252}$
- t = -8.55
- a ward 1 mark

Alternative 1 for (b) (takings logs of both sides)

- $\bullet^2 \quad 252e^{-0.06335t} = 20$
- $\bullet^3 \quad e^{-0.06335t} = \frac{20}{252}$
- $\bullet^4 \quad -0 \cdot 06335t \log_k(e) = \log_k\left(\frac{20}{252}\right)$
 - where k = e or k = 10
- t = 40
- [Note 1]

Solution via graphics calculator

- e^2 252 $e^{-0.06335t} = 20$
- 3 choose to graph $y = 252e^{-0.06335t} 20$
- \bullet^4 a sketch [see below]
- 5 t = 40



Alternative 2

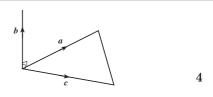
- \bullet^2 252 $e^{-0.06335t} = 20$
- $\bullet^3 \quad \log 252 0.06335t \log e = \log 20$
- \bullet ⁴ 5.53 0.06335t = 2.99
- 5 t = 40

Alternative 3

- $\bullet^2 \quad 252e^{-0.06335t} = 20$
- \bullet^3 $\ln 252 + \ln e^{-0.06335t} = \ln 20$
- \bullet^4 $-0.06335t \ln e = \ln 20 \ln 252$
- 5 t = 40

Note

Vectors a and Care represented by two sides of an equilateral triangle with sides of length 3 units, as shown in the diagram. Vector bis 2 units long and bis perpendicular to both a and Evaluate the scalar product a.(a+b+c).



Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
10		4	Α	G29	CN	05/31

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- •¹ ss: use distributive law
- ² pd: process scalar product
- 3 pd: process scalar product
- d: process scalar product & complete

Primary Method : Give 1 mark for each •

• a.a + a.b + a.c see
• a.a = 9 CAVE
• $a.c = \frac{9}{2}$ [Notes 1,2]
• a.b = 0 and a total of $13\frac{1}{2}$ [Note 3]

Notes

- 1 Treat \underline{a} . Waritten as a^2 as bad form.
- 2 Treat \underline{a} _ . Waritten as ab as an error unless it is subsequently evaluated as a scalar product. Similarly for \underline{a} _ . C
- 3 Using \underline{p} =\$\pm\$| $||q| \sin \theta$ consistently loses 1 mark. (ie max. available is 3)
- 4 When attaching the components

$$c = \begin{pmatrix} 3 \\ 0 \\ 0 \end{pmatrix}, b = \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}, a = \begin{pmatrix} \frac{3}{2} \\ \frac{3\sqrt{3}}{2} \\ 0 \end{pmatrix}$$
, all marks are available.

When attaching the components

$$\mathbf{c} = \begin{bmatrix} 3 \\ 0 \\ 0 \end{bmatrix}, \ \mathbf{b} = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \ \mathbf{a} = \begin{bmatrix} 0 \\ 3 \\ 0 \end{bmatrix}, \text{only } \mathbf{\cdot 1} \text{ is available.}$$

CAVE

a.
$$(a + b + c) = a.a + a.b + a.c$$

followed by
a. $a = 9$
earns •1 and •2.
but
a. $(a + b + c) = a.a + a.b + a.c$
followed by
a. $a = 9$, a. $c = 9$, a. $b = 6$
earns •1 only.

Show that x = -1 is a solution of the cubic equation $x^3 + px^2 + px + 1 = 0$. 11 (a)

Syllabus Code

A21

A22

Hence find the range of values of p for which all the roots of the cubic equation are real.

CN

CN

Calculator class

Source 05/54	

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Grade

Α

- pd: evaluate the function at x = -1
- ss: strategy for finding other factors
- quadratic factor ic:
- ss: strategy for real roots
- substitute ic:
- pd: process
- starts to solve inequation ss:
- ic: complete

Primary Method: Give 1 mark for each .

- f(-1) = -1 + p p + 1 = 01 mark
- $x^2 + (p-1)x + 1 = 0$
- $"b^2 4ac"$ a "d > 0"
- [Notes 3,4]

- $(p-1)^2-4$
- (p-3)(p+1)
- p = 3, p = -1
- $p \le -1, p \ge 3$

[Note 6]

7 marks

7

Notes

Qu.

part

marks

- 1 For alternative method 1, •2
 - ·2 (as is ·3 also) is for interpreting the result of a synthetic division.

Candidates must show some acknowledgement of the result of the synthetic division. Although a statement w.r.t. the zero is preferable, accept something as simple as "underlining" the zero.

- Treat "= 0" missing at •3 as Bad Form
- •4 is only available as a consequence of obtaining a quadratic factor from a division of the cubic.
- Using $b^2 4ac > 0$ loses •4 An ">" must appear at least once somewhere between •4 and •6
- Where errors occur at the •3/•5 stages, then •6,•7,•8 are still available for solving a '3-term' quadratic inequation.
- Evidence for •8 may be a table of values or a sketch
- For candidates who start with $\frac{-b\pm\sqrt{b^2-4ac}}{}$, all marks

are available (subject to working being equivalent to the Primary Method).

8 Wrong disciminant:

Using $b^2 + 4ac$ only •5 (out of the last 5 marks) is available.

Any other expression masquerading as the discriminant loses all of the last 5 marks.

Alternative method 1 for marks 1,2 (starting with synth. division)

f(-1)=0

[Note 1]

etc

Marks should still be recorded as out of 1 and 7

Alternative method 2 for marks 1,2 (quad. factor obtained by inspection)

- f(-1) = -1 + p p + 1 = 0
- $f(x) = (x+1)(x^2 \dots)$ etc

Common Error 1 (marks 5 to 8)

$$(p-1)^2 - 4 \ge 0$$
$$(p-1)^2 > 4$$

$$(p-1)^2 \ge 4$$

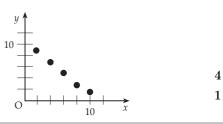
$$p-1 \ge 2$$
$$p > 3$$

award 2 marks out of

[4] The scatter diagram shows 5 pairs of data values for x and y where

 $\Sigma x = 30$, $\Sigma y = 26$, $\Sigma x^2 = 220$, $\Sigma y^2 = 168$ and $\Sigma xy = 120$.

- (a) Find the equation of the regression line.
- (b) Estimate the value of y when x = 5.



Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
S1		4	С	4.4.2	Ca	05/76
	b	1	С	4.4.2	CN	

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- \bullet^1 pd: calculate S_{xy}
- \bullet^2 pd: calculate S_{rr}
- \bullet^3 pd: calculate b
- \bullet^4 pd: calculate a & state equ.
- \bullet^5 ic: use equ. of regression line

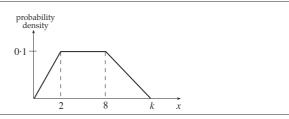
Primary Method: Give 1 mark for each •

- $\bullet^1 \quad S_{xy} = -36$
- $\bullet^2 \quad S_{xx} = 40$
- \bullet^3 b = -0.9
- $a = 10 \cdot 6$ a ny = $\mathbf{d} \cdot 6 0 \cdot 9x$
- 4 marks

• $y_{x=5} = 6.1$

1 mark

- [7] The diagram represents the probability density function for a continuous random variable X.
 - (a) Find the value of k.
 - (b) Find the median.



Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
S4	a	3	A	4.3.1	CN	05/83
	b	2	Α	4.3.5	CN	

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- \bullet^1 ss: state total area = 1
- ic: find expression for total area
- ³ pd: process
- \bullet^4 ss: know total area = 0.5
- pd: process

Primary Method : Give 1 mark for each •

- \bullet^1 area = 1
- \bullet^2 $0 \cdot 1 + 0 \cdot 6 + \frac{1}{2} (k 8) \times 0.1$
 - k 14

3 marks

3

2

- 4 $0 \cdot 1 + (m-2) \times 0.1 = \frac{1}{2}$
- \bullet m=6

2 marks

- [9] (a) Explain briefly the difference between sample standard deviation and range as measures of spread.
 - (b) In statistics mode, a calculator shows the summary statistics for a certain data set.

One data value, $1\cdot 2$, is shown to be erroneous and is deleted. Calculate the sample standard deviation of the new data set of 19 values correct to 3 decimal places.

		_
$\overline{x} =$	2.325	
$S_x =$	0.573883355	
$\sigma_x =$	0.559352304	
$\Sigma x =$	46.5	
$\Sigma x^2 =$	114.37	
n =	20	
$x_{\min} =$	1.2	
$x_{\text{max}} =$	3.2	
		ノ

1

4

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
S3	a	1	В	4.2.11/12	CN	05/79
	b	4	В	4.1.1	Ca	

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- •¹ ic: explanation
- 2 pd: find new $\sum x$
- 3 pd: find new $\sum x^2$
- 4 ss: use formula for S
- 5 pd: process

Primary Method : Give 1 mark for each •

- •¹ SD is a measure of spread about mean whereas $(x_{\rm max}-x_{\rm min})$ is a measure of range. 1 mark
- \bullet^2 $\Sigma x = 45.3$
- 3 $\Sigma x^2 = 112.93$
- \bullet^4 $S = \sqrt{\frac{1}{18} \left(112.93 \frac{45.3^2}{19} \right)}$
- $5 \cdot 0.523$

4 marks

- [10] A large organisation decides to run a mini-lottery for charity.
 - Each participant selects any three different numbers from 1 to 20 inclusive.
 - Every Friday the three winning numbers are drawn at random from the 20.
 - Each participant with these winning numbers shares the jackpot.
 - (a) Find the number of possible combinations and hence find the probability of a particular combination winning a share of the jackpot.
 - (b) Find the probability that someone chooses the winning combination exactly twice within 3 successive weeks.

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
S4	а	2	В	4.2.5, 4.2.3	Ca	05/78
	b	3	Α	4.2.7	Ca	

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SHOWN IN DETAIL IN THE MARKING SCHEME

- •¹ ss: find combination
- pd: calculate probability
- \bullet^3 ic: interpret p(win)
- ss: find combination
- pd: process

Primary Method: Give 1 mark for each •

- No. of outcomes = $\binom{20}{3}$
- 2 $prob = \frac{1}{\binom{20}{3}} = \frac{1}{1140}$

2 marks

3

- 3 $p(L) = \frac{1139}{1149}$
- 4 p(2 wins in 3)

$$= 3 \times \left(\frac{1}{1140}\right)^2 \times \left(\frac{1139}{1140}\right)$$

• 5 2.306×10⁻⁶

3 marks