X100/301

NATIONAL
QUALIFICATIONS
2006

FRIDAY, 19 MAY
9.00 AM – 10.10 AM

MATHEMATICS
HIGHER
Units 1, 2 and 3
Paper 1
(Non-calculator)

Read Carefully

1 **Calculators may NOT be used in this paper.**

2 Full credit will be given only where the solution contains appropriate working.

3 Answers obtained by readings from scale drawings will not receive any credit.

SCOTTISH
QUALIFICATIONS
AUTHORITY

ALL questions should be attempted.

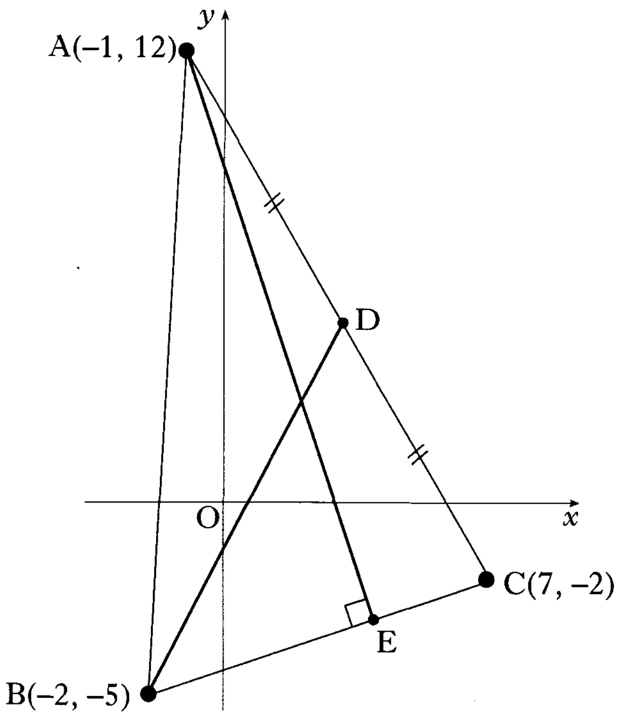
Marks

1. Triangle ABC has vertices A(–1, 12), B(–2, –5) and C(7, –2).

(*a*) Find the equation of the median BD. **3**

(*b*) Find the equation of the altitude AE. **3**

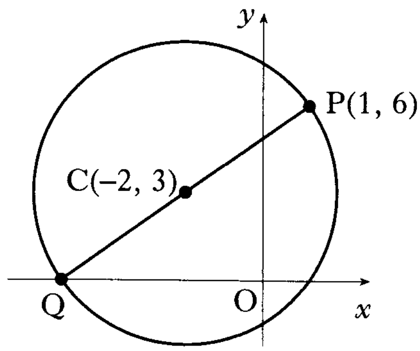
(*c*) Find the coordinates of the point of intersection of BD and AE. **3**



2. A circle has centre C(–2, 3) and passes through P(1, 6).

(*a*) Find the equation of the circle. **2**

(*b*) PQ is a diameter of the circle. Find the equation of the tangent to this circle at Q. **4**



3. Two functions f and g are defined by $f(x) = 2x + 3$ and $g(x) = 2x - 3$, where x is a real number.

(*a*) Find expressions for:

(i) $f(g(x))$;

(ii) $g(f(x))$. **3**

(*b*) Determine the least possible value of the product $f(g(x)) \times g(f(x))$. **2**

[Turn over

Marks

4. A sequence is defined by the recurrence relation $u_{n+1} = 0{\cdot}8u_n + 12$, $u_0 = 4$.

(*a*) State why this sequence has a limit. **1**

(*b*) Find this limit. **2**

5. A function f is defined by $f(x) = (2x - 1)^5$.

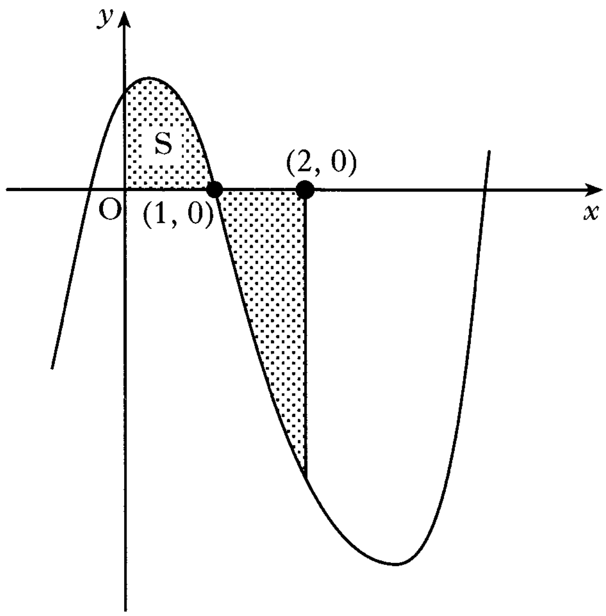
Find the coordinates of the stationary point on the graph with equation $y = f(x)$ and determine its nature. **7**

6. The graph shown has equation $y = x^3 - 6x^2 + 4x + 1$.

The total shaded area is bounded by the curve, the x-axis, the y-axis and the line $x = 2$.

(*a*) Calculate the shaded area labelled S. **4**

(*b*) Hence find the total shaded area. **3**



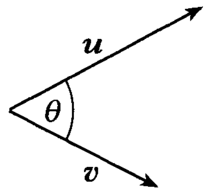
7. Solve the equation $\sin x° - \sin 2x° = 0$ in the interval $0 \leq x \leq 360$. **4**

8. (*a*) Express $2x^2 + 4x - 3$ in the form $a(x + b)^2 + c$. **3**

(*b*) Write down the coordinates of the turning point on the parabola with equation $y = 2x^2 + 4x - 3$. **1**

Marks

9. $\boldsymbol{u}$ and $\boldsymbol{v}$ are vectors given by $\boldsymbol{u} = \begin{pmatrix} k^3 \\ 1 \\ k+2 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 1 \\ 3k^2 \\ -1 \end{pmatrix}$, where $k > 0$.



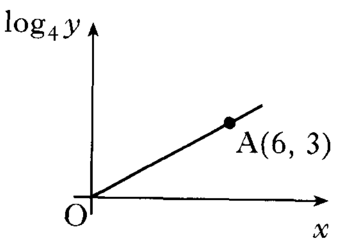
(*a*) If $\boldsymbol{u}.\boldsymbol{v} = 1$, show that $k^3 + 3k^2 - k - 3 = 0$. **2**

(*b*) Show that $(k + 3)$ is a factor of $k^3 + 3k^2 - k - 3$ and hence factorise $k^3 + 3k^2 - k - 3$ fully. **5**

(*c*) Deduce the only possible value of k. **1**

(*d*) The angle between $\boldsymbol{u}$ and $\boldsymbol{v}$ is θ. Find the exact value of $\cos\theta$. **3**

10. Two variables, x and y, are connected by the law $y = a^x$. The graph of $\log_4 y$ against x is a straight line passing through the origin and the point A(6, 3). Find the value of a. **4**



[*END OF QUESTION PAPER*]

X100/303

NATIONAL QUALIFICATIONS 2006

FRIDAY, 19 MAY
10.30 AM – 12.00 NOON

MATHEMATICS
HIGHER
Units 1, 2 and 3
Paper 2

Read Carefully

1 **Calculators may be used in this paper.**

2 Full credit will be given only where the solution contains appropriate working.

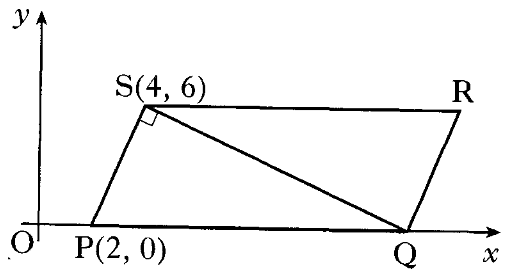
3 Answers obtained by readings from scale drawings will not receive any credit.

ALL questions should be attempted.

Marks

1. PQRS is a parallelogram. P is the point (2, 0), S is (4, 6) and Q lies on the x-axis, as shown.

The diagonal QS is perpendicular to the side PS.



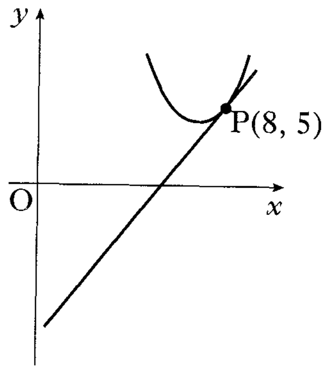
(*a*) Show that the equation of QS is $x + 3y = 22$. **4**

(*b*) Hence find the coordinates of Q and R. **2**

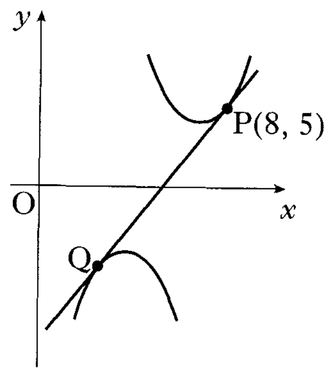
2. Find the value of k such that the equation $kx^2 + kx + 6 = 0$, $k \neq 0$, has equal roots. **4**

3. The parabola with equation $y = x^2 - 14x + 53$ has a tangent at the point P(8, 5).

(*a*) Find the equation of this tangent. **4**



(*b*) Show that the tangent found in (*a*) is also a tangent to the parabola with equation $y = -x^2 + 10x - 27$ and find the coordinates of the point of contact Q. **5**



4. The circles with equations $(x - 3)^2 + (y - 4)^2 = 25$ and $x^2 + y^2 - kx - 8y - 2k = 0$ have the same centre.

Determine the radius of the larger circle. **5**

Marks

5. The curve $y = f(x)$ is such that $\frac{dy}{dx} = 4x - 6x^2$. The curve passes through the point $(-1, 9)$. Express y in terms of x. **4**

6. P is the point $(-1, 2, -1)$ and Q is $(3, 2, -4)$.

   (*a*) Write down $\overrightarrow{PQ}$ in component form. **1**

   (*b*) Calculate the length of $\overrightarrow{PQ}$. **1**

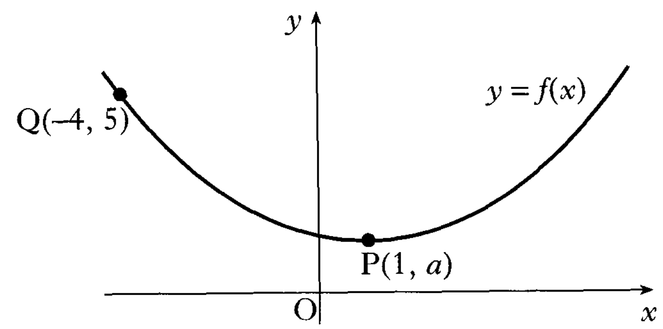
   (*c*) Find the components of a unit vector which is parallel to $\overrightarrow{PQ}$. **1**

7. The diagram shows the graph of a function $y = f(x)$.

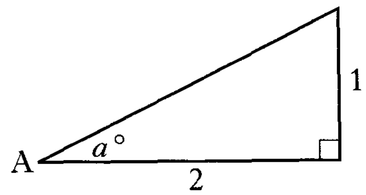
   Copy the diagram and on it sketch the graphs of:

   (*a*) $y = f(x - 4)$; **2**

   (*b*) $y = 2 + f(x - 4)$. **2**



8. The diagram shows a right-angled triangle with height 1 unit, base 2 units and an angle of $a°$ at A.



   (*a*) Find the exact values of:

   (i) $\sin a°$;

   (ii) $\sin 2a°$. **4**

   (*b*) By expressing $\sin 3a°$ as $\sin(2a + a)°$, find the exact value of $\sin 3a°$. **4**

9. If $y = \frac{1}{x^3} - \cos 2x$, $x \neq 0$, find $\frac{dy}{dx}$. **4**

10. A curve has equation $y = 7\sin x - 24\cos x$.

   (*a*) Express $7\sin x - 24\cos x$ in the form $k\sin(x - a)$ where $k > 0$ and $0 \leq a \leq \frac{\pi}{2}$. **4**

   (*b*) Hence find, in the interval $0 \leq x \leq \pi$, the x-coordinate of the point on the curve where the gradient is 1. **3**

Marks

11. It is claimed that a wheel is made from wood which is over 1000 years old.

To test this claim, carbon dating is used.

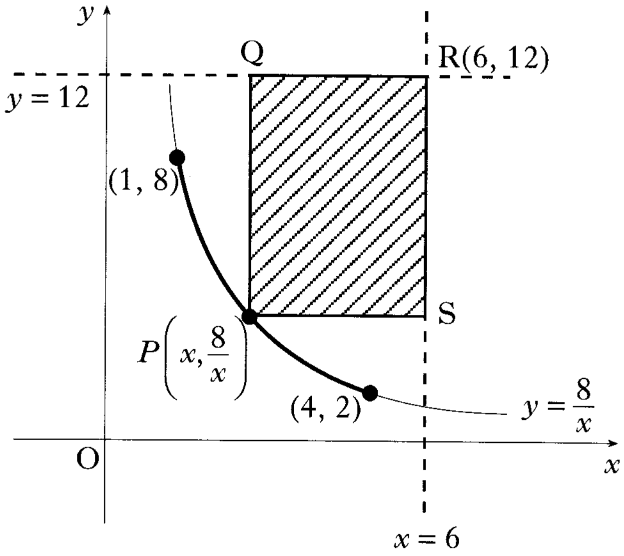
The formula $A(t) = A_0 e^{-0\cdot 000124t}$ is used to determine the age of the wood, where A_0 is the amount of carbon in any living tree, $A(t)$ is the amount of carbon in the wood being dated and t is the age of the wood in years.

For the wheel it was found that $A(t)$ was 88% of the amount of carbon in a living tree.

Is the claim true? **5**

12. PQRS is a rectangle formed according to the following conditions:

- it is bounded by the lines $x = 6$ and $y = 12$
- P lies on the curve with equation $y = \frac{8}{x}$ between (1, 8) and (4, 2)
- R is the point (6, 12).



(*a*) (i) Express the lengths of PS and RS in terms of x, the x-coordinate of P.

(ii) Hence show that the area, A square units, of PQRS is given by

$$A = 80 - 12x - \frac{48}{x}.$$ **3**

(*b*) Find the greatest and least possible values of A and the corresponding values of x for which they occur. **8**

[*END OF QUESTION PAPER*]

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2006 Mathematics

Higher − Paper 1

Finalised Marking Instructions

1. Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than marks deducted for what is wrong.

2. Award one mark for each 'bullet' point. Each error should be underlined in RED at the point in the working where it first occurs, and not at any subsequent stage of the working.

3. The working subsequent to an error must be followed through by the marker with possible full marks for the subsequent working, provided that the difficulty involved is approximately similar. Where, subsequent to an error, the working is eased, a deduction(s) of mark(s) should be made.
   This may happen where a question is divided into parts. In fact, failure to even answer an earlier section does not preclude a candidate from assuming the result of that section and obtaining full marks for a later section.

4. Correct working should be ticked (✓). This is essential for later stages of the SQA procedures. Where working subsequent to an error(s) is correct and scores marks, it should be marked with a crossed tick (✗ or **X**✓). In appropriate cases attention may be directed to work which is not quite correct (e.g. bad form) but which has not been penalised, by underlining with a dotted or wavy line.
   Work which is correct but inadequate to score any marks should be corrected with a double cross tick (✗✗).

5. • The total mark for each section of a question should be entered in red in the **outer** right hand margin, opposite the end of the working concerned.
   • Only the mark should be written, **not** a fraction of the possible marks.
   • These marks should correspond to those on the question paper and these instructions.

6. It is of great importance that the utmost care should be exercised in adding up the marks. Where appropriate, all summations for totals and grand totals must be carefully checked. Where a candidate has scored zero marks for any question attempted, "0" should be shown against the answer.

7. As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Accept answers arrived at by inspection or mentally where it is possible for the answer so to have been obtained. Situations where you may accept such working will normally be indicated in the marking instructions.

8. Do not penalise:
   - working subsequent to a correct answer
   - legitimate variations in numerical answers
   - correct working in the "wrong" part of a question
   - omission of units
   - bad form

9. No piece of work should be scored through without careful checking - even where a fundamental misunderstanding is apparent early in the answer. Reference should always be made to the marking scheme - answers which are widely off-beam are unlikely to include anything of relevance but in the vast majority of cases candidates still have the opportunity of gaining the odd mark or two provided it satisfies the criteria for the mark(s).

10. If in doubt between two marks, give an intermediate mark, but without fractions. When in doubt between consecutive numbers, give the higher mark.

11. In cases of difficulty covered neither in detail nor in principle in the Instructions, attention may be directed to the assessment of particular answers by making a referal to the P.A. Please see the general instructions for P.A. referrals.

12. No marks should be deducted at this stage for careless or badly arranged work. In cases where the writing or arrangement is very bad, a note may be made on the upper left-hand corner of the front cover of the script.

13 Transcription errors: In general, as a consequence of a transcription error, candidates lose the opportunity of gaining either the first ic mark or the first pr mark.

14 Casual errors: In general, as a consequence of a casual error, candidates lose the opportunity of gaining the appropriate ic mark or pr mark.

15 **Do not write any comments on the scripts**. A **revised** summary of acceptable notation is given on page 4.

16 Working that has been crossed out by the candidate cannot receive any credit. If you feel that a candidate has been disadvantaged by this action, make a P.A. Referral.

17 Throughout this paper, unless specifically mentioned, a correct answer with no working receives no credit.

Summary

Throughout the examination procedures many scripts are remarked. It is essential that markers follow common procedures:

1 **Tick** correct working.
2 Put a mark in the **outer right-hand margin to match the marks allocations on the question paper.**
3 Do **no**t write marks as fractions.
4 Put each mark **at the end** of the candidate's response to the question.
5 **Follow through** errors to see if candidates can score marks subsequent to the error.
6 Do **not** write any comments on the scripts.

Higher Mathematics : A Guide to Standard Signs and Abbreviations

Remember - No comments on the scripts. Please use the following and nothing else.

Signs

✓ The tick. You are not expected to tick every line but of course you must check through the whole of a response.

___ ✗ The cross and underline. Underline an error and place a cross at the end of the line.

The tick-cross. Use this to show correct work where you are **following through** subsequent to an error.

∧ The roof. Use this to show something is missing such as a crucial step in a proof or a 'condition' etc.

The tilde. Use this to indicate a minor transgression which is not being penalised (such as bad form).

The double cross-tick. Use this to show correct work but which is inadequate to score any marks. This may happen when working has been eased.

Bullets showing where marks are being allotted may be shown on scripts

		margins
$\frac{dy}{dx} = 4x - 7$ ✓ • $4x - 7 = \underline{0}$ ✗ $x = \frac{7}{4}$ $y = 3\frac{7}{8}$ •		2
$C = \underline{(1, -1)}$ ✗ $m = \frac{3-(-1)}{4-1}$ $m_{rad} = \frac{4}{3}$ • $m_{tgt} = \frac{-1}{\frac{4}{3}}$ $m_{tgt} = -\frac{3}{4}$ • $y - 3 = -\frac{3}{4}(x-2)$ •		3
$x^2 - 3x = 28$ ✓ • $x = 7$ ∧		1
$\sin(x) = 0.75 = inv\sin(0.75) = 48.6°$ ✓ •		1

Remember - No comments on the scripts. No abreviations. No new signs.
Please use the above and nothing else.

All of these are to help us be more consistent and **accurate**.

Note: There is no such thing as a transcription error, a trivial error, a casual error or an insignificant error. These are all mistakes and as a consequence a mark is lost.

Page 5 lists the syllabus coding for each topic. This information is given in the legend underneath the question. The calculator classification is CN(calculator neutral), CR(calculator required) and NC(non-calculator).

1	2		UNIT 1
		A1	determine range/domain
		A2	recognise general features of graphs:poly,exp,log
		A3	sketch and annotate related functions
		A4	obtain a formula for composite function
		A5	complete the square
		A6	interpret equations and expressions
		A7	determine function(poly,exp,log) from graph & vv
		A8	sketch/annotate graph given critical features
		A9	interpret loci such as st.lines,para,poly,circle
		A10	use the notation u_n for the nth term
		A11	evaluate successive terms of a RR
		A12	decide when RR has limit/interpret limit
		A13	evaluate limit
		A14	apply A10-A14 to problems
		G1	use the distance formula
		G2	find gradient from 2 pts,/angle/equ. of line
		G3	find equation of a line
		G4	interpret all equations of a line
		G5	use property of perpendicular lines
		G6	calculate mid-point
		G7	find equation of median, altitude,perp. bisector
		G8	apply G1-G7 to problems eg intersect.,concur.,collin.
		C1	differentiate sums, differences
		C2	differentiate negative & fractional powers
		C3	express in differentiable form and differentiate
		C4	find gradient at point on curve & vv
		C5	find equation of tangent to a polynomial/trig curve
		C6	find rate of change
		C7	find when curve strictly increasing etc
		C8	find stationary points/values
		C9	determinenature of stationary points
		C10	sketch curvegiven the equation
		C11	apply C1-C10 to problems eg optimise, greatest/least
		T1	use gen. features of graphs of $f(x)=k\sin(ax+b)$,
			$f(x)=k\cos(ax+b)$; identify period/amplitude
		T2	use radians inc conversion from degrees & vv
		T3	know and use exact values
		T4	recognise form of trig. function from graph
		T5	interpret trig. equations and expressions
		T6	apply T1-T5 to problems

1	2		UNIT 2
		A15	use the general equation of a parabola
		A16	solve a quadratic inequality
		A17	find nature of roots of a quadratic
		A18	given nature of roots, find a condition on coeffs
		A19	form an equation with given roots
		A20	apply A15-A19 to solve problems
		A21	use Rem Th. For values, factors, roots
		A22	solve cubic and quartic equations
		A23	find intersection of line and polynomial
		A24	find if line is tangent to polynomial
		A25	find intersection of two polynomials
		A26	confiirm and improve on approx roots
		A27	apply A21-A26 to problems
		G9	find C/R of a circle from its equation/other data
		G10	find the equation of a circle
		G11	find equation of a tangent to a circle
		G12	find intersection of line & circle
		G13	find if/when line is tangent to circle
		G14	find if two circles touch
		G15	apply G9-G14 to problems
		C12	find integrals of px^n and sums/diffs
		C13	integrate with negative & fractional powers
		C14	express in integrable form and integrate
		C15	evaluate definite integrals
		C16	find area between curve and x-axis
		C17	find area between two curves
		C18	solve differential equations(variables separable)
		C19	apply C12-C18 to problems
		T7	solve linear & quadratic equations in radians
		T8	apply compound and double angle (c & da) formulae
			in numerical & literal cases
		T9	apply c & da formulae in geometrical cases
		T10	use c & da formulaewhen solving equations
		T11	apply T7-T10 to problems

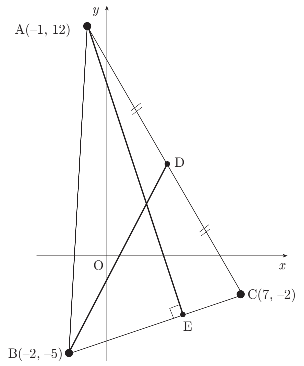
1	2		UNIT 3
		A28	use the laws of logs to simplify/find equiv. expression
		A29	sketch associated graphs
		A30	solve equs of the form $A = Be^{kt}$ for A,B,k or t
		A31	solve equs of the form $\log_b(a) = c$ for a,b or c
		A32	solve equations involving logarithms
		A33	use relationships of the form $y = ax^n$ or $y = ab^x$
		A34	apply A28-A33 to problems
		G16	calculate the length of a vector
		G17	calculate the 3rd given two from A,B and vector AB
		G18	use unit vectors
		G19	use: if **u**, **v** are parallel then $\boldsymbol{v} = k\boldsymbol{u}$
		G20	add, subtract, find scalar mult. of vectors
		G21	simplify vector pathways
		G22	interpret 2D sketches of 3D situations
		G23	find if 3 points in space are collinear
		G24	find ratio which one point divides two others
		G25	given a ratio, find/interpret 3rd point/vector
		G26	calculate the scalar product
		G27	use: if **u**, **v** are perpendicular then $\boldsymbol{v}.\boldsymbol{u}=\boldsymbol{0}$
		G28	calculate the angle between two vectors
		G29	use the distributive law
		G30	apply G16-G29 to problems eg geometry probs.
		C20	differentiate $p\sin(ax+b)$, $p\cos(ax+b)$
		C21	differentiate using the chain rule
		C22	integrate $(ax + b)^n$
		C23	integrate psin(ax+b), pcos(ax+b)
		C24	apply C20-C23 to problems
		T12	solve sim.equs of form $k\cos(a)=p$, $k\sin(a)=q$
		T13	express $p\cos(x)+q\sin(x)$ in form $k\cos(x\pm a)$ etc
		T14	find max/min/zeros of $p\cos(x)+q\sin(x)$
		T15	sketch graph of $y=p\cos(x)+q\sin(x)$
		T16	solve equ of the form $y=p\cos(rx)+q\sin(rx)$
		T17	apply T12-T16 to problems

1 Triangle ABC has vertices A(–1,12), B(–2, –5) and C(7, –2).

(*a*) Find the equation of the median BD. **3**

(*b*) Find the equation of the altitude AE. **3**

(*c*) Find the coordinates of the point of intersection of BD and AE. **3**



Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
1	a,b,c	3,3,3	C	G7, G8	CN	06/01

The primary method m/s is based on the following generic m/s. THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- $\bullet^1$ ic interpret "median"
- $\bullet^2$ ss find gradient
- $\bullet^3$ ic state equation
- $\bullet^4$ ss find gradient
- $\bullet^5$ ss find perpendicular gradient
- $\bullet^6$ ic state equation
- $\bullet^7$ ss start to solve simultaneous equations
- $\bullet^8$ pr solve for one variable
- $\bullet^9$ pr process

Primary Method : Give 1 mark for each ·

- $\bullet^1$ $D=(3,5)$
- $\bullet^2$ $m_{BD}=2$
- $\bullet^3$ $y-5=2(x-3)$ *or* $y+5=2\big(x-(-2)\big)$ *etc* **3 marks**
- $\bullet^4$ $m_{BC}=\frac{1}{3}$ **stated explicitly**
- $\bullet^5$ $m_{alt}=-3$
- $\bullet^6$ $y-12=-3(x-(-1))$ **3 marks**
- $\bullet^7$ $y-5=2(x-3)$ ***and*** $y-12=-3(x-(-1))$ **or equivalent**
- $\bullet^8$ $x=2$
- $\bullet^9$ $y=3$ **3 marks**

Notes

1 For candidates who find two medians $\bullet^1,\bullet^2,\bullet^3$ and $\bullet^7,\bullet^8,\bullet^9$ are available.

2 For candidates who find two altitudes $\bullet^4,\bullet^5,\bullet^6$ and $\bullet^7,\bullet^8,\bullet^9$ are available.

3 For candidates who find (a) altitude and (b) median see common error box number 3.

4 In (a) note that (4, 7) happens to lie on the median but does not qualify as a point to be used in $\bullet^3$.

Notes cont

5 In (b) $\bullet^6$ is only available as a consequence of attempting to find a perpendicular gradient.

6 In (b) candidates who guess the coordinates for E and use these to find the equation AE, can earn no marks in this part.

7 In (c) note that "equating zeros" is only a valid strategy when either the coefficients of x or the coefficients of y are equal.

8 $\bullet^7$ is a strategy mark for juxtaposing the two required equations.

9 See general note at the foot of page 7.

Common Error 1
Finding two medians

- $\bullet^1$ $D=(3,5)$
- $\bullet^2$ $m_{BD}=2$
- $\bullet^3$ $y-5=2(x-3)$
- $\bullet^4$ X
- $\bullet^5$ X
- $\bullet^6$ X
- $\bullet^7$ $y=2x-1$ & $31x+7y=53$
- $\bullet^8$ $x=\dfrac{4}{3}$
- $\bullet^9$ $y=\dfrac{5}{3}$

maximum of 6 marks

Common Error 2
Finding two altitudes

- $\bullet^1$ X
- $\bullet^2$ X
- $\bullet^3$ X
- $\bullet^4$ $m_{BC}=\frac{1}{3}$
- $\bullet^5$ $m_{alt}=-3$
- $\bullet^6$ $y-12=-3(x-(-1))$
- $\bullet^7$ $4x-7y=27$ & $y=-3x+9$
- $\bullet^8$ $x=\dfrac{18}{5}$
- $\bullet^9$ $y=-\dfrac{9}{5}$

maximum of 6 marks

Common Error 3
Finding (a) altitude and (b) median

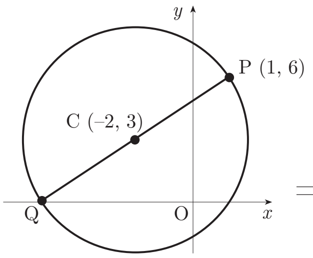
- $\bullet^1$ $m_{AC}=-\frac{7}{4}$
- $X\surd$ $\bullet^2$ $m_{BD}=\frac{4}{7}$
- $\bullet^3$ $y--5=\frac{4}{7}(x--2)$
- $X\surd$ $\bullet^4$ $midpt\ of\ BC=\left(\frac{5}{2},-\frac{7}{2}\right)$
- $\bullet^5$ $m_{AC}=-\frac{31}{7}$
- $\bullet^6$ $y-12=-\frac{31}{7}(x-(-1))$
- $X\surd$ $\bullet^7$ $4x-7y=27$ & $31x+7y=53$
- $X\surd$ $\bullet^8$ $x=\frac{16}{7}$
- $X\surd$ $\bullet^9$ $y=-\frac{125}{49}$

maximum of 5 marks

2 A circle has centre C(–2, 3) and passes through P(1, 6).

(*a*) Find the equation of the circle. **2**

(*b*) PQ is a diameter of the circle. Find the equation of the tangent to this circle at Q. **4**



Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
2	a	2	C	G10	CN	06/54
	b	4	C	G11	CN	

The primary method m/s is based on the following generic m/s.
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- •¹ ic enter coord. of centre in general equation
- •² ss find (radius)²
- •³ ss e.g. use $\overrightarrow{PC} = \overrightarrow{CQ}$ to find Q
- •⁴ pr find gradient of diameter
- •⁵ ss know and use tangent perp. to diameter
- •⁶ ic state equation

Primary Method : Give 1 mark for each •

$(x-a)^2 + (y-b)^2 = r^2$

- •¹ $(x-(-2))^2 + (y-3)^2$
- •² $r^2 = 18$

2 marks

- •³ $Q = (-5, 0)$
- •⁴ $m_{\text{diameter}} = 1$ stated or implied by •5
- •⁵ $m_{\text{tangent}} = -1$
- •⁶ $y - 0 = -\left(x-(-5)\right)$

4 marks

Notes

1 In (a) (√18)² is not acceptable for •².

2 In (b) if the coordinates of Q are estimated (i.e. guessed) then •⁶ can only be awarded if the coordinates are of the form (a, 0) where a < –2.

3 In (b) •⁶ is only available if an attempt has been made to find a perpendicular gradient.

Alternative Method for (a)

For answers of the form $x^2 + y^2 + 2gx + 2fy + c = 0$

- •¹ $x^2 + y^2 + 4x - 6y + c = 0$
- •² $c = -5$

General Notes applicable throughout the marking scheme

There are many instances when follow throughs come into play and these will not always be highlighted for you. The following example is a reminder of what you have to look out for when you are marking.

example
At the •³ stage a candidate start with the wrong coordinates for Q. Then

X	•³	$Q = (-4, 0)$
$X\surd$	•⁴	$m_{\text{diameter}} = \frac{6}{5}$
$X\surd$	•⁵	$m_{\text{tangent}} = -\frac{5}{6}$
$X\surd$	•⁶	$y - 0 = -\frac{5}{6}\left(x-(-4)\right)$

so the candidate loses •³ but gains •⁴, •⁵ and •⁶ as a consequence of following through.
Any error can be followed through and the subsequent marks awarded provided the working has not been eased.
Any deviation from this will be noted in the marking scheme.

3 Two functions f and g are defined on the set of real numbers by

$f(x) = 2x + 3$ and $g(x) = 2x - 3$.

(a) Find an expressions for (i) $f\big(g(x)\big)$ (ii) $g\big(f(x)\big)$. **3**

(b) Determine the least possible value of $f\big(g(x)\big) \times g\big(f(x)\big)$. **2**

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
3	a	3	C	A4	CN	06/07
	b	2	C	A6	CN	

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- $\bullet^1$ ic int. composition
- $\bullet^2$ ic int. composition
- $\bullet^3$ ic int. composition
- $\bullet^4$ pr simplify all functions
- $\bullet^5$ ic int. result

Primary Method : Give 1 mark for each •

$\bullet^1$ $f\big(g(x)\big) = f(2x-3)$	**stated or implied by •2**	
$\bullet^2$ $2(2x-3)+3$		
$\bullet^3$ $g\big(f(x)\big) = 2(2x+3)-3$		**3 marks**
$\bullet^4$ $16x^2 - 9$	**stated explicitly**	
$\bullet^5$ $\text{min.value} = -9$		**2 marks**

Notes

1 In (a) 2 marks are available for finding one of $f\big(g(x)\big)$ *or* $g\big(f(x)\big)$ and the third mark is for the other one.

2 In (a) the finding of $f\big(f(x)\big)$ and $g\big(g(x)\big)$ earns no marks.

3 $\bullet^5$ is only available if $\bullet^4$ has been awarded.

4 In (b) for $\bullet^5$, no justification is necessary. Ignore any comments, rational or irrational.

Alternative Marking 1 [Marks 1-3]

$\bullet^1$ $g\big(f(x)\big) = g(2x+3)$

$\bullet^2$ $2(2x+3)-3$

$\bullet^3$ $f\big(g(x)\big) = 2\big(2x-3\big)+3$

Common Error No.1 for (a) "g and f" transposed.

X	$\bullet^1$	$f\big(g(x)\big) = f(2x+3)$
$\surd\ X$	$\bullet^2$	$2(2x+3)-3$
$\surd\ X$	$\bullet^3$	$g\big(f(x)\big) = 2(2x-3)+3$

Award 2 out of 3

Common Error No.2 for (a)

X	$\bullet^1$	$f\big(g(x)\big) = f(2x+3)$
$\surd\ X$	$\bullet^2$	$2(2x+3)-3$
$\surd$	$\bullet^3$	$g\big(f(x)\big) = 2(2x+3)-3$

Award 2 out of 3

Common Error No.3 for (a) Repeated error

$\surd$	$\bullet^1$	$f\big(g(x)\big) = f(2x-3)$
X	$\bullet^2$	$2(2x+3)-3$
$\surd\ X$	$\bullet^3$	$g\big(f(x)\big) = 2(2x-3)+3$

Award 2 out of 3

4 A sequence is defined by the recurrence relation $u_{n+1} = 0.8u_n + 12,\ u_0 = 4$.

(a) State why the recurrence relation has a limit. **1**

(b) Find this limit. **2**

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
4	a	1	C	A12	NC	06/28
	b	2	C	A13	NC	

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- $\bullet^1$ ic state limit condition
- $\bullet^2$ ss know how to find L
- $\bullet^3$ pr process limt

Primary Method : Give 1 mark for each •

$\bullet^1$	sequence has limit since $-1 < 0.8 < 1$	**1 mark**
$\bullet^2$	$L = 0.8L + 12$	
$\bullet^3$	limit $= 60$	**2 marks**

Notes

For (a)

1 **Accept**

$|0.8| < 1$

$0 < 0.8 < 1$

0.8 lies between -1 and 1

0.8 is a proper fraction

2 **Do NOT accept**

$-1 \le 0.8 \le 1$

$-1 < a < 1$ unless a is clearly identifed/replaced by 0.8 anywhere in the answer.

$0.8 < 1$

In (b)

3 $L = \dfrac{b}{1-a}$ and nothing else gains **no** marks.

4 $L = \dfrac{12}{0.2}$ or $\dfrac{120}{2}$ or $\dfrac{60}{1}$ etc does **NOT** gain $\bullet^3$.

5 An answer of 60 without any working gains **NO** marks.

6 Any calculations based on "wrong" formulae gain **NO** marks.

Alternative Method for (b)

$\bullet^2$ $L = \dfrac{12}{1-0.8}$

$\bullet^3$ $limit = 60$

Bad Form

$\bullet^2$ $L = \dfrac{12}{0.2}$

$\bullet^3$ $limit = 60$

award 2 marks

Common Error 1

X	$\bullet^2$	$L = \dfrac{4}{1-0.8}$
$X\surd$	$\bullet^3$	$limit = 20$

5 A function f is defined by $f(x) = \left(2x-1\right)^5$. Find the coordinates of the stationary point on the graph with equation $y = f(x)$ and determine its nature. **7**

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
5		6	C	C8, C9	NC	06/76
		1	B			

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- $\bullet^1$ ss know to start to differentiate
- $\bullet^2$ pr differentiate
- $\bullet^3$ ss set derivative $= 0$
- $\bullet^4$ pr solve
- $\bullet^5$ pr evaluate
- $\bullet^6$ ic justification
- $\bullet^7$ ic state conclusion

Primary Method : Give 1 mark for each •

- $\bullet^1$ $f'(x) = \ldots\ldots$
- $\bullet^2$ $5(2x-1)^4 \times 2$
- $\bullet^3$ $f'(x) = 0$
- $\bullet^4$ $x = \frac{1}{2}$
- $\bullet^5$ $f(\frac{1}{2}) = 0$
- $\bullet^6$ nature table
- $\bullet^7$ pt of inflexion at $(\frac{1}{2}, 0)$

7 marks

Notes

1 The "= 0" shown at $\bullet^3$ must appear at least once somewhere in the working between $\bullet^1$ and $\bullet^4$ (but not necessarily at $\bullet^3$).

2 $\bullet^4$ is only available as a consequence of solving $f'(x) = 0$.

3 A wrong derivative which eases the working will preclude at least $\bullet^4$ from being awarded.

4 For marks $\bullet^6$ and $\bullet^7$, a nature table is mandatory. The minimum amount of detail that is required is shown here:

	$<\frac{1}{2}$	$\frac{1}{2}$	$>\frac{1}{2}$
$f'(x)$	$+$	0	$+$
	$\cdot^{\cdot^{\cdot}}$	$\cdots$	$\cdot^{\cdot^{\cdot}}$

Candidates who use only $f''(x) = 0$ and try to draw conclusions from this cannot gain $\bullet^6$ or $\bullet^7$.
[$f''(x) = 0$ is a necessary but not sufficient condition for identifying points of inflexion].

5 $\bullet^7$ is **ONLY** available subsequent to a correct nature table for the candidate's own derivative.

6 $\bullet^4$ is lost in each of the following cases for the candidate's solution to the equation at $\bullet^3$.

(i) $x = \frac{1}{2}$ *and* $x =$ *something else*
(ii) *two wrong values for* x
(iii) *guess a value for* x

Only one value for x needs to be followed through for $\bullet^5$, $\bullet^6$ and $\bullet^7$.

Common Error No.1

$\surd$	$\bullet^1$	$f'(x) = \ldots\ldots$
X	$\bullet^2$	$5(2x-1)^4$
$\surd$	$\bullet^3$	$f'(x) = 0$
$X\surd$	$\bullet^4$	$x = \frac{1}{2}$

$\bullet^5$, $\bullet^6$ *and* $\bullet^7$ *are still available*

Common Error No.2

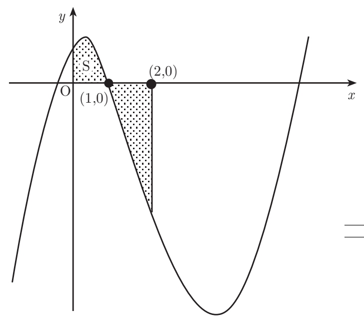
$\surd$	$\bullet^1$	$f'(x) = \ldots\ldots$
X	$\bullet^2$	$\frac{1}{12}(2x-1)^6$
$\surd$	$\bullet^3$	$f'(x) = 0$
$X\surd$	$\bullet^4$	$x = \frac{1}{2}$

$\bullet^5$, $\bullet^6$ *and* $\bullet^7$ *are still available*

6 The graph shown has equation $y = x^3 - 6x^2 + 4x + 1$.
The shaded area is bounded by the curve, the x-axis, the y-axis and the line $x = 2$.

(a) Calculate the shaded area labelled S. **4**

(b) Hence find the total shaded area. **3**



Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
6	a	4	C	C16	NC	06/40
	b	3	B	C16	NC	

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- $\bullet^1$ ss know to integrate
- $\bullet^2$ pr integrate
- $\bullet^3$ ic substitute limits
- $\bullet^4$ pr evaluate
- $\bullet^5$ ic use result from $\bullet^2$ with new limits
- $\bullet^6$ pr evaluate
- $\bullet^7$ ss deal with the "–ve" sign and evaluate total area

Primary Method : Give 1 mark for each $\bullet$

$\bullet^1$ $\int_0^1 \left(x^3 - 6x^2 + 4x + 1\right)dx$ stated or implied by $\bullet^2$

$\bullet^2$ $\frac{1}{4}x^4 - \frac{6}{3}x^3 + \frac{4}{2}x^2 + x$

$\bullet^3$ $\left(\frac{1}{4}.1^4 - 2.1^3 + 2.1^2 + 1\right) - 0$

$\bullet^4$ $\frac{5}{4}$ **or equivalent** **4**

$\bullet^5$ $\int_1^2 ...dx$

$\bullet^6$ $\left(\frac{1}{4}.2^4 - 2.2^3 + 2.2^2 + 2\right) - \left(\frac{1}{4}.1^4 - 2.1^3 + 2.1^2 + 1\right) = -\frac{13}{4}$

$\bullet^7$ $\frac{9}{2}$ **or equivalent** **3**

Notes
for (a)

1 Only a limited number of marks are available to candidates who differentiate –see Common Error No.1.

2 In (a)
candidates who transpose the limits can still earn $\bullet^4$ if the deal with the "–ve" sign appropriately.

3 In (b)
$\bullet^7$ is lost for such statements as $-3\frac{1}{4} = 3\frac{1}{4}$.

4 In (b) using $\int_0^2 ...dx$ earns no marks.

Alternative Method 1 for (b)

$\bullet^5$ $\int_2^1 ...dx$

$\bullet^6$ $\left(\frac{1}{4}.1^4 - 2.1^3 + 2.1^2 + 1\right) - \left(\frac{1}{4}.2^4 - 2.2^3 + 2.2^2 + 2\right)$

$\bullet^7$ $\frac{9}{2}$

Alternative Method 2 for (b)

$\bullet^5$ $-\int_1^2 ...dx$

$\bullet^6$ $-\left(\frac{1}{4}.2^4 - 2.2^3 + 2.2^2 + 2\right) + \left(\frac{1}{4}.1^4 - 2.1^3 + 2.1^2 + 1\right)$

$\bullet^7$ $\frac{9}{2}$

Alternative Method 3 for (b)

$\bullet^5$ $\left|\int_1^2 ...dx\right|$

$\bullet^6$ $\left|\left(\frac{1}{4}.2^4 - 2.2^3 + 2.2^2 + 2\right) - \left(\frac{1}{4}.1^4 - 2.1^3 + 2.1^2 + 1\right)\right|$

$\bullet^7$ $\frac{9}{2}$

Common Error No.1

$\checkmark$	$\bullet^1$	$\int_0^1 \left(x^3 - 6x^2 + 4x + 1\right)dx$
X	$\bullet^2$	$3x^2 - 12x + 4$
X	$\bullet^3$	$\left(3.1^2 - 12.1 + 4\right) - 4$
X	$\bullet^4$	-9
$\checkmark$	$\bullet^5$	$\int_1^2 ...dx$ *or equivalent*
$X\checkmark$	$\bullet^6$	$\left(3.2^2 - 12.2 + 4\right) - \left(3.1^2 - 12.1 + 4\right) = -3$
$X\checkmark$	$\bullet^7$	12

7	Solve the equation $\sin x° - \sin 2x° = 0$ in the interval $0 \le x \le 360$.	**4**

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
7		4	C	T10	NC	06/46

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- $\bullet^1$ ss know to use double angle formula
- $\bullet^2$ pr factorise
- $\bullet^3$ pr solve
- $\bullet^4$ ic know exact values

Primary Method : Give 1 mark for each •

- $\bullet^1$ $\sin(x°) - 2\sin(x°)\cos(x°) = 0$
- $\bullet^2$ $\sin(x°)\big(1 - 2\cos(x°)\big) = 0$
- $\bullet^3$ $\sin(x°) = 0 \textit{ or } \cos(x°) = 0.5$
- $\bullet^4$ $x = 0, 180, 360, \quad 60, 300$ **4**

Alternative Marking Method (Cross marking for •3 and •4)

- $\bullet^1$ $\sin(x°) - 2\sin(x°)\cos(x°) = 0$
- $\bullet^2$ $\sin(x°)\big(1 - 2\cos(x°)\big) = 0$
- $\bullet^3$ $\sin(x°) = 0 \textit{ and } x = 0, 180, 360$
- $\bullet^4$ $\cos(x°) = 0.5 \textit{ and } x = 60, 300$

Notes

1 An "= 0" must appear somewhere between the start and •2 evidence.

2 The inclusion of extra answers which would have been correct with a larger interval should be treated as bad form and NOT penalised.

3 The omission of a correct answer (e.g. 0) means the candidates loses a mark (•4 in the Primary Method).

4 Candidates may embark on a journey with the wrong formula for sin(2x°). With an equivalent level of difficulty it may still be worth a maximum of 3 marks. See Common Error No.1.

5 Candidates who draw a sketch of $y = \sin(x°)$ and $y = \sin(2x°)$ giving 0,180,360 may be awarded •1 and •3.

Alternative Method Division by sin(x)

- $\bullet^1$ $\sin(x°) - 2\sin(x°)\cos(x°) = 0$
- $\bullet^2$ $\textit{either } \sin(x°) = 0 \textit{ or } \sin(x°) \ne 0$
- $\bullet^3$ $\sin(x°) = 0 \Rightarrow x = 0, 180, 360$
- $\bullet^4$ $\cos(x°) = 0.5 \Rightarrow x = 60, 300$

Common Error No.1

$X \quad \bullet^1 \ \sin(x°) - \left(1 - 2\sin^2(x°)\right) = 0$

$2\sin^2(x°) + \sin(x°) - 1 = 0$

$X\checkmark \quad \bullet^2 \ \left(2\sin(x°) - 1\right)\left(\sin(x°) + 1\right) = 0$

$X\checkmark \quad \bullet^3 \ \sin(x°) = \frac{1}{2} \textit{ or } \sin(x°) = -1$

$X\checkmark \quad \bullet^4 \ x = 30, 150, \quad x = 270$

award 3 marks

Common Error No.2

$\sin(x°) - \sin^2(x°) = 0$

$X \quad \bullet^1$

$X\checkmark \quad \bullet^2 \ \sin(x°)\left(1 - \sin(x°)\right) = 0$

$X \quad \bullet^3 \ \sin(x°) = 0 \textit{ or } \sin(x°) = 1$

$X\checkmark \quad \bullet^4 \ x = 0, 180, 360, \quad 90$

award 2 marks

Common Error No.3

$\sin(x) - \sin(2x) = 0$

$\sin(x) = 0, \ \sin(2x) = 0$

etc

gains NO marks

8	(a)	Express $2x^2 + 4x - 3$ in the form $a(x+b)^2 + c$.	3
	(b)	Write down the coordinates of the turning point on the parabola with equation $y = 2x^2 + 4x - 3$.	1

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
8	a	3	B	A5	NC	06/32
	b	1	C	A6	NC	

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- $\bullet^1$ ss know how to complete (deal with the "a")
- $\bullet^2$ pr process the value of "b"
- $\bullet^3$ pr process the value of "c"
- $\bullet^4$ ic interpret equation of parabola

Primary Method : Give 1 mark for each •

- $\bullet^1$ $a = 2$
- $\bullet^2$ $b = 1$
- $\bullet^3$ $c = -5$ **3**
- $\bullet^4$ $(-1, -5)$ **1**

Alternative Method 1 for (a)

- $\bullet^1$ $2(x^2 + 2x)$
- $\bullet^2$ $2(x+1)^2$
- $\bullet^3$ $2(x+1)^2 - 5$
- $\bullet^4$ $(-1, -5)$

Note

1 Alternative Method 1 should be used for assessing part marks/follow throughs.

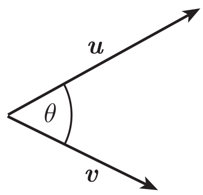
2 For $\bullet^4$, no justification is required.
Candidates may choose to differentiate etc. but may still earn only one mark for the correct answer.

3 For $\bullet^4$, accept (–b, c).

Alternative Method 2 for (a) : Comparing coefficients

- $\bullet^1$ $2x^2 + 4x - 3 = ax^2 + 2abx + ab^2 + c \quad \Rightarrow a = 2$
- $\bullet^2$ $2ab = 4 \quad \Rightarrow b = 1$
- $\bullet^3$ $ab^2 + c = -3 \quad \Rightarrow c = -5$
- $\bullet^4$ $(-1, -5)$

9 $\boldsymbol{u}$ and $\boldsymbol{v}$ are vectors given by $\boldsymbol{u} = \begin{pmatrix} k^3 \\ 1 \\ k+2 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 1 \\ 3k^2 \\ -1 \end{pmatrix}$, where $k > 0$.



(a) If $\boldsymbol{u}.\boldsymbol{v} = 1$ show that $k^3 + 3k^2 - k - 3 = 0$. **2 marks**

(b) Show that $(k+3)$ is a factor of $k^3 + 3k^2 - k - 3$ and hence factorise $k^3 + 3k^2 - k - 3$ fully. **5 marks**

(c) Deduce the only possible value of k. **1 mark**

(d) The angle between $\boldsymbol{u}$ and $\boldsymbol{v}$ is θ. Find the exact value of $\cos\theta$. **3 marks**

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
8	a	2	C	G26	CN	05/10
	b	5	C	A21	NC	
	c	1	C	A6	CN	
	d	3	C	G28	NC	

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- •1 pr find scalar product
- •2 ic complete proof
- •3 ss know to use $k = -3$
- •4 pr complete evaluation and conclusion
- •5 ic start to find quadratic factor
- •6 ic complete quadratic factor
- •7 pr factorise completely
- •8 ic interpret k
- •9 ic interpret vectors
- •10 pr find magnitudes
- •11 ss use formula

Primary Method : Give 1 mark for each •

- •1 $\boldsymbol{u}.\boldsymbol{v} = k^3.1 + 1.\left(3k^2\right) + \left(k+2\right).(-1)$ stated or implied by •2 before completion
- •2 $k^3 + 3k^2 - k - 2 = 1$ *and complete* **2 marks**
- •3 know to use $k = -3$
- •4 $-27 + 27 - (-3) - 3 = 0 \Rightarrow x + 3$ *is a factor*
- •5 $(k+3)(k^2 \ ....)$
- •6 $(k+3)\left(k^2 - 1\right)$
- •7 $(k+3)(k+1)(k-1)$ **stated explicitly** **5 marks**
- •8 $k = 1$ **1 mark**
- •9 $\boldsymbol{u} = \begin{pmatrix} 1 \\ 1 \\ 3 \end{pmatrix}, \boldsymbol{v} = \begin{pmatrix} 1 \\ 3 \\ -1 \end{pmatrix}$ **stated or implied by •10**
- •10 $|\,\boldsymbol{u}\,| = \sqrt{11}$ *and* $|\,\boldsymbol{v}\,| = \sqrt{11}$
- •11 $\cos\theta = \dfrac{1}{11}$ **3 marks**

N.B.
•9 and •10 may be cross-marked.

Notes

1 No explanation is required for k but the chosen value must follow from the working for •6 or •7. **Do not accept √1.**

2 In primary method (•4) and alternative (•5) candidates must show some acknowledgement of the resulting "zero". Although a statement w.r.t. the zero is preferable, accept something as simple as "underlining" the zero.

3 Only numerical values are acceptable for •9, •10 and •11; answers are acceptable in unsimplified form eg $\cos\theta = \frac{1}{\sqrt{11} \times \sqrt{11}}$

Alternative method 1 (marks 3–7) Long Division

•3
$$\begin{array}{r|llll} & k^2 & & -1 & \\ \hline k+3 & k^3 & +3k^2 & -k & -3 \\ & k^3 & +3k^2 & & \\ \hline & ..... & .... & .... -k & -3 \end{array}$$

•4
$$-k \quad -3$$

- •5 *remainder is zero so* $(k+3)$ *is a factor*
- •6 $k^2 - 1$
- •7 $(k+3)(k+1)(k-1)$ **stated explicitly**

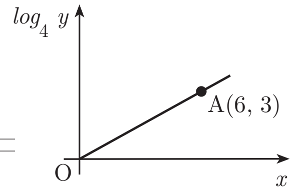
Alternative method 2 (marks 3–7) Synthetic Division

•3
$$\begin{array}{r|l} -3 & \\ \hline & \end{array}$$

•4
$$\begin{array}{r|rrrr} -3 & 1 & 3 & -1 & -3 \\ & & -3 & 0 & 3 \\ \hline & 1 & 0 & -1 & 0 \end{array}$$

- •5 $"f(-3)" = 0$ *so* $(k+3)$ *is a factor*
- •6 $\left(k^2 - 1\right)$
- •7 $(k+3)(k+1)(k-1)$ **stated explicitly**

10 Two variables, x and y, are connected by the law $y = a^x$. A graph of $\log_4(y)$ against x is a straight line passing through the origin and the point A(6,3). Find the value of a. **4**



Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
10		4	A	A33	NC	06/91

The primary method m/s is based on the following generic m/s.
THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- •¹ ss know to take logarithms
- •² ic substitute known point
- •³ pr solve
- •⁴ pr solve

Primary Method : Give 1 mark for each •

- •¹ $\log_4(y) = \log_4(a^x)$
- •² $3 = \log_4(a^6)$
- •³ $a^6 = 4^3$
- •⁴ $a = 2$

4 marks

Note

1 $m = \frac{1}{2}$ and nothing else gains no marks.

2 For •⁴, a correct answer without any legitimate evidence gains **NO** marks.

3 For •⁴, ignore the inclusion of a negative answer.

Alternative Method 1

- •¹ $\log_4(y) = \log_4(a^x)$
- •² $3 = 6\log_4(a)$
- •³ $\log_4(a) = \frac{1}{2}$
- •⁴ $a = 2$

Alternative Method 2

- •¹ $\log_4(y) = mx + c$
- •² $m = \frac{1}{2}$, $c = 0$
- •³ $y = 4^{\frac{1}{2}x}$
- •⁴ $y = \left(4^{\frac{1}{2}}\right)^x = 2^x \Rightarrow a = 2$

Common Error 1

$\surd$	•¹	$\log_4(y) = \log_4(a^x)$
X	•²	$\log_4(3) = \log_4(a^6)$
X	•³	$3 = a^6$
X	•⁴	$a = 3^{\frac{1}{6}}$

Alternative Method 3

- •¹ At A $\log_4(y) = 3$
- •² $y = 4^3$
- •³ $a^6 = 4^3$
- •⁴ $a = 2$

Common Error 2

X	•¹	$\log_4(y) = x$
X	•²	$--$
$X\surd$	•³	$y = 4^x$
X	•⁴	$a = 4$

Alternative Method 4

- •¹ $\log_4(y) = \log_4(a^x)$
- •² $\log_4(y) = x\log_4(a)$
- •³ $\log_4(a) = \frac{1}{2}$
- •⁴ $a = 4^{\frac{1}{2}} = 2$

SCOTTISH
QUALIFICATIONS
AUTHORITY

2006 Mathematics

Higher − Paper 2

Finalised Marking Instructions

1. Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than marks deducted for what is wrong.

2. Award one mark for each 'bullet' point. Each error should be underlined in RED at the point in the working where it first occurs, and not at any subsequent stage of the working.

3. The working subsequent to an error must be followed through by the marker with possible full marks for the subsequent working, provided that the difficulty involved is approximately similar. Where, subsequent to an error, the working is eased, a deduction(s) of mark(s) should be made.
   This may happen where a question is divided into parts. In fact, failure to even answer an earlier section does not preclude a candidate from assuming the result of that section and obtaining full marks for a later section.

4. Correct working should be ticked (✓). This is essential for later stages of the SQA procedures. Where working subsequent to an error(s) is correct and scores marks, it should be marked with a crossed tick (✗ or **X**✓). In appropriate cases attention may be directed to work which is not quite correct (e.g. bad form) but which has not been penalised, by underlining with a dotted or wavy line.
   Work which is correct but inadequate to score any marks should be corrected with a double cross tick (✗✗).

5. • The total mark for each section of a question should be entered in red in the **outer** right hand margin, opposite the end of the working concerned.
   • Only the mark should be written, **not** a fraction of the possible marks.
   • These marks should correspond to those on the question paper and these instructions.

6. It is of great importance that the utmost care should be exercised in adding up the marks. Where appropriate, all summations for totals and grand totals must be carefully checked. Where a candidate has scored zero marks for any question attempted, "0" should be shown against the answer.

7. As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Accept answers arrived at by inspection or mentally where it is possible for the answer so to have been obtained. Situations where you may accept such working will normally be indicated in the marking instructions.

8. Do not penalise:
   - working subsequent to a correct answer
   - legitimate variations in numerical answers
   - correct working in the "wrong" part of a question
   - omission of units
   - bad form

9. No piece of work should be scored through without careful checking - even where a fundamental misunderstanding is apparent early in the answer. Reference should always be made to the marking scheme - answers which are widely off-beam are unlikely to include anything of relevance but in the vast majority of cases candidates still have the opportunity of gaining the odd mark or two provided it satisfies the criteria for the mark(s).

10. If in doubt between two marks, give an intermediate mark, but without fractions. When in doubt between consecutive numbers, give the higher mark.

11. In cases of difficulty covered neither in detail nor in principle in the Instructions, attention may be directed to the assessment of particular answers by making a referal to the P.A. Please see the general instructions for P.A. referrals.

12. No marks should be deducted at this stage for careless or badly arranged work. In cases where the writing or arrangement is very bad, a note may be made on the upper left-hand corner of the front cover of the script.

13 Transcription errors: In general, as a consequence of a transcription error, candidates lose the opportunity of gaining either the first ic mark or the first pr mark.

14 Casual errors: In general, as a consequence of a casual error, candidates lose the opportunity of gaining the appropriate ic mark or pr mark.

15 **Do not write any comments on the scripts**. A **revised** summary of acceptable notation is given on page 4.

16 Working that has been crossed out by the candidate cannot receive any credit. If you feel that a candidate has been disadvantaged by this action, make a P.A. Referral.

17 Throughout this paper, unless specifically mentioned, a correct answer with no working receives no credit.

Summary

Throughout the examination procedures many scripts are remarked. It is essential that markers follow common procedures:

1 **Tick** correct working.
2 Put a mark in the **outer right-hand margin to match the marks allocations on the question paper.**
3 Do **no**t write marks as fractions.
4 Put each mark **at the end** of the candidate's response to the question.
5 **Follow through** errors to see if candidates can score marks subsequent to the error.
6 Do **not** write any comments on the scripts.

Higher Mathematics : A Guide to Standard Signs and Abbreviations

Remember - No comments on the scripts. Please use the following and nothing else.

Signs

✓ The tick. You are not expected to tick every line but of course you must check through the whole of a response.

✗ The cross and underline. Underline an error and place a cross at the end of the line.

The tick-cross. Use this to show correct work where you are **following through** subsequent to an error.

∧ The roof. Use this to show something is missing such as a crucial step in a proof or a 'condition' etc.

The tilde. Use this to indicate a minor transgression which is not being penalised (such as bad form).

The double cross-tick. Use this to show correct work but which is inadequate to score any marks. This may happen when working has been eased.

Bullets showing where marks are being allotted may be shown on scripts

margins

$\frac{dy}{dx} = 4x - 7$ ✓ •

$4x - 7 = \underline{0}$ ✗

$x = \frac{7}{4}$

$y = 3\frac{7}{8}$ • 2

$C = \underline{(1,-1)}$ ✗

$m = \dfrac{3-(-1)}{4-1}$

$m_{rad} = \frac{4}{3}$ •

$m_{tgt} = \dfrac{-1}{\frac{4}{3}}$

$m_{tgt} = -\frac{3}{4}$ •

$y - 3 = -\frac{3}{4}(x-2)$ • 3

$x^2 - 3x = 28$ ✓ •

$x = 7$ ∧ 1

$\sin(x) = 0.75 = inv\sin(0.75) = 48.6°$ 1

✓ •

Remember - No comments on the scripts. No abreviations. No new signs.
Please use the above and nothing else.

All of these are to help us be more consistent and **accurate**.

Note: There is no such thing as a transcription error, a trivial error, a casual error or an insignificant error. These are all mistakes and as a consequence a mark is lost.

Page 5 lists the syllabus coding for each topic. This information is given in the legend underneath the question. The calculator classification is CN(calculator neutral), CR(calculator required) and NC(non-calculator).

1	2		UNIT 1
		A1	determine range/domain
		A2	recognise general features of graphs:poly,exp,log
		A3	sketch and annotate related functions
		A4	obtain a formula for composite function
		A5	complete the square
		A6	interpret equations and expressions
		A7	determine function(poly,exp,log) from graph & vv
		A8	sketch/annotate graph given critical features
		A9	interpret loci such as st.lines,para,poly,circle
		A10	use the notation u_n for the nth term
		A11	evaluate successive terms of a RR
		A12	decide when RR has limit/interpret limit
		A13	evaluate limit
		A14	apply A10-A14 to problems
		G1	use the distance formula
		G2	find gradient from 2 pts,/angle/equ. of line
		G3	find equation of a line
		G4	interpret all equations of a line
		G5	use property of perpendicular lines
		G6	calculate mid-point
		G7	find equation of median, altitude,perp. bisector
		G8	apply G1-G7 to problems eg intersect.,concur.,collin.
		C1	differentiate sums, differences
		C2	differentiate negative & fractional powers
		C3	express in differentiable form and differentiate
		C4	find gradient at point on curve & vv
		C5	find equation of tangent to a polynomial/trig curve
		C6	find rate of change
		C7	find when curve strictly increasing etc
		C8	find stationary points/values
		C9	determinenature of stationary points
		C10	sketch curvegiven the equation
		C11	apply C1-C10 to problems eg optimise, greatest/least
		T1	use gen. features of graphs of $f(x)=k\sin(ax+b)$,
			$f(x)=k\cos(ax+b)$; identify period/amplitude
		T2	use radians inc conversion from degrees & vv
		T3	know and use exact values
		T4	recognise form of trig. function from graph
		T5	interpret trig. equations and expressions
		T6	apply T1-T5 to problems

1	2		UNIT 2
		A15	use the general equation of a parabola
		A16	solve a quadratic inequality
		A17	find nature of roots of a quadratic
		A18	given nature of roots, find a condition on coeffs
		A19	form an equation with given roots
		A20	apply A15-A19 to solve problems
		A21	use Rem Th. For values, factors, roots
		A22	solve cubic and quartic equations
		A23	find intersection of line and polynomial
		A24	find if line is tangent to polynomial
		A25	find intersection of two polynomials
		A26	confiirm and improve on approx roots
		A27	apply A21-A26 to problems
		G9	find C/R of a circle from its equation/other data
		G10	find the equation of a circle
		G11	find equation of a tangent to a circle
		G12	find intersection of line & circle
		G13	find if/when line is tangent to circle
		G14	find if two circles touch
		G15	apply G9-G14 to problems
		C12	find integrals of px^n and sums/diffs
		C13	integrate with negative & fractional powers
		C14	express in integrable form and integrate
		C15	evaluate definite integrals
		C16	find area between curve and x-axis
		C17	find area between two curves
		C18	solve differential equations(variables separable)
		C19	apply C12-C18 to problems
		T7	solve linear & quadratic equations in radians
		T8	apply compound and double angle (c & da) formulae
			in numerical & literal cases
		T9	apply c & da formulae in geometrical cases
		T10	use c & da formulaewhen solving equations
		T11	apply T7-T10 to problems

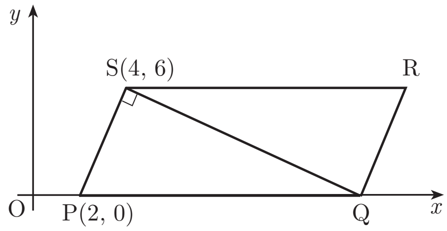
1	2		UNIT 3
		A28	use the laws of logs to simplify/find equiv. expression
		A29	sketch associated graphs
		A30	solve equs of the form $A = Be^{kt}$ for A,B,k or t
		A31	solve equs of the form $\log_b(a) = c$ for a,b or c
		A32	solve equations involving logarithms
		A33	use relationships of the form $y = ax^n$ or $y = ab^x$
		A34	apply A28-A33 to problems
		G16	calculate the length of a vector
		G17	calculate the 3rd given two from A,B and vector AB
		G18	use unit vectors
		G19	use: if $\mathbf{u}$, $\mathbf{v}$ are parallel then $\mathbf{v} = k\mathbf{u}$
		G20	add, subtract, find scalar mult. of vectors
		G21	simplify vector pathways
		G22	interpret 2D sketches of 3D situations
		G23	find if 3 points in space are collinear
		G24	find ratio which one point divides two others
		G25	given a ratio, find/interpret 3rd point/vector
		G26	calculate the scalar product
		G27	use: if $\mathbf{u}$, $\mathbf{v}$ are perpendicular then $\mathbf{v.u=0}$
		G28	calculate the angle between two vectors
		G29	use the distributive law
		G30	apply G16-G29 to problems eg geometry probs.
		C20	differentiate $p\sin(ax+b)$, $p\cos(ax+b)$
		C21	differentiate using the chain rule
		C22	integrate $(ax + b)^n$
		C23	integrate $p\sin(ax+b)$, $p\cos(ax+b)$
		C24	apply C20-C23 to problems
		T12	solve sim.equs of form $k\cos(a)=p$, $k\sin(a)=q$
		T13	express $p\cos(x)+q\sin(x)$ in form $k\cos(x\pm a)$ etc
		T14	find max/min/zeros of $p\cos(x)+q\sin(x)$
		T15	sketch graph of $y=p\cos(x)+q\sin(x)$
		T16	solve equ of the form $y=p\cos(rx)+q\sin(rx)$
		T17	apply T12-T16 to problems

1 PQRS is a parallelogram. P is the point (2, 0), S is (4, 6) and Q lies on the x-axis, as shown.

The diagonal QS is perpendicular to the side PS.

(a) Show that the equation of QS is $x + 3y = 22$. **4**

(b) Hence find the coordinates of Q and R. **2**



Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
1	a,b	4,2	C	G8	CN	06/05

The primary method m/s is based on the following generic m/s.
THIS GENERIC M/S MAY BE USED AS AN EQUIVALENCE GUIDE BUT ONLY WHERE A CANDIDATE DOES NOT USE THE PRIMARY METHOD OR ANY ALTERNATIVE METHOD SHOWN IN DETAIL IN THE MARKING SCHEME

- •1 pr find gradient from two points
- •2 ss use $m_1 m_2 = -1$
- •3 ic state equation of the line
- •4 ic completes proof
- •5 ic interpret diagram
- •6 ic interpret diagram

Primary Method : Give 1 mark for each •

- •1 $m_{PS} = 3$
- •2 $m_{QS} = -\dfrac{1}{3}$
- •3 $y - 6 = -\frac{1}{3}(x - 4)$
- •4 completes proof **4 marks**
- •5 $Q = (22, 0)$
- •6 $R = (24, 6)$ **2 marks**

Notes

In (a)

1 In the Primary method, •3 is only available if an attempt has been made to find and use a perpendicular gradient.

2 In the Primary method and the Alt. method 1, •4 is only available for reaching the required equation.

3 To gain •4, some evidence of completion needs to be shown

e.g. $y - 6 = -\frac{1}{3}(x - 4)$

$3(y - 6) = -(x - 4)$

$x + 3y = 22$

4 Sometimes candidates manage to find R first. Provided the coordinates of R are of the form (?, 6), only then is •6 available as a follow through.

5 •5 and •6 are available to candidates who use their own erroneous equation for QS.

Alternative Method 1

- •1 $m_{PS} = 3$
- •2 $m_{QS} = -\frac{1}{3}$
  $y = -\frac{1}{3}x + c$
- •3 $6 = -\frac{1}{3} \times 4 + c$
- •4 completes proof
- •5 $Q = (22, 0)$
- •6 $R = (24, 6)$

Alternative Method 2

$Let\ Q = (q, 0)$

- •1 $(q - 2)^2 = 2^2 + 6^2 + (q - 4)^2 + 6^2$
- •2 $q = 22$
- •3 $Q = (22, 0)\ and\ R = (24, 6)$
- •4 $m_{QS} = -\frac{1}{3}$
- •5 $y - 0 = -\frac{1}{3}(x - 22)$
- •6 $leading\ to\ 3y + x = 22$

N.B.
The coordinates of Q can also be arrived at by right-angled trig.
Use the alt. method 2 marking scheme with •1 replaced by appropriate trig. work.
The only acceptable value for q is 22.

General Notes applicable throughout the marking scheme

There are many instances when follow throughs come into play and these will not always be highlighted for you. The following example is a reminder of what you have to look out for when you are marking.

example
At the •5 stage a candidate may switch the coordinates round so we have

- •5 X $Q(0, 22)$
- •6 $X\checkmark$ $R(2, 28)$ $repeated\ error$

so the candidate loses •5 for switching the coordinates but gains •6 as a consequence of following through.
Any error can be followed through and the subsequent marks awarded provided the working has not been eased.
Any deviation from this will be noted in the marking scheme.

2 Find the value of k such that the equation $kx^2 + kx + 6 = 0$, $k \neq 0$, has equal roots. **4**

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
2		4	C	A18	CN	06/new

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- $\bullet^1$ ss know to use "discriminant = 0"
- $\bullet^2$ ic interpret a, b, c
- $\bullet^3$ pr substitute & factorise
- $\bullet^4$ ic interpret solution

Primary Method : Give 1 mark for each •

- $\bullet^1$ $"b^2 - 4ac" = 0$
- $\bullet^2$ $a = k, b = k, c = 6$
- $\bullet^3$ $k(k - 24)$
- $\bullet^4$ $\left[\begin{array}{l} k = 0 \quad \textit{and} \quad k = 24 \\ \therefore \quad k = 24 \end{array}\right.$

4 marks

Notes

1 The evidence for $\bullet^1$ and/or $\bullet^2$ may not appear until the working immediately preceding the evidence for $\bullet^3$.
i.e. a candidate may simply start

$\checkmark \bullet^1, \checkmark \bullet^2$ $\quad k^2 - 4 \times k \times 6 = 0$
$\checkmark \bullet^3$ $\quad k(k - 24)$

or

$\checkmark \bullet^2$ $\quad k^2 - 4 \times k \times 6$
$\checkmark \bullet^1, \checkmark \bullet^3$ $\quad k(k - 24) = 0$

2 The "= 0" has to appear at least once, at the $\bullet^1$ stage or at the $\bullet^3$ stage.

3 In the Primary method, candidates who do not deal with the root k = 0 cannot obtain $\bullet^4$.
[see Common Errors 1 and 2]
Minimum evidence for $\bullet^4$ would be scoring out "k = 0" or "k = 24" underlined.

4 Some candidates may start with the quadratic formula. Apply the marking scheme to the part underneath the square root sign.

5 The use of any expression masquerading as the discriminant can only gain $\bullet^2$ at most.

Alternative Method 1 (completing the square)

- $\bullet^1$ $\left(x + \frac{1}{2}\right)^2 + \ldots\ldots$
- $\bullet^2$ $\left(x + \frac{1}{2}\right)^2 - \frac{1}{4} + \frac{6}{k} = 0$
- $\bullet^3$ $\textit{equal roots} \Rightarrow -\frac{1}{4} + \frac{6}{k} = 0$
- $\bullet^4$ $k = 24$

Acceptable alternative for •4

$\checkmark$	$\bullet^1$	$"b^2 - 4ac" = 0$
$\checkmark$	$\bullet^2$	$a = k, b = k, c = 6$
$\checkmark$	$\bullet^3$	$k(k - 24)$
$\checkmark$	$\bullet^4$	$\cancel{k = 0}$ *or* 24

Common Error 1 at the •4 stage

$\checkmark$	$\bullet^1$	$"b^2 - 4ac" = 0$
$\checkmark$	$\bullet^2$	$a = k, b = k, c = 6$
$\checkmark$	$\bullet^3$	$k(k - 24)$
X	$\bullet^4$	$k = 0$ *or* 24

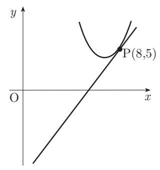
Common Error 2 at the •4 stage

$\checkmark$	$\bullet^1$	$"b^2 - 4ac" = 0$
$\checkmark$	$\bullet^2$	$a = k, b = k, c = 6$
$\checkmark$	$\bullet^3$	$k(k - 24)$
X	$\bullet^4$	$k = 24$

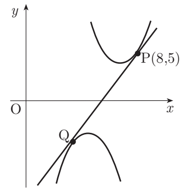
Common Error 3 Division by k

$\checkmark$	$\bullet^1$	$"b^2 - 4ac" = 0$
$\checkmark$	$\bullet^2$	$a = k, b = k, c = 6$
X	$\bullet^3$	$k^2 - 24k = 0$
		$k^2 = 24k$
X	$\bullet^4$	$k = 24$

3 The parabola with equation $y = x^2 - 14x + 53$ has a tangent at the point P(8,5).



(a) Find the equation of this tangent. **4**



(b) Show that the tangent found in (a) is also a tangent to the parabola with equation $y = -x^2 + 10x - 27$ and find the coordinates of the point of contact Q. **5**

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
3	a	4	C	C5	CN	06/26
	b	5	C	A24	CN	

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- •1 ss know to differentiate
- •2 pr differentiate
- •3 pr evaluate gradient
- •4 ic state equation of tangent
- •5 ss arrange in standard form
- •6 ss substitute into quadratic
- •7 pr process
- •8 ic factorise & interpret
- •9 ic state coordinates

Primary Method : Give 1 mark for each •

- •1 $\frac{dy}{dx} =$
- •2 $2x - 14$
- •3 $m = 2$ **stated or implied by •4**
- •4 $y - 5 = 2(x - 8)$ **4 marks**

- •5 $y = 2x - 11$
- •6 $2x - 11 = -x^2 + 10x - 27$
- •7 $x^2 - 8x + 16 = 0$
- •8 $(x - 4)^2 = 0 \Rightarrow \textit{equal roots so tgt}$
- •9 $\mathrm{Q} = (4, -3)$ **5 marks**

Notes

In (a)

1 •4 is only available if an attempt has been made to find the gradient from differentiation.

In (b)

2 •6 is only available for a numerical value of m.

3 An "= 0" must occur somewhere in the working between •7 and •8.

4 •8 is awarded for drawing a conclusion from the candidate's quadratic equation.

5 Candidates may substitute the equation of the parabola into the equation of the line. This is a perfectly acceptable approach.

Common Error 1

√	•1	$\frac{dy}{dx} =$
√	•2	$2x - 14$
X	•3	$2x - 14 = 0$ *so* $x = 7$ *so* $m = 7$
X	•4	$y - 5 = 7(x - 8)$
X√	•5	$y = 7x - 51$
X√	•6	$7x - 51 = -x^2 + 10x - 27$
X√	•7	$x^2 - 3x - 24 = 0$
X√	•8	$b^2 - 4ac = 105 \Rightarrow \textit{line is not tgt}$
X	•9	$--$

so award 6 marks

Alternative Marking 1 [Marks 8]

- •8 $b^2 - 4ac = 64 - 4 \times 16 = 0 \Rightarrow \textit{line is a tangent}$

Alternative Method 1 for (b)

- •5 $2x = y + 11$
- •6 $4y = -(y^2 + 22y + 121) + 20y + 220 - 108$
- •7 $y^2 + 6x + 9 = 0$
- •8 $(y + 3)^2 = 0 \Rightarrow \textit{equal roots so tgt}$
- •9 $\mathrm{Q} = (4, -3)$

Alternative Method 2 for (b)

- •5 *Find the equ. of the tgt to 2nd curve with grad. 2* **stated or implied by •6**
- •6 $-2x + 10 = 2$
- •7 $\mathrm{Q} = (4, -3)$
- •8 $y - (-3) = 2(x - 4)$
- •9 $y = 2x - 11$ *which is the same equ. as (a)* **stated explicitly**

4 The circles with equations $(x-3)^2+(y-4)^2=25$ and $x^2+y^2-kx-8y-2k=0$. have the same centre. Determine the radius of the larger circle. **5**

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
4		5	C	G9	CN	06/55

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- $\bullet^1$ ic state centre of circle 1
- $\bullet^2$ ss equate x-coordinates, find k.
- $\bullet^3$ ic find radius of circle 1
- $\bullet^4$ ic substitute into the radius formula
- $\bullet^5$ ic process radius formula and compare.

Primary Method : Give 1 mark for each •

- $\bullet^1$ $C_1=(3,4)$
- $\bullet^2$ $k=6$
- $\bullet^3$ $R_1=5$
- $\bullet^4$ $R_2=\sqrt{(-3)^2+(-4)^2-(-12)}$ **or equivalent**
- $\bullet^5$ $\sqrt{37}>5$ *or* "2nd circle"

5 marks

Notes

1 $\bullet^2$ requires no justification.

2 Evidence for $\bullet^3$ may appear for the first time at the $\bullet^5$ stage.

3 If $R_1 = 5$ is clearly stated at the $\bullet^3$ stage, then it does not have to appear at the $\bullet^5$ stage for the conclusion to be drawn.

4 For any formula masquerading as the radius formula (e.g. see Common Error 2) , $\bullet^4$ and $\bullet^5$ are NOT available.

Alternative Method 1

- $\bullet^1$ $x^2+y^2-6x-8y+25=25$
- $\bullet^2$ $k=6$
- $\bullet^3$ $R_1=5$
- $\bullet^4$ $R_2=\sqrt{(-3)^2+(-4)^2-(-12)}$ **or equivalent**
- $\bullet^5$ $\sqrt{37}>5$ *or* "2nd circle"

Common Error 1

$\checkmark$	$\bullet^1$	$C_1=(3,4)$
$\checkmark$	$\bullet^2$	$k=6$
$\checkmark$	$\bullet^3$	$R_1=5$
X	$\bullet^4$	$R_2=\sqrt{(-3)^2+(-4)^2-12}$
$X\checkmark$	$\bullet^5$	$\sqrt{13}<5$ *or* "1st circle"

Common Error 2

$\checkmark$	$\bullet^1$	$C_1=(3,4)$
$\checkmark$	$\bullet^2$	$k=6$
$\checkmark$	$\bullet^3$	$R_1=5$
X	$\bullet^4$	$R_2=\sqrt{(-3)^2+(-4)^2+(12)^2}$
X	$\bullet^5$	$13>5$ *or* "2nd circle"

5 The curve $y = f(x)$ is such that $\frac{dy}{dx} = 4x - 6x^2$. The curve passes through the point (–1, 9). Express y in terms of x. **4**

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
5		4	C/B	C18	CN	06/37

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- $\bullet^1$ ss know to integrate
- $\bullet^2$ pr integrate
- $\bullet^3$ ic substitute values
- $\bullet^4$ pr process constant

Primary Method : Give 1 mark for each •

- $\bullet^1$ $y = \int \ldots$ **stated or implied by •2**
- $\bullet^2$ $\frac{4}{2}x^2 - \frac{6}{3}x^3$
- $\bullet^3$ $9 = 2(-1)^2 - 2(-1)^3 + c$
- $\bullet^4$ $y = 2x^2 - 2x^3 + 5$ **stated explicitly**

4 marks

Notes

1 The equation “y = ...” must appear somewhere in the solution.

Common Error 1 Missing “equation”

√ $\bullet^1$ $y = \int ..$
√ $\bullet^2$ $\frac{4}{2}x^2 - \frac{6}{3}x^3$
√ $\bullet^3$ $9 = 2(-1)^2 - 2(-1)^3 + c$
X $\bullet^4$ $c = 5$
award 3 marks

Common Error 2 : Not using (–1, 9)

√ $\bullet^1$ $y = \int ..$
√ $\bullet^2$ $\frac{4}{2}x^2 - \frac{6}{3}x^3$
X $\bullet^3$ $2(-1)^2 - 2(-1)^3 + c = 0$
X $\bullet^4$ $y = 2x^2 - 2x^3 - 4$
award 2 marks

Alternative Marking

$\bullet^1$ $y = \int \ldots$

$\bullet^2$ $\frac{4}{2}x^2 - \frac{6}{3}x^3$

$\bullet^3$ $\left[\begin{array}{l} y = 2x^2 - 2x^3 + c \\ and \\ 9 = 2(-1)^2 - 2(-1)^3 + c \end{array}\right.$ **stated explicitly**

$\bullet^4$ $c = 5$

6 P is the point (–1, 2, –1) and Q is (3, 2, –4).

(*a*) Write down $\overrightarrow{PQ}$ in component form. **1**

(*b*) Calculate the length of $\overrightarrow{PQ}$. **1**

(*c*) Find the components of a unit vector which is parallel to $\overrightarrow{PQ}$. **1**

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
6	a	1	C	G17	CN	06/59
	b	1	C	G16		
	c	1	B	G18		

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- •[1] ic state vector components
- •[2] pr find the length of a vector
- •[3] ic state unit vector

Primary Method : Give 1 mark for each •

•[1] $\overrightarrow{PQ} = \begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix}$ **1 mark**

•[2] $|\overrightarrow{PQ}| = 5$ **1 mark**

•[3] $\begin{pmatrix} \frac{4}{5} \\ 0 \\ -\frac{3}{5} \end{pmatrix}$ **1 mark**

Note

In (a)

1 It is perfectly acceptable to write the components as a row vector eg $\overrightarrow{PQ} = \begin{pmatrix} 4 & 0 & -3 \end{pmatrix}$.

Treat $\overrightarrow{PQ} = (4, 0, -3)$ as bad form (i.e. not penalised).

In (b)

2 •[2] is not awarded for an unsimplified $\sqrt{25}$.

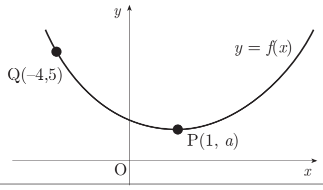
3 Beware of misappropriate use of the scalar product where, by coincidence, $\boldsymbol{p}.\boldsymbol{q} = 5$.

In (c)

4 Accept $\frac{1}{5}\begin{pmatrix} 4 \\ 0 \\ -3 \end{pmatrix}$ for •[3].

7 The diagram shows the graph of a function $y = f(x)$.

Copy the diagram and on it sketch the graphs of



(a) $y = f(x-4)$ **2**

(b) $y = 2 + f(x-4)$ **2**

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
7	a	2	C	A3	CN	06/new
	b	2	C	A3		

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- •¹ ic know translate parallel to x-axis, +ve dir.
- •² ic annotate points
- •³ ic know translate parallel to y-axis, +ve dir.
- •⁴ ic annotate points

Primary Method : Give 1 mark for each •

- •¹ translate 4 units right and annotate one point
- •² annotate the other point [P'$(5, a)$ Q'$(0,5)$]

2 marks

- •³ translate (a) 2 units up and annotate one point
- •⁴ annotate the other point [P"$(5, a+2)$ Q"$(0,7)$]

2 marks

Alternative Method

- •¹ translate 4 units right and annotate one point
- •² annotate the other point [P'$(5, a)$ Q'$(0,5)$]
- •³ translate original $\begin{pmatrix}4\\2\end{pmatrix}$ and annotate one point
- •⁴ annotate the other point [P"$(5, a+2)$ Q"$(0,7)$]

Notes

For (a)

1 A translation of $\begin{pmatrix}-4\\0\end{pmatrix}$ earns a maximum of 1 mark with both points clearly annotated and f(x) retaining its shape.

2 Any other translation gains no marks.

In the Primary method

For (b)

3 •³ and •⁴ are only available for applying the translation to the resultant graph from (a).

4 A translation of $\begin{pmatrix}0\\-2\end{pmatrix}$ earns a maximum of 1 mark with both points clearly annotated and the resultant graph from (a) retaining its shape.

5 Any other translation gains no marks.

In the Alternative method

For (b)

6 A translation of $\begin{pmatrix}4\\-2\end{pmatrix}, \begin{pmatrix}-4\\2\end{pmatrix}$ *or* $\begin{pmatrix}-4\\-2\end{pmatrix}$ applied to the original graph earns a maximum of 1 mark with both points clearly annotated and the resultant graph retaining its original shape.

7 Any other translation gains no marks.

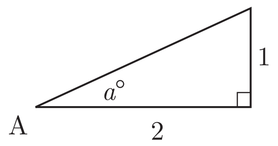
In either method

For (a) and (b)

8 For the annotated points, accept a superimposed grid or clearly labelled axes.

9 A candidate may choose to use two separate diagrams. This is acceptable.

8 The diagram shows a right-angled triangle with height 1 unit, base 2 units and an angle of $a°$ at A.



(*a*) Find the exact values of

(*i*) $\sin a°$

(*ii*) $\sin 2a°$. **4**

(*b*) By expressing $\sin 3a°$ as $\sin(2a+a)°$, find the exact value of $\sin 3a°$. **4**

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
8	a	4	C	T9	CN	06/44
	b	4	B	T8	CN	

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- $\bullet^1$ ic interpret diagram for sin(a°)
- $\bullet^2$ ss use double angle formula for sin(2A)
- $\bullet^3$ ic interpret diagram for cos($a°$)
- $\bullet^4$ pr substitute and complete
- $\bullet^5$ ss use compound angle formula
- $\bullet^6$ pr use double angle formula for cos(2A)
- $\bullet^7$ ic substitute
- $\bullet^8$ pr complete

Primary Method : Give 1 mark for each •

$\bullet^1$ $\sin(a°) = \dfrac{1}{\sqrt{5}}$

$\bullet^2$ $\sin(2a°) = 2\sin(a°)\cos(a°)$

$\bullet^3$ $\cos(a°) = \dfrac{2}{\sqrt{5}}$

$\bullet^4$ $\sin(2a°) = \dfrac{4}{5}$ **4 marks**

$\bullet^5$ $\sin(3a°) = \sin(2a°)\cos(a°) + \cos(2a°)\sin(a°)$

$\bullet^6$ $\cos(2a°) = \dfrac{3}{5}$

$\bullet^7$ $\sin(3a°) = \dfrac{4}{5}.\dfrac{2}{\sqrt{5}} + \dfrac{3}{5}.\dfrac{1}{\sqrt{5}}$

$\bullet^8$ $\sin(3a°) = \dfrac{11}{5\sqrt{5}}$ **4 marks**

Note

1 Calculating approximate angles using arcsin and arccos gains no credit.

2 There are 3 processing marks $\bullet^4$, $\bullet^6$ and $\bullet^8$. None of these are available for an answer > 1.

3 sin(2a) = 0.8 and cos(2a) = 0.6 are the only two decimal fractions which may receive any credit.

4 Some candidates may double the height of the triangle and then call the base angle 2a. This error is equivalent to Common Error 1 illustrated on the right.

Common Error 1 An example of Incorrect formulae

$\checkmark$	$\bullet^1$	$\sin(a°) = \dfrac{1}{\sqrt{5}}$
X	$\bullet^2$	$\sin(2a°) = 2\sin(a°)$
X	$\bullet^4$	$\sin(2a°) = \dfrac{2}{\sqrt{5}}$
$\checkmark$	$\bullet^5$	$\sin(3a°) = \sin(2a°)\cos(a°) + \cos(2a°)\sin(a°)$
$\checkmark$	$\bullet^3$	$\cos(a°) = \dfrac{2}{\sqrt{5}}$
X	$\bullet^6$	$\cos(2a°) = \dfrac{4}{\sqrt{5}}$
$X\checkmark$	$\bullet^7$	$\sin(3a°) = \dfrac{2}{\sqrt{5}}.\dfrac{2}{\sqrt{5}} + \dfrac{4}{\sqrt{5}}.\dfrac{1}{\sqrt{5}}$
X	$\bullet^8$	$\sin(3a°) = \dfrac{8}{5}$

Common Error 2
An example based on a numerical error in Pythagoras

X	$\bullet^1$	$\sin(a°) = \dfrac{1}{\sqrt{3}}$
$\checkmark$	$\bullet^2$	$\sin(2a°) = 2\sin(a°)\cos(a°)$
$X\checkmark$	$\bullet^3$	$\cos(a°) = \dfrac{2}{\sqrt{3}}$
X	$\bullet^4$	$\sin(2a°) = \dfrac{4}{3}$
$\checkmark$	$\bullet^5$	$\sin(3a°) = \sin(2a°)\cos(a°) + \cos(2a°)\sin(a°)$
X	$\bullet^6$	$\cos(2a°) = 2\cos^2(a°) - 1 = \dfrac{5}{3}$ *or equivalent*
$X\checkmark$	$\bullet^7$	$\sin(3a°) = \dfrac{4}{3}.\dfrac{2}{\sqrt{3}} + \dfrac{5}{3}.\dfrac{1}{\sqrt{3}}$
X	$\bullet^8$	$\sin(3a°) = \dfrac{13}{3\sqrt{3}}$

9	$y = \dfrac{1}{x^3} - \cos 2x,\ x \neq 0,$ find $\dfrac{dy}{dx}$.	4

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
8		4	C/B	C3,C20	CN	06/79

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- •[1] ss express in differentiable form
- •[2] pr differentiate a term with a negative power
- •[3] pr start to process a compound derivative
- •[3] pr complete compound derivative

Primary Method : Give 1 mark for each •

- •[1] x^{-3}
- •[2] $-3x^{-4}$
- •[3] $+\sin 2x$
- •[4] $\times 2$

4 marks

Notes

1 For clearly integrating, correctly or otherwise, only •[1] is available.

2 If you cannot decide whether a candidate has attempted to differentiate or integrate, assume they have attempted to differentiate.

10 A curve has equation $y = 7\sin x - 24\cos x$.

(a) Express $7\sin x - 24\cos x$ in the form $k\sin(x-a)$ where $k>0$ and $0 \le a \le \frac{\pi}{2}$. **4**

(b) Hence find, in the interval $0 \le x \le \pi$, the x-coordinate of the point on the curve where the gradient is 1. **3**

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
10	a	4	C	T13	CR	06/97
	b	3	A/B	T17	CR	

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- •1 ss expand
- •2 ic compare coefficients
- •3 pr process k
- •4 pr process a
- •5 ic state result
- •6 ss set derivative = gradient
- •7 pr process 'x' from the derivative

Primary Method : Give 1 mark for each •

- •1 $k\sin(x)\cos(a) - k\cos(x)\sin(a)$ **stated explicitly**
- •2 $k\cos(a) = 7,\ k\sin(a) = 24$ **stated explicitly**
- •3 $k = 25$
- •4 $a = 1.29$ **4 marks**

- •5 $25\sin(x-1.29)$
- •6 $\frac{dy}{dx} = 25\cos(x-1.29) = 1$
- •7 $x = 2.82$ **3 marks**

Notes

In (a)

1 $k\big(\sin(x)\cos(a) - \cos(x)\sin(a)\big)$ is acceptable for •1.

2 Treat $k\sin(x)\cos(a) - \cos(x)\sin(a)$ as bad form if •2 is gained.

3 No justification is required for •3.

4 •3 is not available for an unsimplified $\sqrt{625}$.

5 $25\big(\sin(x)\cos(a) - \cos(x)\sin(a)\big)$ is acceptable evidence for •1 and •3.

6 Candidates may use any form of the wave equation to start with as long as their final answer is in the form $k\sin(x-a)$. If it is not, then •4 is not available.

7 •4 is only available for
(i) an answer in radians which rounds to 1.3 OR
(ii) an answer given as a multiple of π *e.g.* $\frac{37}{90}\pi$.

8 $k\cos(a) = 7$ *and* $k\sin(a) = -24$ leading to a = 4.99 can only gain •4 if a comment intimating that this answer is not in the given interval is given.

In (b)

9 In (b) candidates have a choice of two starting points.
They can either start from $y = 25\sin(x-1.29)$ as shown in the Primary method OR
they can start from $\frac{dy}{dx} = 7\cos(x) + 24\sin(x)$. Either of these starting positions may be awarded •5.

10 Candidates who work in degrees will lose •6 for attempting to differentiate .

11 •7 is only available as a consequence of solving $\frac{dy}{dx} = 1$. Do not penalise "extra" solutions at the •7 stage (e.g. 6.04).

Common Error 1 Working in degrees

✓	•1	$25\big(\sin(x)\cos(a) - \cos(x)\sin(a)\big)$
✓	•2	$k\cos(a) = 7,\ k\sin(a) = 24$
✓	•3	$k = 25$
X	•4	$a = 73.7$
✓	•5	$25\sin(x-73.7)$
X	•6	$\frac{dy}{dx} = 25\cos(x-73.7) = 1$
✓X	•7	$x = 161.4$

Award (a) 3 marks and (b) 2 marks

11 It is claimed that a wheel is made from wood which is over 1000 years old.
To test this claim, carbon dating is used.

The formula $A(t) = A_0 e^{-0.000124t}$ is used to determine the age of the wood, where A_0 is the amount of carbon in any living tree, $A(t)$ is the amount of carbon in the wood being dated and t is the age of the wood in years For the wheel it was found that $A(t)$ was 88% of the amount of carbon in a living tree.
Is the claim true? **5**

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
11		5	A/B	A30	CR	06/36

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- $\bullet^1$ ic interpret information
- $\bullet^2$ ic substitute
- $\bullet^3$ ss take logarithms
- $\bullet^4$ pr process
- $\bullet^5$ ic interpret result

Primary Method : Give 1 mark for each •

- $\bullet^1$ $A(t) = 0.88A_0$ **stated or implied by $\bullet^2$**
- $\bullet^2$ $e^{-0.000124t} = 0.88$
- $\bullet^3$ $\ln\left(e^{-0.000124t}\right) = \ln\left(0.88\right)$ **stated or implied by $\bullet^4$**
- $\bullet^4$ $-0.000124t = \ln(0.88)$
- $\bullet^5$ $t = 1031$ years so claim valid

5 marks

Notes

1 Candidates may choose a numerical value for A_0 at the start of their solution. Accept this situation.

2 $\bullet^5$ is only available if $\bullet^4$ has been awarded.

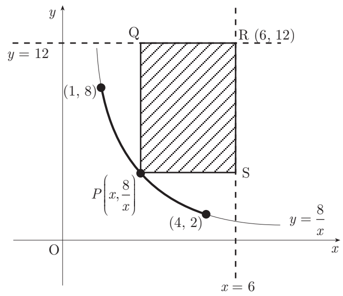
3 In following through from an error, $\bullet^5$ is only available for a positive value of t.

Alternative Method 1 Graph and Calculator Solution

- $\bullet^1$ $A(1000) = A_0 e^{-0.000124\times1000}$
- $\bullet^2$ $0.883A_0$ *and* 1000 year old piece of wood contains 88.3% carbon.
- $\bullet^3$ try a point where t > 1030
  e.g. $A(1050)$ getting $0.878A_0$
- $\bullet^4$ sketch of y=$A_0 e^{-0.000124t}$ showing
  1. a monotonic decreasing function
  2. points representing eg (1000, 88.3%) etc
- $\bullet^5$ observation that the point lies between the two plotted values for t and so claim valid.

12 PQRS is a rectangle formed according to the following conditions:

- it is bounded by the lines $x = 6$ and $y = 12$
- P lies on the curve with equation $y = \frac{8}{x}$ between (1, 8) and (4, 2)
- R is the point (6, 12).



(*a*) (i) Express the lengths of PS and RS in terms of x, the x-coordinate of P.

(ii) Hence show that the area, A square units, of PQRS is given by $A = 80 - 12x - \frac{48}{x}$. **3**

(*b*) Find the greatest and least possible values of A and the corresponding values of x for which they occur. **8**

Qu.	part	marks	Grade	Syllabus Code	Calculator class	Source
12	a	3	A	C12	CN	06/20
	b	9	A/B	C12		

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- •1 ic interpret diagram to find PS
- •2 ic interpret diagram to find RS
- •3 ic complete proof
- •4 ic express in differentiable form
- •5 ss know to set derivative to zero
- •6 pr differentiate
- •7 pr process equation
- •8 pr evaluate area at the turning point
- •9 pr evaluate area at the end point
- •10 pr evaluate area at the end point
- •11 ic state conclusion

Primary Method : Give 1 mark for each •

- •1 $PS = 6 - x$
- •2 $RS = 12 - \frac{8}{x}$
- •3 $Area = \left(6 - x\right)\left(12 - \frac{8}{x}\right)$ and complete **3 marks**
- •4 $48x^{-1}$
- •5 $\frac{dA}{dx} = 0$
- •6 $-12 + 48x^{-2}$
- •7 $x = 2$
- •8 $A(2) = 32$
- •9 $A(1) = 20$
- •10 $A(4) = 20$
- •11 $\max.A = 32 \textit{ at } x = 2$ ***and*** $\min.A = 20 \textit{ at } x = 1 \textit{ or } x = 4$ **8 marks**

Notes

1 For •3 there needs to be clear evidence that candidates have multiplied out the brackets in order to complete the proof.

2 An " = 0 " must appear somewhere in the working between •4 and •7.

3 At the •7 stage, ignore the omission or inclusion of x = –2.

4 •8 has to be as a consequence of solving $\frac{dA}{dx} = 0$.

5 •11 is only available if both end points have been considered.