X100/12/02

NATIONAL QUALIFICATIONS 2014

TUESDAY, 6 MAY
1.00 PM – 2.30 PM

MATHEMATICS
HIGHER
Paper 1
(Non-calculator)

Read carefully

Calculators may NOT be used in this paper.

Section A – Questions 1–20 (40 marks)

Instructions for completion of **Section A** are given on Page two.

For this section of the examination you must use an **HB pencil**.

Section B (30 marks)

1 Full credit will be given only where the solution contains appropriate working.

2 Answers obtained by readings from scale drawings will not receive any credit.

Read carefully

1 Check that the answer sheet provided is for **Mathematics Higher (Section A)**.

2 For this section of the examination you must use an **HB pencil** and, where necessary, an eraser.

3 Check that the answer sheet you have been given has **your name**, **date of birth**, **SCN** (Scottish Candidate Number) and **Centre Name** printed on it.

Do not change any of these details.

4 If any of this information is wrong, tell the Invigilator immediately.

5 If this information is correct, **print** your name and seat number in the boxes provided.

6 The answer to each question is A, B, C or D. Decide what your answer is, then, using your pencil, put a horizontal line in the space provided (see sample question below).

7 There is **only one correct** answer to each question.

8 Rough working should **not** be done on your answer sheet.

9 At the end of the exam, put the **answer sheet for Section A inside the front cover of your answer book**.

Sample Question

A curve has equation $y = x^3 - 4x$.

What is the gradient at the point where x = 2?

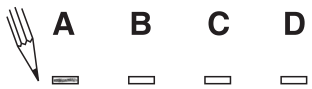
A $\quad 8$

B $\quad 1$

C $\quad 0$

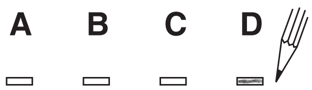
D $\quad -4$

The correct answer is **A—**8. The answer **A** has been clearly marked in **pencil** with a horizontal line (see below).



Changing an answer

If you decide to change your answer, carefully erase your first answer and, using your pencil, fill in the answer you want. The answer below has been changed to **D**.



FORMULAE LIST

Circle:

The equation $x^2 + y^2 + 2gx + 2fy + c = 0$ represents a circle centre $(-g, -f)$ and radius $\sqrt{g^2 + f^2 - c}$.

The equation $(x - a)^2 + (y - b)^2 = r^2$ represents a circle centre (a, b) and radius r.

Scalar Product: $\mathbf{a.b} = |\mathbf{a}||\mathbf{b}| \cos \theta$, where θ is the angle between **a** and **b**

or $\mathbf{a.b} = a_1b_1 + a_2b_2 + a_3b_3$ where $\mathbf{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ and $\mathbf{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$.

Trigonometric formulae:

$$\begin{aligned} \sin(A \pm B) &= \sin A \cos B \pm \cos A \sin B \\ \cos(A \pm B) &= \cos A \cos B \mp \sin A \sin B \\ \sin 2A &= 2\sin A \cos A \\ \cos 2A &= \cos^2 A - \sin^2 A \\ &= 2\cos^2 A - 1 \\ &= 1 - 2\sin^2 A \end{aligned}$$

Table of standard derivatives:

$f(x)$	$f'(x)$
$\sin ax$	$a \cos ax$
$\cos ax$	$-a \sin ax$

Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a}\cos ax + c$
$\cos ax$	$\frac{1}{a}\sin ax + c$

[Turn over

SECTION A

ALL questions should be attempted.

1. A sequence is defined by the recurrence relation $u_{n+1} = \frac{1}{3}u_n + 1$, with $u_2 = 15$.

What is the value of u_4?

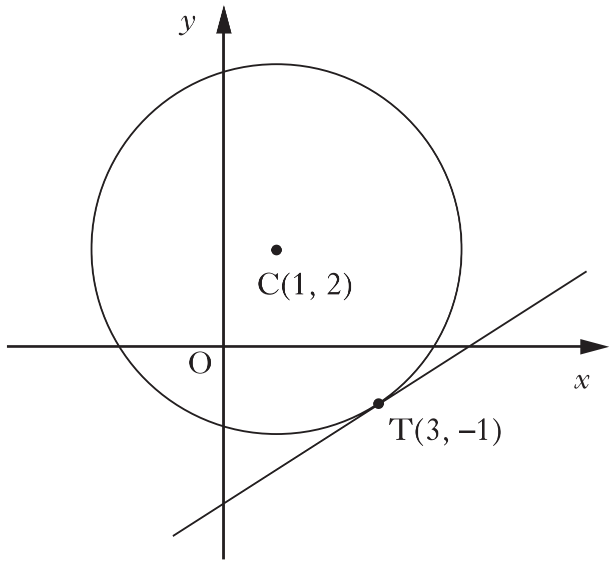
A $2\frac{1}{9}$

B $2\frac{1}{3}$

C 3

D 30

2. The diagram shows a circle with centre C(1, 2) and the tangent at T(3, –1).



What is the gradient of this tangent?

A $\frac{1}{4}$

B $\frac{2}{3}$

C $\frac{3}{2}$

D 4

3. If $\log_4 12 - \log_4 x = \log_4 6$, what is the value of x?

A 2

B 6

C 18

D 72

4. If $3\sin x - 4\cos x$ is written in the form $k\cos(x - a)$, what are the values of $k\cos a$ and $k\sin a$?

	$k\cos a$	$k\sin a$
A	-3	4
B	3	-4
C	4	-3
D	-4	3

5. Find $\int (2x+9)^5\, dx$.

A $10\,(2x + 9)^4 + c$

B $\frac{1}{4}\,(2x + 9)^4 + c$

C $10\,(2x + 9)^6 + c$

D $\frac{1}{12}\,(2x + 9)^6 + c$

[Turn over

6. Given that $\boldsymbol{u} = \begin{pmatrix} -3 \\ 1 \\ 0 \end{pmatrix}$ and $\boldsymbol{v} = \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$, find $2\boldsymbol{u} - 3\boldsymbol{v}$ in component form.

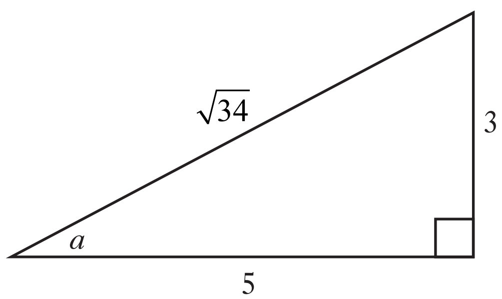
A $\begin{pmatrix} -9 \\ 5 \\ -6 \end{pmatrix}$

B $\begin{pmatrix} -9 \\ -1 \\ -4 \end{pmatrix}$

C $\begin{pmatrix} -3 \\ -1 \\ 6 \end{pmatrix}$

D $\begin{pmatrix} 11 \\ -5 \\ 4 \end{pmatrix}$

7. A right-angled triangle has sides and angles as shown in the diagram.



What is the value of $\sin 2a$?

A $\frac{8}{17}$

B $\frac{3}{\sqrt{34}}$

C $\frac{15}{17}$

D $\frac{6}{\sqrt{34}}$

8. What is the derivative of $\left(4-9x^4\right)^{\frac{1}{2}}$?

A $-\frac{9}{2}\left(4-9x^4\right)^{-\frac{1}{2}}$

B $\frac{1}{2}\left(4-9x^{-4}\right)^{-\frac{1}{2}}$

C $2\left(4-9x^4\right)^{-\frac{1}{2}}$

D $-18x^3\left(4-9x^4\right)^{-\frac{1}{2}}$

9. $\sin x+\sqrt{3}\cos x$ can be written as $2\cos\left(x-\frac{\pi}{6}\right)$.

The maximum value of $\sin x+\sqrt{3}\cos x$ is 2.

What is the maximum value of $5\sin 2x+5\sqrt{3}\cos 2x$?

A 20

B 10

C 5

D 2

10. A sequence is defined by the recurrence relation

$u_{n+1}=(k-2)u_n+5$ with $u_0=3$.

For what values of k does this sequence have a limit as $n\rightarrow\infty$?

A $-3<k<-1$

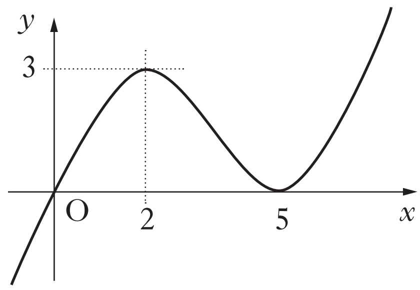
B $-1<k<1$

C $1<k<3$

D $k<3$

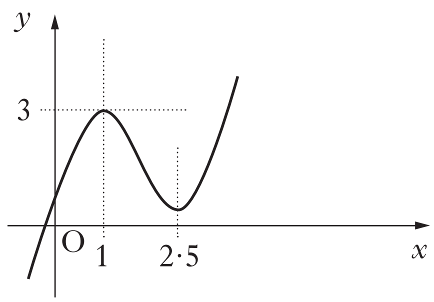
[Turn over

11. The diagram shows part of the graph of $y = f(x)$.

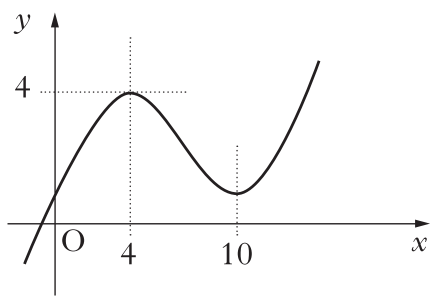


Which of the following diagrams could be the graph of $y = 2f(x) + 1$?

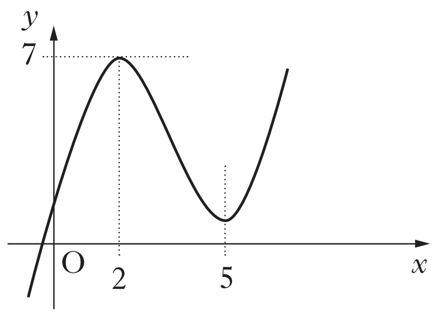
A



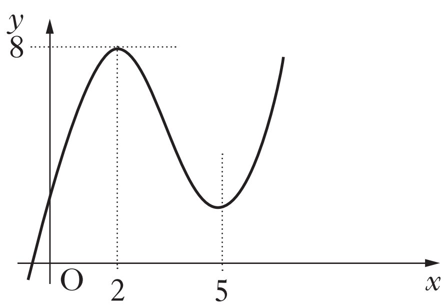
B



C



D



12. A function f, defined on a suitable domain, is given by $f(x)=\dfrac{6x}{x^2+6x-16}$.

What restrictions are there on the domain of f?

A $x \neq -8$ or $x \neq 2$

B $x \neq -4$ or $x \neq 4$

C $x \neq 0$

D $x \neq 10$ or $x \neq 16$

13. What is the value of $\sin\left(\dfrac{\pi}{3}\right)-\cos\left(\dfrac{5\pi}{4}\right)$?

A $\dfrac{\sqrt{3}}{2}-\dfrac{1}{\sqrt{2}}$

B $\dfrac{\sqrt{3}}{2}+\dfrac{1}{\sqrt{2}}$

C $\dfrac{1}{2}-\dfrac{1}{\sqrt{2}}$

D $\dfrac{1}{2}+\dfrac{1}{\sqrt{2}}$

14. The vectors $\boldsymbol{u}=\begin{pmatrix}1\\k\\k\end{pmatrix}$ and $\boldsymbol{v}=\begin{pmatrix}-6\\2\\5\end{pmatrix}$ are perpendicular.

What is the value of k?

A $\dfrac{-6}{7}$

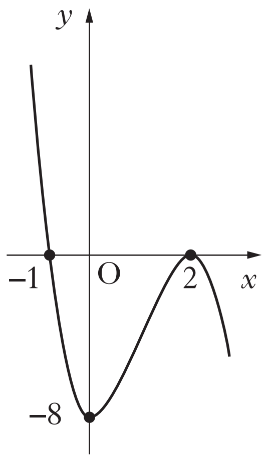
B -1

C 1

D $\dfrac{6}{7}$

[Turn over

15. The diagram shows a cubic curve passing through $(-1, 0)$, $(2, 0)$ and $(0, -8)$.



What is the equation of the curve?

A $\quad y = -2(x + 1)^2(x + 2)$

B $\quad y = -2(x + 1)(x - 2)^2$

C $\quad y = 4(x + 1)(x - 2)$

D $\quad y = -8(x + 1)(x - 2)^2$

16. The unit vectors $\boldsymbol{a}$ and $\boldsymbol{b}$ are such that $\boldsymbol{a}.\boldsymbol{b} = \frac{2}{3}$. Determine the value of $\boldsymbol{a}.(\boldsymbol{a} + 2\boldsymbol{b})$.

A $\quad \frac{2}{3}$

B $\quad \frac{4}{3}$

C $\quad \frac{7}{3}$

D $\quad 3$

17. $3x^2 + 12x + 17$ is expressed in the form $3(x + p)^2 + q$.

What is the value of q?

A $\quad 1$

B $\quad 5$

C $\quad 17$

D $\quad -19$

18. What is the value of $1 - 2\sin^2 15°$?

A $\frac{1}{2}$

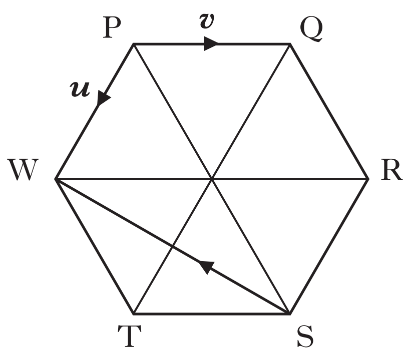
B $\frac{3}{4}$

C $\frac{\sqrt{3}}{2}$

D $\frac{7}{8}$

19. The diagram shows a regular hexagon PQRSTW.

$\overrightarrow{PW}$ and $\overrightarrow{PQ}$ represent vectors $\boldsymbol{u}$ and $\boldsymbol{v}$ respectively.



What is $\overrightarrow{SW}$ in terms of $\boldsymbol{u}$ and $\boldsymbol{v}$?

A $-\boldsymbol{u} - 2\boldsymbol{v}$

B $-\boldsymbol{u} - \boldsymbol{v}$

C $\boldsymbol{u} - \boldsymbol{v}$

D $\boldsymbol{u} + 2\boldsymbol{v}$

20. Evaluate $2 - \log_5 \frac{1}{25}$.

A -3

B 0

C $\frac{3}{2}$

D 4

[*END OF SECTION A*]

SECTION B

Marks

ALL questions should be attempted.

21. A curve has equation $y = 3x^2 - x^3$.

(*a*) Find the coordinates of the stationary points on this curve and determine their nature. **6**

(*b*) State the coordinates of the points where the curve meets the coordinate axes and sketch the curve. **2**

22. For the polynomial $6x^3 + 7x^2 + ax + b$,

- $x + 1$ is a factor
- 72 is the remainder when it is divided by $x - 2$.

(*a*) Determine the values of a and b. **4**

(*b*) Hence factorise the polynomial completely. **3**

23. (*a*) Find P and Q, the points of intersection of the line $y = 3x - 5$ and the circle C_1 with equation $x^2 + y^2 + 2x - 4y - 15 = 0$. **4**

(*b*) T is the centre of C_1.

Show that PT and QT are perpendicular. **3**

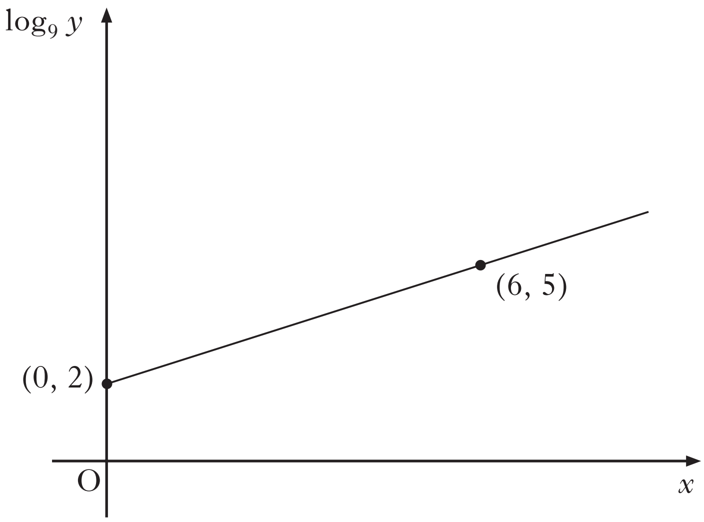
(*c*) A second circle C_2 passes through P, Q and T.

Find the equation of C_2. **3**

24. Two variables, x and y, are related by the equation *Marks*

$$y = ka^x.$$

When $\log_9 y$ is plotted against x, a straight line passing through the points (0, 2) and (6, 5) is obtained, as shown in the diagram.



Find the values of k and a. **5**

[*END OF SECTION B*]

[*END OF QUESTION PAPER*]

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X100/12/03

NATIONAL QUALIFICATIONS 2014

TUESDAY, 6 MAY
2.50 PM – 4.00 PM

MATHEMATICS
HIGHER
Paper 2

Read carefully

1 **Calculators may be used in this paper.**

2 Full credit will be given only where the solution contains appropriate working.

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LI

FORMULAE LIST

Circle:

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Table of standard derivatives:

$f(x)$	$f'(x)$
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Table of standard integrals:

$f(x)$	$\int f(x)dx$
$\sin ax$	$-\frac{1}{a} \cos ax + c$
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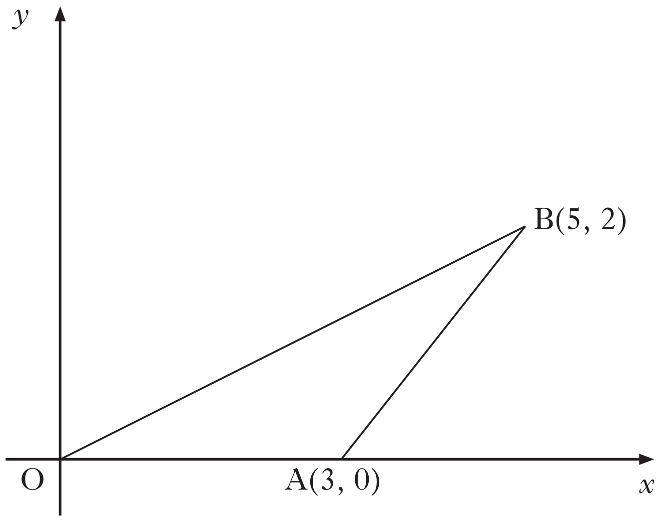
ALL questions should be attempted. *Marks*

1. A(3, 0), B(5, 2) and the origin are the vertices of a triangle as shown in the diagram.



(*a*) Obtain the equation of the perpendicular bisector of AB. **4**

(*b*) The median from A has equation $y + 2x = 6$.

Find T, the point of intersection of this median and the perpendicular bisector of AB. **2**

(*c*) Calculate the angle that AT makes with the positive direction of the x-axis. **2**

2. A curve has equation $y = x^4 - 2x^3 + 5$.

Find the equation of the tangent to this curve at the point where $x = 2$. **4**

3. Functions f and g are defined on suitable domains by

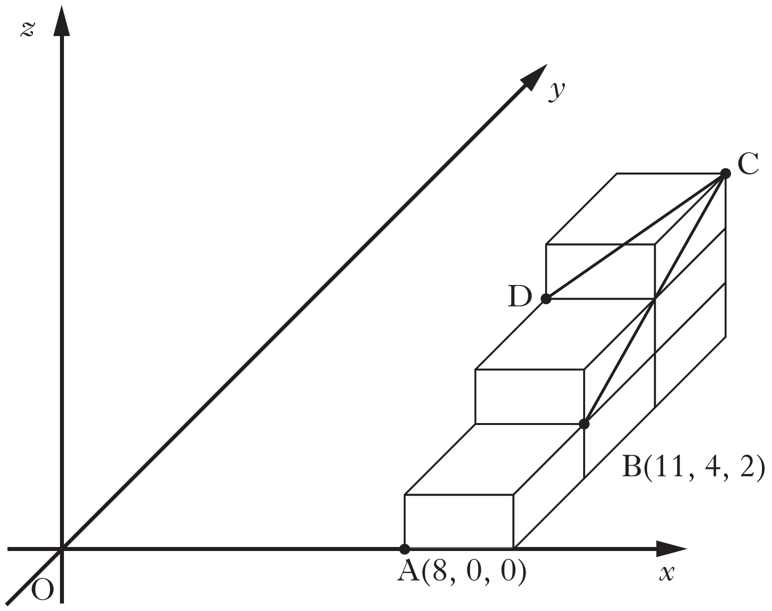
$$f(x) = x(x - 1) + q \quad \text{and} \quad g(x) = x + 3.$$

(*a*) Find an expression for $f(g(x))$. **2**

(*b*) Hence, find the value of q such that the equation $f(g(x)) = 0$ has equal roots. **4**

[Turn over

4. Six identical cuboids are placed with their edges parallel to the coordinate axes as shown in the diagram.



A and B are the points (8, 0, 0) and (11, 4, 2) respectively.

(*a*) State the coordinates of C and D. **2**

(*b*) Determine the components of $\overrightarrow{CB}$ and $\overrightarrow{CD}$. **2**

(*c*) Find the size of the angle BCD. **5**

5. Given that $\int_4^t (3x+4)^{-\frac{1}{2}}\,dx = 2$, find the value of t. **5**

6. Solve the equation

$$\sin x - 2\cos 2x = 1 \qquad \text{for } 0 \leq x < 2\pi.$$ **5**

7. Land enclosed between a path and a railway line is being developed for housing.

This land is represented by the shaded area shown in Diagram 1.

- The path is represented by a parabola with equation $y = 6x - x^2$.
- The railway is represented by a line with equation $y = 2x$.
- One square unit in the diagram represents $300\,\text{m}^2$ of land.

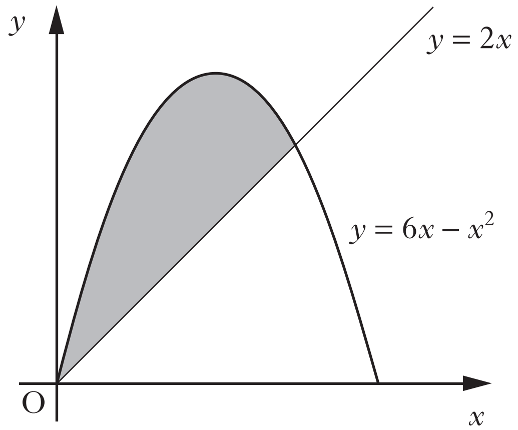


Diagram 1

(*a*) Calculate the area of land being developed. **5**

(*b*) A road is built parallel to the railway line and is a tangent to the path as shown in Diagram 2.

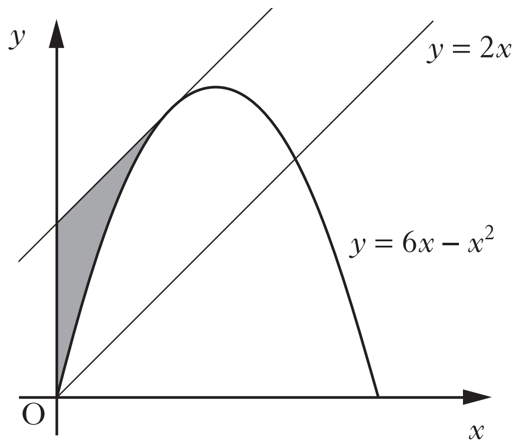


Diagram 2

It is decided that the land, represented by the shaded area in Diagram 2, will become a car park.

Calculate the area of the car park. **5**

[Turn over

Marks

8. Given that the equation

$$x^2 + y^2 - 2px - 4py + 3p + 2 = 0$$

represents a circle, determine the range of values of p. **5**

9. Acceleration is defined as the rate of change of velocity.

An object is travelling in a straight line. The velocity, v m/s, of this object, t seconds after the start of the motion, is given by $v(t) = 8\cos(2t - \frac{\pi}{2})$.

(*a*) Find a formula for $a(t)$, the acceleration of this object, t seconds after the start of the motion. **3**

(*b*) Determine whether the velocity of the object is increasing or decreasing when $t = 10$. **2**

(*c*) Velocity is defined as the rate of change of displacement.

Determine a formula for $s(t)$, the displacement of the object, given that $s(t) = 4$ when $t = 0$. **3**

[*END OF QUESTION PAPER*]

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2014 Mathematics

Higher

Finalised Marking Instructions

General Comments

These marking instructions are for use with the 2014 Higher Mathematics Examination.

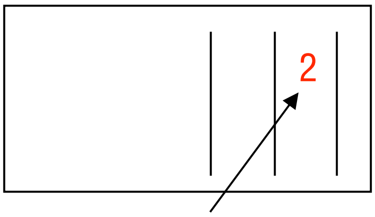
For each question the marking instructions are in two sections, namely Illustrative Scheme and Generic Scheme. The Illustrative Scheme covers methods which are commonly seen throughout the marking. The Generic Scheme indicates the rationale for which each mark is awarded. In general markers should use the Illustrative Scheme and only use the Generic Scheme where a candidate has used a method not covered in the Illustrative Scheme.

All markers should apply the following general marking principles throughout their marking:

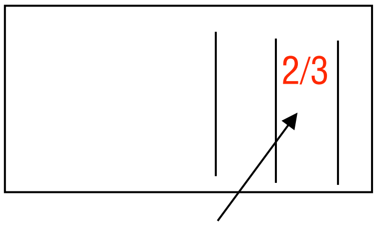
1 Marks must be assigned in accordance with these marking instructions. In principle, marks are awarded for what is correct, rather than deducted for what is wrong.

2 Award one mark for each •. There are no half marks.

3 The mark awarded for each part of a question should be entered in the outer right hand margin, opposite the end of the working concerned. The marks should correspond to those on the question paper and these marking instructions. Only the mark, as a whole number, should be written.



Marks in this column whole numbers only



Do not record marks on scripts in this manner.

4 Where a candidate has not been awarded any marks for an attempt at a question, or part of a question, 0 should be written in the right hand margin against their answer. It should not be left blank. If absolutely no attempt at a question, or part of a question, has been made, ie a completely empty space, then NR should be written in the outer margin.

5 IT IS ESSENTIAL that every page of a candidate's script should be checked for working. Unless blank, every page which is devoid of a marking symbol should have a tick placed in the bottom right hand margin.

6 Where the solution to part of a question is fragmented and continues later in the script, the marks should be recorded at the end of the solution. This should be indicated with a down arrow (↓), in the margin, at the earlier stages.

7 Working subsequent to an error must be followed through, with possible full marks for the subsequent working, provided that the level of difficulty involved is approximately similar. Where, subsequent to an error, the working for a follow through mark has been eased, the follow through mark cannot be awarded.

8 As indicated on the front of the question paper, full credit should only be given where the solution contains appropriate working. Throughout this paper, unless specifically mentioned in the marking instructions, a correct answer with no working receives no credit.

9 Marking Symbols

No comments or words should be written on scripts. Please use the following symbols and those indicated on the welcome letter and from comment 6 on the previous page.

A tick should be used where a piece of working is correct and gains a mark. Markers must check through the whole of a response, ticking the work only where a mark is awarded.

At the point where an error occurs, the error should be underlined and a cross used to indicate where a mark has not been awarded. If no mark is lost the error should only be underlined, ie a cross is only used where a mark is not awarded.

A cross-tick should be used to indicate "correct" working where a mark is awarded as a result of follow through from an error.

A double cross-tick should be used to indicate correct working which is irrelevant or insufficient to score any marks. This should also be used for working which has been eased.

A tilde should be used to indicate a minor error which is not being penalised, eg bad form.

This should be used where a candidate is given the benefit of the doubt.

A roof should be used to show that something is missing, such as part of a solution or a crucial step in the working.

These will help markers to maintain consistency in their marking and are essential for the later stages of SQA procedures.

The examples below illustrate the use of the marking symbols.

Example 1

$y = x^3 - 6x^2$

$\dfrac{dy}{dx} = 3x^2 - 12$

$3x^2 - 12 = 0$

$x = 2$

$y = -16$

•1

•2

•3

•4

•5

Example 2

A(4,4,0), B(2,2,6), C(2,2,0)

$\overrightarrow{AB} = \mathbf{b} + \mathbf{a} = \begin{pmatrix} 6 \\ 6 \\ 6 \end{pmatrix}$ •1

$\overrightarrow{AC} = \begin{pmatrix} 6 \\ 6 \\ 0 \end{pmatrix}$ •2

Example 3

$3\sin x - 5\cos x$

$k\sin x\cos a - \cos x\sin a$ •1

$k\cos a = 3,\ k\sin a = 5$ •2

Example 4

4	1	−5	2	8	•1
		4	−4	−8	
	1	1	−2	0	•2

Since the remainder is 0, $x-4$ must be a factor. •3

$(x^2 - x - 2)$ •4

$(x-4)(x+1)(x-2)$ •5

$x = 4$ or $x = -1$ or $x = 2$ •6

10 In general, as a consequence of an error perceived to be trivial, casual or insignificant, eg $6 \times 6 = 12$, candidates lose the opportunity of gaining a mark. But note example 4 in comment 9 and the second example in comment 11.

11 Where a transcription error (paper to script or within script) occurs, the candidate should be penalised, eg

This is a transcription error and so the mark is not awarded.

Eased as no longer a solution of a quadratic equation.

$x^2 + 5x + 7 = 9x + 4$ ✓

$x - 4x + 3 = 0$ ✗

$x = 1$ ✗

Exceptionally this error is not treated as a transcription error as the candidate deals with the intended quadratic equation. The candidate has been given the benefit of the doubt.

$x^2 + 5x + 7 = 9x + 4$ ✓

$x - 4x + 3 = 0$ ✓

$(x-3)(x-1) = 0$

$x = 1$ or 3 ✓

12 Cross marking

Where a question results in two pairs of solutions, this technique should be applied, but only if indicated in the detailed marking instructions for the question.

Example: Point of intersection of line with curve

Illustrative Scheme:		Cross marked:	
$\bullet^5$	$x = 2,\ x = -4$	$\bullet^5$	$x = 2,\ y = 5$
$\bullet^6$	$y = 5,\ y = -7$	$\bullet^6$	$x = -4,\ y = -7$

Markers should choose whichever method benefits the candidate, but **not** a combination of both.

13 In final answers, numerical values should be simplified as far as possible.

Examples: $\frac{15}{12}$ should be simplified to $\frac{5}{4}$ or $1\frac{1}{4}$ $\qquad$ $\frac{43}{1}$ should be simplified to 43

$\frac{15}{0{\cdot}3}$ should be simplified to 50 $\qquad$ $\frac{4/5}{3}$ should be simplified to $\frac{4}{15}$

$\sqrt{64}$ must be simplified to 8

The square root of perfect squares up to and including 100 must be known.

14 Commonly Observed Responses (COR) are shown in the marking instructions to help mark common and/or non-routine solutions. CORs may also be used as a guide in marking similar non-routine candidate responses.

15 Unless specifically mentioned in the marking instructions, the following should not be penalised:

- Working subsequent to a **correct** answer;
- Correct working in the wrong part of a question;
- Legitimate variations in numerical answers, eg angles in degrees rounded to nearest degree;
- Omission of units;
- Bad form;
- Repeated error within a question, but not between questions or papers.

16 In any 'Show that . . .' question, where the candidate has to arrive at a required result, the last mark of that part is not available as a follow through from a previous error.

17 All working should be carefully checked, even where a fundamental misunderstanding is apparent early in the candidate's response. Marks may still be available later in the question so reference must be made continually to the marking instructions. All working must be checked: the appearance of the correct answer does not necessarily indicate that the candidate has gained all the available marks.

18 Scored out working which has not been replaced should be marked where still legible. However, if the scored out working has been replaced, only the work which has not been scored out should be marked.

19 Where a candidate has made multiple attempts using the same strategy, mark all attempts and award the lowest mark.
Where a candidate has tried different strategies, apply the above ruling to attempts within each strategy and then award the highest resultant mark. For example:

Strategy 1 attempt 1 is worth 3 marks	Strategy 2 attempt 1 is worth 1 mark
Strategy 1 attempt 2 is worth 4 marks	Strategy 2 attempt 2 is worth 5 marks
From the attempts using strategy 1, the resultant mark would be 3.	From the attempts using strategy 2, the resultant mark would be 1.

In this case, award 3 marks.

20 It is of great importance that the utmost care should be exercised in totalling the marks.

A tried and tested procedure is as follows:

Step 1 Manually calculate the total from the candidate's script.

Step 2 Check this total using the grid issued with these marking instructions.

Step 3 Electronically enter the marks and obtain a total, which should now be compared to the manual total.

This procedure enables markers to identify and rectify any errors in data entry before submitting each candidate's marks.

21 **The candidate's script for Paper 2 should be placed inside the script for Paper 1, and** the **candidate's total score (ie** Paper 1 Section B + Paper 2) written in the space provided on the front cover of the script for Paper 1.

Paper 1 Section A

Question	Answer
1	C
2	B
3	A
4	D
5	D
6	A
7	C
8	D
9	B
10	C
11	C
12	A
13	B
14	D
15	B
16	C
17	B
18	C
19	A
20	D

Summary		
	A	4
	B	5
	C	6
	D	5

Question		Generic Scheme	Illustrative Scheme	Max Mark
21	a			
		•1 ss know to differentiate and one term correct	•1 $= 6x\ldots$ or $= \ldots. - 3x^2$	
		•2 ss the other term correct and set derivative to 0	•2 $6x - 3x^2 = 0$ stated explicitly	
		•3 pd solve $\frac{dy}{dx} = 0$	•3 $x = 0$ and 2	
		•4 pd evaluate y coordinates	•4 $y = 0$ and 4	
		•5 pd justify nature of stationary points	•5 use 2nd derivative or nature table	
		•6 ic interpretation	•6 min. at $(0,0)$ and max. at $(2,4)$	6

Notes:

1. •2 is not available for statements such as '$\frac{dy}{dx} = 0$' with no other working.
2. Accept $3x^2 - 6x = 0$ for •2.
3. For candidates using a nature table, the minimum response for •5 is:

   x values 0 and 2; $\frac{dy}{dx}$ or expression $6x - 3x^2$; signs and zeroes; shape.

		0		2	
$\frac{dy}{dx}$	−	0	+	0	−
	↘	—	↗	—	↘

4. For candidates who differentiate correctly but then solve $\frac{dy}{dx} = 0$ incorrectly, •4 may be awarded as a follow through mark. •5 and •6 are not available if a nature table has been used, but may be awarded where candidates have used the 2nd derivative.
5. For candidates who differentiate incorrectly •3 and •4 may be awarded as follow through marks. •5 and •6 are not available if a nature table has been used, but may be awarded where candidates have used the 2nd derivative.
6. At •6 stage accept min at $x = 0$ and max at $x = 2$.
7. Candidates who find the x-coordinates of the SPs correctly but correctly process only one of these to determine its nature, gain •6 but not •5.

Commonly Observed Responses:

Candidate A

$\frac{d^2y}{dx^2}=6-6x$

at $x=0$, $\frac{d^2y}{dx^2}>0$, at $x=2$, $\frac{d^2y}{dx^2}<0$ $\bullet^5$ ✓

hence minimum SP at $x=0$, maximum SP at $x=2$ $\bullet^6$ ✓

Candidate B

$\frac{dy}{dx}=6x-3x^2=0$ $\bullet^1$ ✓ $\bullet^2$ ✓

$3x(3-x)=0$

$x=0, x=3$ $\bullet^3$ ✗

$y=0, y=0$ $\bullet^4$ ✓

Case (i)

$\frac{d^2y}{dx^2}=6-6x$

$x=0 \Rightarrow \frac{d^2y}{dx^2}>0 \Rightarrow$ Minimum SP $\bullet^5$ ✓

$x=3 \Rightarrow \frac{d^2y}{dx^2}<0 \Rightarrow$ Maximum SP $\bullet^6$ ✓

Case (ii)

x	$\rightarrow$	0	$\rightarrow$	3	$\rightarrow$
$\frac{dy}{dx}$	$-$	0	?	?	+

$\bullet^5$ ✗ $\bullet^6$ ✗

? inconsistent. Different signs for $6x-3x^2$ or $3x(3-x)$

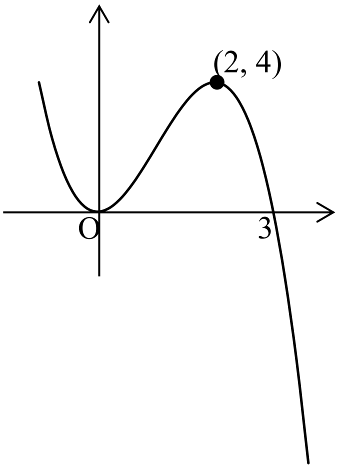
21	b			
$\bullet^7$	pd	find intercepts	$\bullet^7$ $3x^2-x^3=0$ and $(3,0)$ or $x=3$; $(0,0)$ [may appear in part a]	2
$\bullet^8$	ic	sketch	$\bullet^8$ sketch	

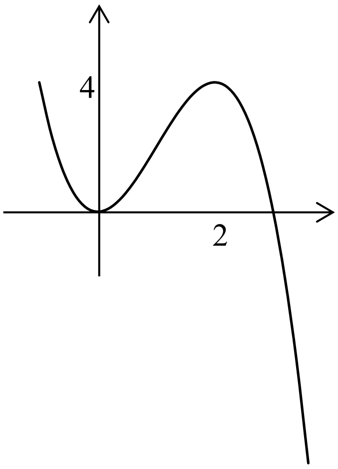
Notes:

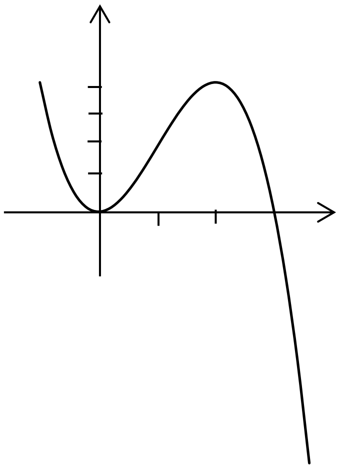
8. $\bullet^7$ accept $3x^2-x^3=0$ and correctly annotated diagram with 0, 3 and no other intercepts marked on sketch.
9. The minimum required for $\bullet^8$ is a cubic curve, consistent with the SPs found in part (a) and appropriate number of x intercepts appearing on their sketch. It must be possible to determine the coordinates of the SPs from the sketch.

Commonly Observed Responses:

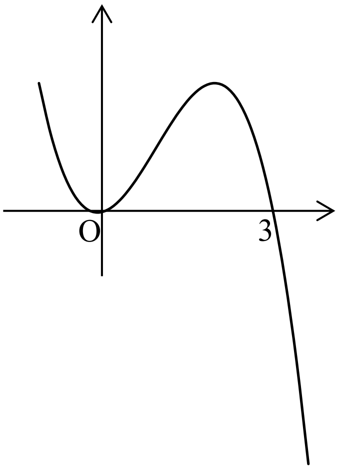
The following are acceptable for $\bullet^{8}$

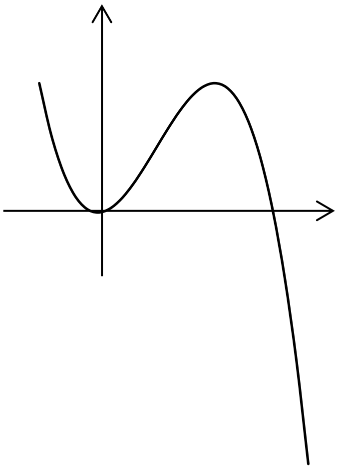


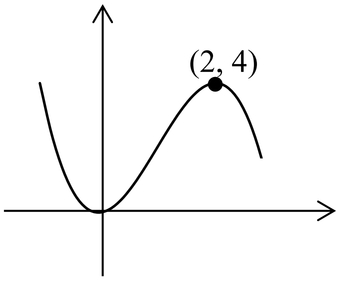


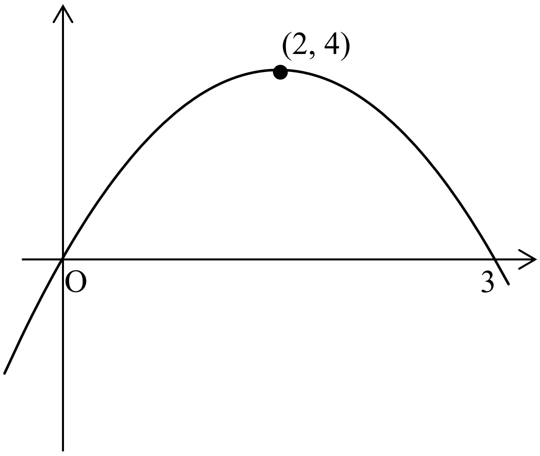


Do not accept the following for $\bullet^{8}$









Question		Generic Scheme	Illustrative Scheme	Max Mark
22	a			
		•[1] ss know to use $x=-1$ and obtain an equation	•[1] $6(-1)^3+7(-1)^2+a(-1)+b=0$	4
		•[2] ss know to use $x=2$ and obtain an equation	•[2] $6(2)^3+7(2)^2+a(2)+b=72$	
		•[3] pd process equations to find one value	•[3] $a=-1$ or $b=-2$	
		•[4] pd find the other value	•[4] $b=-2$ or $a=-1$	

Alternative Method for •[1] and •[2]

•[1]

$$\begin{array}{r|cccc} -1 & 6 & 7 & a & b \\ & & -6 & -1 & -a+1 \\ \hline & 6 & 1 & a-1 & b-a+1=0 \end{array}$$

•[2]

$$\begin{array}{r|cccc} 2 & 6 & 7 & a & b \\ & & 12 & 38 & 2a+76 \\ \hline & 6 & 19 & a+38 & 2a+b+76=72 \end{array}$$

Note:

1. An incorrect value at •[3] should be followed through for the possible award of •[4]. However, if the equations are such that no solution exists, then •[3] and •[4] are not available.

Commonly Observed Responses:

Candidate A

•[1] x

$$\begin{array}{r|cccc} 1 & 6 & 7 & a & b \\ & & 6 & 13 & a+13 \\ \hline & 6 & 13 & a+13 & a+b+13=0 \end{array}$$

•[2] ✓ repeated error

$$\begin{array}{r|cccc} -2 & 6 & 7 & a & b \\ & & -12 & 10 & -2a-20 \\ \hline & 6 & -5 & a+10 & -2a+b-20=72 \end{array}$$

Solving to get $a=-35,\ b=22$ •[3] ✓ •[4] ✓

Leading to, in part (b), $\Rightarrow 6x^3+7x^2-35x+22=(x-1)(6x^2+13x-22)$

•[5] ✓ •[6] ✓ •[7] ^

Question		Generic Scheme		Illustrative Scheme	Max Mark
22	b				
$\bullet^5$	ss	substitute for a and b and know to divide by $x+1$	$\bullet^5$	$(6x^3+7x^2-x-2)\div(x+1)$ Stated or implied by $\bullet^6$	
$\bullet^6$	pd	obtain quadratic factor	$\bullet^6$	$(x+1)(6x^2+x-2)$	
$\bullet^7$	pd	complete factorisation	$\bullet^7$	$(x+1)(3x+2)(2x-1)$	3

Notes:

2. For candidates who substitute $a=-1$ into the correct quotient from part (a), $\bullet^5$, $\bullet^6$ and $\bullet^7$ are available.
3. Candidates who use incorrect values obtained in part (a) may gain $\bullet^5$, $\bullet^6$ and $\bullet^7$
4. Where the quadratic factor obtained is irreducible, candidates must clearly demonstrate that $b^2-4ac<0$ to gain $\bullet^7$.
5. Do not penalise the inclusion of '$=0$' or for solving for x.
6. Candidates who use values, ex nihilo, for a and b can gain $\bullet^5$, if division is correct, but $\bullet^6$ and $\bullet^7$ are only available if $(x+1)$ is a factor of the resulting expression.

Commonly Observed Responses:

Candidate B

22a no solution

22b $a=-4, b=-5$ ex nihilo

$(6x^3+7x^2-4x-5)\div(x+1)$ $\bullet^5$ ✓

$$\begin{array}{r|rrrr} -1 & 6 & 7 & -4 & -5 \\ & & -6 & -1 & 5 \\ \hline & 6 & 1 & -5 & 0 \end{array}$$

$(x+1)(6x^2+x-5)$ $\bullet^6$ ✓

$(x+1)(6x-5)(x+1)$ $\bullet^7$ ✓

Candidate C

22a no solution

22b $a=2, b=3$ ex nihilo

$(6x^3+7x^2+2x+3)\div(x+1)$ $\bullet^5$ ✓

$$\begin{array}{r|rrrr} -1 & 6 & 7 & 2 & 3 \\ & & -6 & -1 & -1 \\ \hline & 6 & 1 & 1 & 2 \end{array}$$

$\Rightarrow$ $(x+1)$ is not a factor

$\bullet^6$ and $\bullet^7$ are not available

Candidate D

22a no solution

22b $a=4, b=3$ ex nihilo

$(6x^3+7x^2+4x+3)\div(x+1)$ $\bullet^5$ ✓

$$\begin{array}{r|rrrr} -1 & 6 & 7 & 4 & 3 \\ & & -6 & -1 & -3 \\ \hline & 6 & 1 & 3 & 0 \end{array}$$

$(x+1)(6x^2+x+3)$ $\bullet^6$ ✓

$b^2-4ac=1-72=-71$

$-71<0$ so does not factorise $\bullet^7$ ✓

Question		Generic Scheme			Illustrative Scheme	Max Mark
23	a					
		•1	ss	substitute $3x-5$	•1 $x^2+(3x-5)^2+2x-4(3x-5)-15=0$	
		•2	pd	express in standard quadratic form	•2 $10x^2-40x+30=0$	
		•3	pd	find x-coordinates	•3 $x=1$ | $x=3$	
		•4	pd	find y-coordinates	•4 $y=-2$ | $y=4$ •3 •4	4

Notes:

1. '$=0$' must appear at •1 or •2 for mark •2 to be awarded.
2. If $x=\frac{1}{3}(y+5)$ is substituted at •1 then $10y^2-20y-80=0$ is obtained at •2.
3. Special Case: In cases where $x=1$ and $x=3$ do not appear as a result of •1 and •2, but are substituted into the equation of the line to obtain the y values, if the candidate then checks that both points lie on the circle, $^3\!/_4$ marks are awarded. If, in addition, the candidate makes a statement to the effect that a line can only cut a circle in, at most, 2 points, then $^4\!/_4$ marks are awarded. Otherwise, $^0\!/_4$ marks.
4. •3 and •4 are not available for any attempt to solve a quadratic equation of the form $ax^2+bx=c$

Commonly Observed Responses:

Candidate A

$x^2+(3x-5)^2+2x-4(3x-5)-15=0$	•1 ✓
$10x^2-40x+40=0$	•2 ✗
$x=2$ and $y=1$	•3 ✓ •4 ^

Question		Generic Scheme			Illustrative Scheme	Max Mark
23	b					
		•5	ss	state centre	•5 $(-1,2)$	
		•6	pd	calculate gradients	•6 $m=-2$, $m=\frac{1}{2}$	
		•7	ic	communicate result	•7 demonstrates $m_1\times m_2=-2\times\frac{1}{2}=-1$ $\Rightarrow$ PT is perpendicular to QT [or other appropriate statement]	
					Alternative Method	
		•5	ss	state centre	•5 $(-1,2)$	
		•6	pd	calculate vectors	•6 eg $\begin{pmatrix}-2\\4\end{pmatrix}$ and $\begin{pmatrix}-4\\-2\end{pmatrix}$	
		•7	ic	communicate result	•7 $\begin{pmatrix}-2\\4\end{pmatrix}.\begin{pmatrix}-4\\-2\end{pmatrix}=-2\times-4+4\times-2=0$ $\Rightarrow$ PT is perpendicular to QT [or other appropriate statement]	3

Notes:

4. Other valid strategies:
   a Converse of Pythagoras' Theorem:
   - •[6] process lengths, $PT = QT = \sqrt{20}$, $PQ = \sqrt{40}$
   - •[7] apply converse and communicate result clearly.

   b Cosine Rule:
   - •[6] process lengths, •[7] apply cosine rule to obtain angle 90° and communicate result clearly.

Commonly Observed Responses:

Candidate B		Candidate C	
$T(-1,2)$	•[5] ✓	$T(-1,2)$	•[5] ✓
$m = \frac{1}{2}, m = -2$	•[6] ✓	$m_1 = \frac{1}{2}, m_2 = -2$	•[6] ✓
$m_1 \times m_2 = -1$	•[7] ^	$m_1 \times m_2 = -1$	•[7] ✓
No link between required condition and gradients found.		Required condition is linked to gradients found.	

23	c		
•[8] ss knows to find and states centre •[9] pd calculate radius •[10] ic state equation of circle		•[8] centre (2, 1) •[9] radius $= \sqrt{10}$ •[10] $(x-2)^2 + (y-1)^2 = 10$	3
		Alternative Method	
		$x^2 + y^2 + 2gx + 2fy + c = 0$	
•[8] ss substitute points into general equation of circle		•[8] $25 + 6g + 8f + c = 0$ $5 + 2g - 4f + c = 0$ $5 - 2g + 4f + c = 0$	
•[9] pd find f or g or c		•[9] $f = -1$, or $g = -2$, or $c = -5$	
•[10] ic state values of f, g and c		•[10] $f = -1, g = -2, c = -5$	

Notes:

5. $(\sqrt{10})^2$ must be simplified to gain •[10]
6. For candidates who find P and Q correctly in part (a), award •[8] if centre $(2,1)$ appears without working.
7. For the mid-point of PQ being (2,1), •[8] is available unless subsequent working indicates that this is not the intended centre.
8. •[9] is only available as a result of PQ being a diameter, or using a valid strategy to find the centre eg midpoint of PQ or point of intersection of the perpendicular bisectors of PT and TQ. •[10] is still available.
9. Where an incorrect centre or an incorrect radius appear ex nihilo •[10] is not available.

Question		Generic Scheme	Illustrative Scheme	Max Mark
24				
			Method 1	
		•[1] ss take $\log_9$ of both sides of the equation	•[1] $\log_9 y = \log_9 ka^x$	
		•[2] pd apply laws of logarithms	•[2] $\log_9 y = \log_9 k + \log_9 a^x$	
		•[3] pd apply laws of logarithms	•[3] $\log_9 y = \log_9 k + x\log_9 a$	
		•[4] pd find k	•[4] $\log_9 k = 2, k = 81$ or $k = 9^2 = 81$	
		•[5] pd find a	•[5] $\log_9 a = \frac{1}{2}, a = 3$ or $a = 9^{1/2} = 3$	5
			Method 2	
		•[1] ss know to use equation of the line	•[1] $\log_9 y = \frac{1}{2}x + 2$	
		•[2] pd write in exponential form	•[2] $y = 9^{\frac{1}{2}x+2}$	
		•[3] pd apply laws of indices	•[3] $y = 9^{\frac{1}{2}x}9^2$	
		•[4] pd find k	•[4] $k = 81$	
		•[5] pd find a	•[5] $a = 3$	

Notes:

1. Candidates who start with •[3] $\log_9 y = \log_9 k + x\log_9 a$ also gain •[1] and •[2].
2. In Method 1, base 9 must appear by •[4] stage, for •[1] to be awarded.
3. For $k = 81$ and $a = 3$ with spurious or no working, •[4] and •[5] are not available.

Commonly Observed Responses:

Candidate A

$\log y = \log ka^x$ $\quad\bullet^1$ ✗ $\quad$ See Note 2

$\log y = \log k + \log a^x$ $\quad\bullet^2$ ✓

$\log y = \log k + x\log a$ $\quad\bullet^3$ ✓

$k = 81$ $\quad\bullet^4$ ✗ $\quad$ No evidence of which base is being used.

$a = 3$ $\quad\bullet^5$ ✓ $\quad$ Answers at both $\bullet^4$ and $\bullet^5$ are consistent with using base 9.

Candidate B : A combination of Method 1 and Method 2.

M2 $\quad \log_9 y = \frac{1}{2}x + 2$ $\quad\bullet^1$ ✓

M1 $\quad \log_9 y = \log_9 ka^x$

$\Rightarrow \log_9 y = x\log_9 a + \log 9k$ $\quad\bullet^2$ ✓ $\quad\bullet^3$ ✓

equating gradients and intercepts

$\log_9 a = \frac{1}{2}$

$a = 9^{\frac{1}{2}} = 3$ $\quad\bullet^4$ ✓

$\log_9 k = 2$ $\quad\bullet^5$ ✓

$k = 9^2 = 81$

Candidate C

at $(0,2)$ $\log_9 y = 2$

$\Rightarrow y = 9^2 = 81$ $\quad\bullet^1$ ✓

Substitute into equation

$81 = ka^0$

$\Rightarrow k = 81$ $\quad\bullet^2$ ✓

at $(6,5)$ $\log_9 y = 5$

$\Rightarrow y = 9^5$ $\quad\bullet^3$ ✓

substitute into equation

$9^5 = ka^6$

$\Rightarrow 9^5 = 81a^6$ $\quad\bullet^4$ ✓

$\Rightarrow a^6 = 9^3 = 3^6$

$\Rightarrow a = 3$ $\quad\bullet^5$ ✓

Paper 2

Question		Generic Scheme		Illustrative Scheme	Max Mark
1	a				
$\bullet^1$	ss	find gradient of AB	$\bullet^1$	$m_{AB}=1$	
$\bullet^2$	pd	find perpendicular gradient	$\bullet^2$	$m_{perp}=-1$ stated or implied by $\bullet^4$	
$\bullet^3$	pd	find midpoint of AB	$\bullet^3$	$(4,1)$ stated or implied by $\bullet^4$	
$\bullet^4$	pd	obtain equation	$\bullet^4$	$y-1=-1(x-4)$	4

Notes:

1. $\bullet^4$ is only available as a consequence of using a perpendicular gradient and a midpoint.
2. The gradient must appear in simplified form at $\bullet^4$ stage for $\bullet^4$ to be awarded.

Commonly Observed Responses:

Candidate A

$m_{\text{AB}}=-1$ $\bullet^1$ ✗

$m_{\text{perp}}=1$ $\bullet^2$ ✓1

$(4,1)$ $\bullet^3$ ✓

$y-1=1(x-4) \Rightarrow y=x-3$ $\bullet^4$ ✓1

Leading to part (b)

$y-x=-3$
$y+2x=6$ $\bullet^5$ ✓1

$(3,0)$ $\bullet^6$ ✓1

$\bullet^7$ and $\bullet^8$ are not available as $\text{A}=\text{T}=(3,0)$

Question		Generic Scheme	Illustrative Scheme	Max Mark
1	b			
		•5 ss know to solve simultaneously	•5 $y+2x=6$ $y+x=5$	
		•6 pd solve correctly for x and y	•6 $x=1,\ y=4$	2

Commonly Observed Responses:

Candidate B

Part (a) $y-1=-1(x-4)$ •4 ✓

$y=-x+3$ error

Part (b) $y+2x=6$ and $y+x=3$ •5 ✓

$x=3,\ y=0$ •6 ✓ correct strategy used, pd mark not available

Question		Generic Scheme	Illustrative Scheme	Max Mark
1	c			
		•7 ss know and use $m=\tan\theta$	•7 $\tan\theta=-2$	
		•8 pd calculate angle	•8 $116{\cdot}6^{\circ}$ accept 117^{0} or $2{\cdot}03$ radians	2

Commonly Observed Responses:

Candidate C

$m_{\text{AT}}=-\dfrac{1}{2}$

base angle $=26{\cdot}6^{\circ}$ •7 ✗

$\Rightarrow$ angle $=90+26{\cdot}6=116{\cdot}6^{\circ}$ •8 ✗

Candidate D

$m_{\text{AT}}=2$ •7 ✗

angle $=\tan^{-1}(2)=63{\cdot}4^{\circ}$ •8 ✓

Candidate E:

Part (a)

$m_{\text{AB}}=\dfrac{2-0}{5-3}=\dfrac{2}{8}=\dfrac{1}{4}$ •1 ✗

$m_{\text{perp}}=-4$ •2 ✓

Midpoint of AB (4, 1) •3 ✓

$y-1=-4(x-1)$ •4 ✓

$y+4x-5$

Part (b)

$y+4x-5=0$
$y+2x+6=0$ •5 ✗ $\Rightarrow$ $y+2x=-6$
$y+4x=-5$ •6 ✗

$\Rightarrow 2x=1,\ \ x=\dfrac{1}{2},\ y=-7$

•5 is a strategy mark. The correct strategy is to solve the given equation with the equation from part (a) simultaneously. •5 is not awarded as the given equation has not been used.

The equation obtained at stage •4, has been rearranged incorrectly in part (b). The next pd mark, •6, is therefore not awarded.

Question		Generic Scheme	Illustrative Scheme	Max Mark
2				
		$\bullet^1$ ss know to and differentiate	$\bullet^1$ $4x^3 - 6x^2$	
		$\bullet^2$ ic find gradient	$\bullet^2$ 8	
		$\bullet^3$ pd find y-coordinate	$\bullet^3$ 5	
		$\bullet^4$ ic state equation of tangent	$\bullet^4$ $y-5=8(x-2)$	4

Notes:

1. $\bullet^4$ is only available if an attempt has been made to find the gradient from differentiation and calculating the y-coordinate by substitution into the original equation.

Commonly Observed Responses:

Candidate A

$\bullet^1$ ✓ $\bullet^2$ ✓ $\bullet^3$ ✓

using $y = mx + c$

$x=2,\ y=5,\ m=8$

$\Rightarrow 5 = 8\times 2 + c$

$\Rightarrow c = -11$ $\bullet^4$ ✓

$y = 8x - 11$

Question		Generic Scheme	Illustrative Scheme	Max Mark
3	a			
		●[1] ic interpret notation ●[2] pd a correct expression	●[1] $f(x+3)$ stated or implied by ●[2] ●[2] $=(x+3)(x+2)+q$ OR $=(x+3)^2-(x+3)+q$ or equivalent	2

Notes:

1. Special Case: ●[1] is for substituting $(x+3)$ for x thus, treat $x+3(x+3-1)+q$ as bad form.

Commonly Observed Responses:

Candidate A	Candidate B
$f(g(x))=x+3(x+3-1)+q$ ●[1] ✓ $=x^2+5x+6+q$ ●[2] ✓ ●[3] ✓	$f(g(x))=x+3(x+3-1)+q$ ●[1] ✓ $=4x+6+q$ ●[2] ✗

Candidate C	Candidate D
$f(g(x))=x+3(x+3-1)+q$ ●[1] ✓ $=(x+3)^2-x+3+q$ $x^2+5x+6+q=0$ ●[2] ✓ ●[3] ✓	$f(g(x))=(x+3)(x+3-1)+q$ ●[1] ✓ ●[2] ✓ $=(x+3)^2-x+3+q$ $x^2+5x+12+q=0$ ●[3] ✗

Candidate E: using $g(f(x))$

part (a)	part (b)
$g(f(x))=g(x(x-1)+q)$ ●[1] ✗	$x^2-x+q+3=0$ ●[3] ✓ (eased)
$=x(x-1)+q+3$ ●[2] ✓	$b^2-4ac=(-1)^2-4\times1\times(q+3)$ ●[4] ✓
Leading to …..	$1-4q-12=0$ ●[5] ✓
	$q=-\frac{11}{4}$ ●[6] ✓

Question		Generic Scheme	Illustrative Scheme	Max Mark
3	b			
		Method 1	**Method 1**	
		•3 pd write in standard quadratic form	•3 $x^2 + 5x + 6 + q = 0$	
		•4 ic use discriminant	•4 $b^2 - 4ac = 5^2 - 4\times1\times(6+q)$	
		•5 pd simplify and equate to zero	•5 $\Rightarrow 25 - 24 - 4q = 0$	
		•6 pd find value of q	•6 $q = \frac{1}{4}$	4
		Method 2	**Method 2**	
		•3 pd write in standard quadratic form	•3 $x^2 + 5x + 6 + q = 0$	
		•4 ic complete the square	•4 $\left(x+\frac{5}{2}\right)^2 - \frac{25}{4} + 6 + q = 0$	
		•5 pd equate to zero	•5 $-\frac{25}{4} + 6 + q = 0$	
		•6 pd find value of q	•6 $q = \frac{1}{4}$	
		Method 3	**Method 3**	
		•3 pd write in standard quadratic form	•3 $f(g(x)) = x^2 + 5x + 6 + q = 0$	
		•4 ic geometric interpretation	•4 equal roots so touches x-axis at SP	
		•5 pd differentiates to obtain x	•5 $\Rightarrow \frac{dy}{dx} = 2x + 5 = 0$ $x = -\frac{5}{2}$	
		•6 pd find value of q	•6 $\frac{25}{4} - \frac{25}{2} + 6 + q = 0$ $q = \frac{1}{4}$	

Notes:

2. Do not penalise the omission of ‘$=0$’ at •3.
3. In Method 1 $a=1$, $b=5$, $c=6+q$ is sufficient for •3.
4. Candidates who assume ‘$=0$’ and follow through to a correct value of q, •6 is still available. In Methods 1 and 2 ‘$=0$’ must appear at •4 or •5 for •5 to be awarded.
5. If the expression obtained at •3 is not a quadratic then •3, •4, •5 and •6 are not available.

Question		Generic Scheme	Illustrative Scheme	Max Mark
Throughout this question treat coordinates written as components, and vice versa, as bad form.				
4	a			
$\bullet^1$	pd	states coordinates of C	$\bullet^1$ C(11,12,6)	
$\bullet^2$	pd	states coordinates of D	$\bullet^2$ $D(8,8,4)$	2

Notes:

1. Accept $x=11$, $y=12$ and $z=6$ for $\bullet^1$ and $x=8$, $y=8$ and $z=4$ for $\bullet^2$.
2. For candidates who write the coordinates as Cartesian triples and omit brackets in both cases, $\bullet^2$ is not available.

Question		Generic Scheme	Illustrative Scheme	Max Mark
4	b			
$\bullet^3$	pd	finds $\overrightarrow{CB}$	$\bullet^3$ $\begin{pmatrix} 0 \\ -8 \\ -4 \end{pmatrix}$	
$\bullet^4$	pd	finds $\overrightarrow{CD}$	$\bullet^4$ $\begin{pmatrix} -3 \\ -4 \\ -2 \end{pmatrix}$	2

Notes:

3. For candidates who find both $\overrightarrow{BC}$ and $\overrightarrow{DC}$, only $\bullet^4$ is available (repeated error).

Question		Generic Scheme	Illustrative Scheme	Max Mark
4	c	.		
$\bullet^5$	ss	know to use scalar product applied to the correct angle	$\bullet^5$ $\cos B\hat{C}D = \dfrac{\overrightarrow{CB}.\overrightarrow{CD}}{\lvert\overrightarrow{CB}\rvert\lvert\overrightarrow{CD}\rvert}$ stated or implied by $\bullet^9$	
$\bullet^6$	pd	find scalar product	$\bullet^6$ 40	
$\bullet^7$	pd	find $\lvert\overrightarrow{CB}\rvert$	$\bullet^7$ $\sqrt{80}$	
$\bullet^8$	pd	find $\lvert\overrightarrow{CD}\rvert$	$\bullet^8$ $\sqrt{29}$	
$\bullet^9$	pd	find angle	$\bullet^9$ $33{\cdot}9^{\circ}$	5

Notes:

4. $\bullet^5$ is not available for candidates who choose to evaluate an incorrect angle.
5. $\bullet^9$ accept 33·8 to 34 degrees or 0·59 to 0·6 radians.
6. If candidates do not attempt $\bullet^9$, then $\bullet^5$ is only available if the formula quoted relates to the labelling in the question.
7. $\bullet^9$ is only available as a result of using a valid strategy.
8. $\bullet^5$ is not available for candidates who write $\cos\theta = \dfrac{40}{\sqrt{80}\times\sqrt{29}}$. Some reference to the labelling of the diagram **must** be made within their solution to part (c), to indicate they are attempting to find the correct angle.

Commonly Observed Responses:	
Candidate A: Cosine Rule $\cos B\hat{C}D = \dfrac{CB^2 + CD^2 - BD^2}{2 \times CB \times CD}$ •5 ✓ $CB = \sqrt{80},\ CD = \sqrt{29},\ BD = \sqrt{29}$ •6 ✓ •7 ✓ •8 ✓ $33 \cdot 9^{\circ}$ or $0 \cdot 59$ radians •9 ✓	Candidate B $\cos B\hat{C}D = \dfrac{\overrightarrow{BC}.\overrightarrow{CD}}{\lvert\overrightarrow{BC}\rvert \times \lvert\overrightarrow{CD}\rvert}$ •5 X $\overrightarrow{BC}.\overrightarrow{CD} = -40$ •6 ✓ $\lvert\overrightarrow{BC}\rvert = \sqrt{80}$, $\lvert\overrightarrow{CD}\rvert = \sqrt{29}$ •7 ✓ •8 ✓ $146 \cdot 1^{\circ}$ or $2 \cdot 55$ radians •9 ✓
Candidate C $\cos B\hat{O}D = \dfrac{\overrightarrow{OB}.\overrightarrow{OD}}{\lvert\overrightarrow{OB}\rvert \times \lvert\overrightarrow{OD}\rvert}$ •5 X $\overrightarrow{OB}.\overrightarrow{OD} = 128$ •6 ✓ $\lvert\overrightarrow{OB}\rvert = \sqrt{141}$, $\lvert\overrightarrow{OD}\rvert = 12$ •7 ✓ •8 ✓ $26 \cdot 1^{\circ}$ or $0 \cdot 46$ radians •9 ✓	Candidate D $\cos C\hat{B}D = \dfrac{\overrightarrow{BC}.\overrightarrow{BD}}{\lvert\overrightarrow{BC}\rvert \times \lvert\overrightarrow{BD}\rvert}$ •5 X $\overrightarrow{BC}.\overrightarrow{BD} = 40$ •6 ✓ $\lvert\overrightarrow{BC}\rvert = \sqrt{80}$, $\lvert\overrightarrow{BD}\rvert = \sqrt{29}$ •7 ✓ •8 ✓ $33 \cdot 9^{\circ}$ or $0 \cdot 59$ radians •9 ✓
Candidate E $\cos B\hat{O}C = \dfrac{\overrightarrow{OB}.\overrightarrow{OC}}{\lvert\overrightarrow{OB}\rvert \times \lvert\overrightarrow{OC}\rvert}$ •5 X $\overrightarrow{OB}.\overrightarrow{OC} = 181$ •6 ✓ $\lvert\overrightarrow{OB}\rvert = \sqrt{141}$, $\lvert\overrightarrow{OC}\rvert = \sqrt{301}$ •7 ✓ •8 ✓ $28 \cdot 5^{\circ}$ or $0 \cdot 50$ radians •9 ✓	Candidate F $\cos B\hat{C}D = \dfrac{\overrightarrow{BC}.\overrightarrow{DC}}{\lvert\overrightarrow{BC}\rvert \times \lvert\overrightarrow{DC}\rvert}$ •5 ✓ this is an acceptable form for the scalar product.

Question		Generic Scheme			Illustrative Scheme		Max Mark
5							
		•1	ss	start to integrate	•1	$\frac{1}{\frac{1}{2}}(\ldots\ldots)^{\frac{1}{2}}$	
		•2	pd	complete integration	•2	$\ldots\times\frac{1}{3}$	
		•3	pd	process limits	•3	$\frac{2}{3}(3t+4)^{\frac{1}{2}}-\frac{2}{3}(3(4)+4)^{\frac{1}{2}}$	
		•4	pd	start to solve equation	•4	$(3t+4)^{\frac{1}{2}}=7$	
		•5	pd	solve for t	•5	$t=15$	5

Notes:

1. •3 is awarded for correct substitution leading to $F(t)-F(4)$ where $F(x)$ is the candidates attempt
2. to integrate $(3x+4)^{-\frac{1}{2}}$. For substituting into the original function •3 is unavailable.
3. •5 is only available as a consequence of squaring both sides of an equation.
4. The integral obtained must contain a non integer power for •4 and •5 to be available.
5. Do not penalise the inclusion of '$+c$'.
6. Incorrect expansion of $(\ldots)^{-1/2}$ at stage •1, only •3 is available as follow through. Incorrect expansion of $(\ldots)^{1/2}$ at stage •4, •4 and •5 are not available.

Commonly Observed Responses:

Candidate A: Forgetting the $\frac{1}{3}$

$\left[2(3x+4)^{\frac{1}{2}}\right]_4^t=2$ •1 ✓ •2 X

$\left(2(3t+4)^{\frac{1}{2}}\right)-\left(2(3(4)+4)^{\frac{1}{2}}\right)=2$ •3 ✓

$(3t+4)^{\frac{1}{2}}=5$ •4 ✓

$t=7$ •5 ✓

Candidate B

$\left[\frac{1}{6}(3x+4)^{\frac{1}{2}}\right]_4^t=2$ •1 X •2 ✓

$\left(\frac{1}{6}(3t+4)^{\frac{1}{2}}\right)-\left(\frac{1}{6}(3(4)+4)^{\frac{1}{2}}\right)=2$ •3 ✓

$(3t+4)^{\frac{1}{2}}=16$ •4 ✓

$t=84$ •5 ✓

Candidate C

$\left[\frac{(3x+4)^{\frac{1}{2}}}{\frac{1}{2}}\times 3\right]_4^t=2$ •1 ✓ •2 X

$\left[\frac{2}{3}(3x+4)^{\frac{1}{2}}\right]_4^t=2$

$\left[\frac{2}{3}(3t+4)^{\frac{1}{2}}\right]-\left[\frac{2}{3}(3(4)+4)^{\frac{1}{2}}\right]=2$ •3 X

$(3t+4)^{\frac{1}{2}}=7$ •4 ✓

$t=15$ •5 ✓

Candidate D

$\left[-\frac{3}{2}(3x+4)^{-3/2}\right]_4^t=2$ •1 X •2 X

$-\frac{3}{2}(3t+4)^{-3/2}-\left(-\frac{3}{2}\times 16^{-3/2}\right)=2$ •3 ✓

$(3t+4)^{3/2}=-\frac{192}{253}$ •4 ✓

decimal equivalent not accepted

$t=-1\cdot 056$ •5 ✓

Question		Generic Scheme	Illustrative Scheme	Max Mark
6				
	$\bullet^1$ ss	use correct double angle formula	$\bullet^1$ $\sin x - 2(1-2\sin^2 x)$ stated or implied by $\bullet^2$	
	$\bullet^2$ ss	arrange in standard quadratic form	$\bullet^2$ $4\sin^2 x + \sin x - 3 = 0$	
	$\bullet^3$ ss	start to solve	$\bullet^3$ $(4\sin x - 3)(\sin x + 1) = 0$ OR $\dfrac{-1\pm\sqrt{(1)^2-4\times4\times(-3)}}{2\times4}$	
	$\bullet^4$ ic	reduce to equations in $\sin x$ only	$\bullet^4$ $\sin x = \frac{3}{4}$ and $\sin x = -1$	
	$\bullet^5$ pd	process to find solutions in given domain	$\bullet^5$ $0\cdot848$, $2\cdot29$ and $\frac{3\pi}{2}$ OR $\bullet^4$ $\sin x = \frac{3}{4}$ and $x = 0\cdot848,\ 2\cdot29$ $\bullet^5$ $\sin x = -1$, and $x = \frac{3\pi}{2}$	5

Notes:

1. $\bullet^1$ is not available for simply stating $\cos 2A = 1 - 2\sin^2 A$ with no further working.
2. In the event of $\cos^2 x - \sin^2 x$ or $2\cos^2 x - 1$ being substituted for $\cos 2x$, $\bullet^1$ cannot be awarded until the equation reduces to a quadratic in $\sin x$.
3. Substituting $1-2\sin^2 A$ or $1-2\sin^2\alpha$ for $\cos 2\alpha$ at $\bullet^1$ stage should be treated as bad form provided the equation is written in terms of x at stage $\bullet^2$. Otherwise, $\bullet^1$ is not available.
4. '$=0$' must appear by $\bullet^3$ stage for $\bullet^2$ to be awarded. However, for candidates using the quadratic formula to solve the equation, '$=0$' must appear at $\bullet^2$ stage for $\bullet^2$ to be awarded.
5. Candidates may express the equation obtained at $\bullet^2$ in the form $4s^2+s-3=0$ or $4x^2+x-3=0$. In these cases, award $\bullet^3$ for $(4s-3)(s+1)=0$ or $(4x-3)(x+1)=0$. However, $\bullet^4$ is only available if $\sin x$ appears explicitly at this stage.
6. $\bullet^4$ and $\bullet^5$ are only available as a consequence of solving a quadratic equation.
7. $\bullet^3$, $\bullet^4$ and $\bullet^5$ are not available for any attempt to solve a quadratic written in the form $ax^2+bx=c$.
8. $\bullet^5$ is not available to candidates who work in degrees and do not convert their solutions into radian measure.
9. $\sin x + 4\sin^2 x - 3 = 0$ does not gain $\bullet^2$, unless $\bullet^3$ is awarded.

Commonly Observed Responses:

Candidate A

•1 ✓ •2 ✓

$(4s-3)(s+1)=0$ •3 ✓

$s=\frac{3}{4},\ s=-1$ •4 X

$x=0\cdot848,\ 2\cdot29$ and $\frac{3\pi}{2}$ •5 ✓

Candidate B

•1 ✓

$4\sin^2 x+\sin x-3=0$ •2 ✓

$5\sin x-3=0$ •3 X

$\sin x=\frac{3}{5}$ •4 X

$x=0\cdot644,\ 2\cdot50$ •5 $\cancel{\checkmark}$

Candidate C

•1 ✓

$\sin x-2(1-2\sin^2 x)=1$

$\sin x-2+4\sin^2 x=1$

$4\sin^2 x+\sin x=3$ •2 $\cancel{\checkmark}$

$\sin x(4\sin x+1)=3$ •3 $\cancel{\checkmark}$

$\sin x=3,\quad 4\sin x+1=3$ •4 X

no solution, $\sin x=\frac{1}{2}$

$x=\frac{\pi}{6},\ \frac{5\pi}{6}$ •5 $\cancel{\checkmark}$

Candidate D

•1 ✓

$\sin x-2(1-2\sin^2 x)=1$

$4\sin^2 x+\sin x-3=0$ •2 ✓

$4\sin^2 x+\sin x=3$

$\sin x(4\sin x+1)=3$ •3 $\cancel{\checkmark}$

$\sin x=3,\quad 4\sin x+1=3$ •4 X

no solution, $\sin x=\frac{1}{2}$

$x=\frac{\pi}{6},\ \frac{5\pi}{6}$ •5 $\cancel{\checkmark}$

Candidate E: Reading $\cos 2x$ as $\cos^2 x$

$\sin x-2\cos^2 x=1$ •1 X

$\sin x-2(1-\sin^2 x)=1$

$2\sin^2 x+\sin x-3=0$ •2 $\cancel{\checkmark}$

$(2\sin x+3)(\sin x-1)=0$ •3 $\cancel{\checkmark}$

$\sin x=-\frac{3}{2},\quad \sin x=1$ •4 $\cancel{\checkmark}$

no solution, $x=\frac{\pi}{2}$ •5 $\cancel{\checkmark}$

Question		Generic Scheme		Illustrative Scheme	Max Mark
7	a				
$\bullet^1$		ss	know to and find intersection of line and curve	$\bullet^1$ $2x = 6x - x^2 \Rightarrow x = 0, x = 4$	
$\bullet^2$		ic	use "upper – lower"	$\bullet^2$ $\int\left(\left(6x - x^2\right) - 2x\right)dx$	
$\bullet^3$		pd	integrate	$\bullet^3$ $2x^2 - \frac{1}{3}x^3$	
$\bullet^4$		pd	substitute limits and evaluate	$\bullet^4$ $10\frac{2}{3}$	
$\bullet^5$		pd	evaluate area developed	$\bullet^5$ $10\frac{2}{3} \times 300 = 3200\,\text{m}^2$	5

Notes:

1. '0' appearing as the lower limit of the integral is sufficient evidence for $x = 0$ at $\bullet^1$ stage.
2. $\bullet^5$ is only available as a consequence of multiplying an exact answer at $\bullet^4$ stage.
3. The omission of dx at $\bullet^2$ should not be penalised.
4. Where a candidate differentiates one or both terms $\bullet^3$, $\bullet^4$ and $\bullet^5$ are unavailable.
5. Do not penalise the inclusion of '$+c$'.
6. Accept $\int\left(4x - x^2\right)dx$ for $\bullet^2$.

Commonly Observed Responses:

Candidate A

$\int_0^4 \left(2x-\left(6x-x^2\right)\right)dx$ •2 ✗

$=\frac{1}{3}x^3-2x^2$ •3 ✓

$=-10\frac{2}{3}$ cannot be negative so $=10\frac{2}{3}$ •4 ✗ however $\ldots.=-10\frac{2}{3}$ so Area $=10\frac{2}{3}$ •4 ✓

Area = 3200m^2 •5 ✓

Candidate B

$2x=6x-x^2 \Rightarrow x=0,\ 4$ •1 ✓

Shaded area

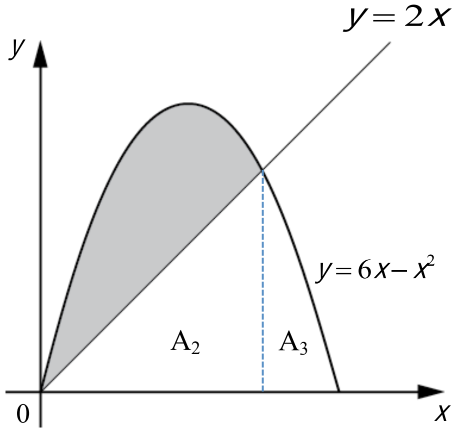
= area under parabola – ($A_2 + A_3$)

$=\int_0^6 \left(6x-x^2\right)dx-\left[A_2+\int_4^6 (6x-x^2)dx\right]$ •2 ✓

Stated or implied by •4

Area under parabola = 36, A_2 = 16 and $A_3 = \frac{28}{3}$ •3 ✓

Shaded area $=36-\left(16+\frac{28}{3}\right)=\frac{32}{3}$ •4 ✓



Candidate C

Part (a)

$x=0,\ x=6$ •1 ✗

$\int\left(\left(6x-x^2\right)-2x\right)dx$ •2 ✓

$\left[2x^2-\frac{1}{3}x^3\right]_0^6$ •3 ✓

$\left(2\times6^2-\frac{1}{3}\times6^3\right)-(0)=0$ •4 ✗

$\Rightarrow$ Area $=0\times300=0$ m^2 •5 ✓

Question		Generic Scheme	Illustrative Scheme	Max Mark
7	b			
		●6 ss set derivative to 2	●6 $6-2x=2$	
		●7 pd find point of contact	●7 $x=2,\ y=8$	
		●8 pd find equation of road	●8 $y=2x+4$	
		●9 ss find correct integral	●9 $\left[\left(x^2+4x\right)-\left(3x^2-\frac{1}{3}x^3\right)\right]_0^2$	
		●10 ic calculate area	●10 800m^2	5

Notes:

6. For candidates who omit ' m^2 ' at both ●5 and ●10 stages, ●10 is not available.
7. Candidates who arrive at an incorrect equation at ●8, or produce an equation ex nihilo, must use an equation of the form $y=2x+c$ with $c>0$, for ●9 and ●10 to be available.
8. $y=2x+4$ must appear explicitly or as part of the integrand for ●8 to be awarded.
9. ●10 is only available as a result of a valid strategy at the ●9 stage, ie $\int(\text{line})-(\text{quadratic})$ and lower limit = 0 and upper limit < 3.

Commonly Observed Responses:

Candidate D: Alternative Method

Line has equation of the form $y=2x+c$, $y=2x+c$ and $y=6x-x^2$

intersect where $x^2-4x+c=0$ ●6 ✓

tangency $\Rightarrow$ 1 point of intersection

$\Rightarrow b^2-4ac=0$ ●7 ✓

$16-4c=0$ ●8 ✓

$c=4$

Continue as above.

Question		Generic Scheme	Illustrative Scheme	Max Mark
8				
		$\bullet^1$ pd correct values	$\bullet^1$ $g=-p$, $f=-2p$, $c=3p+2$	
		$\bullet^2$ ss substitute and rearrange	$\bullet^2$ $5p^2-3p-2$	
		$\bullet^3$ ic knowing condition	$\bullet^3$ $g^2+f^2-c>0$	
		$\bullet^4$ pd factorise and solve	$\bullet^4$ $(5p+2)(p-1)=0 \Rightarrow p=-\frac{2}{5},\ p=1$	
		$\bullet^5$ ic correct range	$\bullet^5$ $p<-\frac{2}{5},\ p>1$	5

Notes:

1. Candidates who state the coordinates of the centre, $(p,2p)$ and state the radius, $r=\sqrt{\ldots-(3p+2)}$ gain $\bullet^1$.
2. Accept $(-p)^2+(-2p)^2-(3p+2)$ or $p^2+(2p)^2-(3p+2)$. If brackets are omitted $\bullet^1$ may only be awarded if subsequent working is correct.
3. Do not accept $(-p)^2+(2p)^2-(3p+2)$ or $(p)^2+(-2p)^2-(3p+2)$ for $\bullet^1$.
4. Do not accept $g^2+f^2-c\geq 0$ for $\bullet^3$.
5. For a candidate who uses $c=2$ and follows through to get $p<-\sqrt{\frac{2}{5}},\ p>\sqrt{\frac{2}{5}}$, award $\bullet^2$, $\bullet^3$ and $\bullet^5$.
6. Evidence for $\bullet^3$ may appear at $\bullet^5$ stage.
7. $\bullet^4$ and $\bullet^5$ can only be awarded for solving a quadratic inequation.

Commonly Observed Responses:

Candidate A		Candidate B	
$g=-2p,\ f=-4p,\ $ c=3p+2	$\bullet^1$ ✗	$(x-p)^2-p^2+(y-2p)^2-4p^2+3p+2=0$	
$20p^2-3p-2$	$\bullet^2$ ✓	$(x-p)^2+(y-2p)^2$	$\bullet^1$ ✓
$g^2+f^2-c>0$	$\bullet^3$ ✓	$=5p^2-3p-2$	$\bullet^2$ ✓
$(4p+1)(5p-2)=0 \Rightarrow p=-\frac{1}{4},\ p=\frac{2}{5}$	$\bullet^4$ ✓	$5p^2-3p-2>0$	$\bullet^3$ ✓
$p<-\frac{1}{4},\ p>\frac{2}{5}$	$\bullet^5$ ✓	$(5p+2)(p-1)>0$	$\bullet^4$ ✓
		$p<-\frac{2}{5},\ p>1$	$\bullet^5$ ✓

Question		Generic Scheme	Illustrative Scheme	Max Mark
9	a			
		•1 ss know to differentiate	•1 $a = v'(t)$	
		•2 pd differentiates trig. function	•2 $-8\sin\left(2t-\frac{\pi}{2}\right)\ldots\ldots$	
		•3 pd applies chain rule	•3 $\ldots\ldots\times 2$ and complete $a(t) = -16\sin\left(2t-\frac{\pi}{2}\right)$	3

Commonly Observed Responses:

Candidate A: Alternative Method

Part (a)

$v(t) = 8\cos\left(2t-\frac{\pi}{2}\right) = 8\sin 2t$

$v'(t) = \ldots\ldots$ •1 ✓

$= 8\cos 2t\ldots$ •2 ✓

$= \ldots\ldots\times 2$ •3 ✓

Part (b)

$v'(10) = 16\cos 20 = 6\cdot 53$ •4 ✓

$> 0, \Rightarrow$ velocity is increasing •5 ✓

Part (c)

$s(t) = \int v(t)\,dt$ •6 ✓

$s(t) = -4\cos 2t + c$ •7 ✓

$4 = -4 + c \Rightarrow c = 8$

$\Rightarrow s(t) = -4\cos 2t + 8$ •8 ✓

or $\Rightarrow s(t) = 8 - 4\cos 2t$

Candidate B: Candidates who misinterpret the process for rate of change.

Part (a)

$a(t) = \int 8\cos\left(2t-\frac{\pi}{2}\right)dt$

$= 4\sin\left(2t-\frac{\pi}{2}\right) + c$

Wrong process award $0/3$

Part (b)

If $t = 10$, $a = 4\sin\left(20-\frac{\pi}{2}\right) + c$

$= -1\cdot 63 + c$

Cannot evaluate award $0/2$

Part (c)

$s = v'(t)$

$s(t) = -16\sin\left(2t-\frac{\pi}{2}\right)$

Award $2/3$

Candidate C

Part (a)

$a = v'(t)$ or equivalent •1

$a = 4\sin\left(2t-\frac{\pi}{2}\right)$ •2 ✗ •3 ✗

Part (b)

$a(10) = 4\sin\left(20-\frac{\pi}{2}\right) = -1\cdot 63$ •4

<0, So decreasing •5 ✓

Only as a consequence of •1 in part (a)

Question		Generic Scheme			Illustrative Scheme	Max Mark
9	b					
		$\bullet^4$	ss	know to and evaluate $a(10)$	$\bullet^4$ $a(10)=6\cdot53$	
		$\bullet^5$	ic	interpret result	$\bullet^5$ $a(10)>0$ therefore increasing	2

Notes:

1. $\bullet^5$ is available only as a consequence of substituting into a derivative.
2. $\bullet^4$ and $\bullet^5$ are not available to candidates who work in degrees.
3. $\bullet^2$ and $\bullet^3$ may be awarded if they appear in the working for 9(b). However, $\bullet^1$ requires a clear link between acceleration and $V(t)$.

Question		Generic Scheme			Illustrative Scheme	Max Mark
9	c					
		$\bullet^6$	ic	know to integrate	$\bullet^6$ $s(t)=\int v(t)\,dt$	
		$\bullet^7$	pd	integrate correctly	$\bullet^7$ $s(t)=4\sin\left(2t-\frac{\pi}{2}\right)+c$	
		$\bullet^8$	ic	determine constant and complete	$\bullet^8$ $c=8$ so $s(t)=4\sin\left(2t-\frac{\pi}{2}\right)+8$	3

Notes:

4. $\bullet^7$ and $\bullet^8$ are not available to candidates who work in degrees. However, accept $\int 8\cos(2t-90)\,dt$ for $\bullet^6$.

[END OF MARKING INSTRUCTIONS]